High Order Time Discretization Of The Wave Equation By Nabla-P Scheme. Application to the Reverse Time Migration

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Abstract

Reverse Time Migration (RTM) is one of the most widely used techniques for Seismic Imaging, but it induces very high computational cost since it is based on many successive solutions to the full wave equation. High-Order Discontinuous Galerkin Methods (DGM), coupled with High Performance Computing techniques, can be used to solve accurately this equation in complex geophysic media without increasing the computational burden. However, to fully exploit the high-order space discretization, it is necessary to use a high-order time discretization. In this work, we propose a new high order time scheme, the so-called Nabla-p scheme. This scheme does not increase the storage costs since it is a single step method and does not require the storage of auxiliary unknowns. Numerical results show that it requires less storage than the ADER scheme for a given accuracy and that it can be efficiently implemented in an RTM algorithm.

Introduction

Geophysical exploration is undertaken on more and more complex media and we need advanced numerical methods to accurately image the subsurface. Indeed, Seismic Imaging algorithms, such as for instance Reverse Time Migration (RTM), generate high computational burden since they are iterative algorithms that require many successive solutions to the wave equation. To reduce these computational cost, we use High-Order Discontinuous Galerkin Methods, which are very accurate even with coarse meshes and can be combined with explicit time schemes. However, to take fully advantage of high-order space discretization, it is necessary to combine DGM with high order time discretization. This can be achieved by using DG-ADER methods [4], which are an extension of the Modified Equation Technique. They are single step methods, i.e., they only require to store the solution at the previous time step. Nevertheless, even when using advanced methods like DG-ADER schemes, we still have to store a huge number of unknowns. We then propose a new single step method, called Nabla-p schemes, which can be seen as an alternative to DG-ADER schemes. The original idea consists in inverting the discretization order, which introduce high order operators in space which require an appropriate space discretization. Fortunately, DG method are well adapted to deal with high order operators. This has already been succesfully applied to the second order formulation of the acoustic wave equation and we focus here on the first order formulation of acoustic and elastodynamic wave equation. Numerical results show that the additional cost induced by the computation of the high order operator is counterbalanced by the accuracy of the method. Indeed, for a given accuracy, it allows for much coarser meshes than ADER, which considerably reduces the storage and the computational time.

1 Discretization of the wave equation

To simplify the presentation, we focus on the acoustic wave equation but the method can be applied to the elastodynamic wave equation too. We consider the following system in a bounded domain $\Omega \subset \mathbf{R}^n$, $n = 1, 2, 3$:

$$\begin{cases} \rho(\mathbf{x}) \dfrac{\partial \mathbf{v}(\mathbf{x},t)}{\partial t} + & \boldsymbol{\nabla} p(\mathbf{x},t) & = 0 \quad \text{in} \quad \Omega \times [0,T] \\ \dfrac{1}{\mu(\mathbf{x})} \dfrac{\partial p(\mathbf{x},t)}{\partial t} + & \nabla \cdot (\mathbf{v}(\mathbf{x},t)) & = 0 \quad \text{in} \quad \Omega \times [0,T] \end{cases} \tag{1}$$

where ρ and μ are respectively the density and the compressibility modulus of Ω, p is the scalar pressure and $\mathbf{v}$ the velocity vector. For the sake of simplicity, we omit the source function, the initial and the boundary conditions. By applying a DGM, we obtain the semi-discretized schemes:

$$\begin{cases} \dfrac{dV}{dt} + \mathcal{M}_v^{-1} K_p P = \dfrac{dV}{dt} - \mathcal{A}_p P = 0 \\ \dfrac{dP}{dt} + \mathcal{M}_p^{-1} K_v V = \dfrac{dP}{dt} - \mathcal{A}_v V = 0 \end{cases} \tag{2}$$

where the mass matrices $\mathcal{M}_v$, $\mathcal{M}_p$ are easily invertible since they are diagonal and the stiffness matrices K_v, K_v are sparse. One of the most efficient way to discretize this system is to use ADER Method [4].

This method is equivalent to the Modified Equation Technique (MET) [2], [3], when using the same time step and the same order for the time discretization in the whole domain. Using the fourth order ADER scheme, we obtain:

$$\left\{\begin{aligned} \frac{V^{n+1}-V^n}{\Delta t} &= \mathcal{A}_p P^{n+1/2} + \frac{\Delta t^2}{24}\mathcal{A}_p\mathcal{A}_v\mathcal{A}_p P^{n+1/2} \\ \frac{P^{n+3/2}-P^{n+1/2}}{\Delta t} &= \mathcal{A}_v V^{n+1} + \frac{\Delta t^2}{24}\mathcal{A}_v\mathcal{A}_p\mathcal{A}_v V^{n+1} \end{aligned}\right.$$

This scheme requires three times more multiplications by the stiffness matrices than the second order LF, but the stability condition is multiplied by almost three. However, for higher order, the increase of the stability condition does not counterbalance rising multiplications. We propose here an alternative to ADER by applying the MET to the continuous wave equation (1). We then obtain the semi-discretized scheme:

$$\left\{\begin{aligned} \frac{v(\mathbf{x})^{n+1}-v(\mathbf{x})^n}{\Delta t} &= -\boldsymbol{\nabla} p(\mathbf{x})^{n+\frac{1}{2}} - \frac{\Delta t^2}{24}\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot\boldsymbol{\nabla} p(\mathbf{x})^{n+\frac{1}{2}} \\ \frac{p(\mathbf{x})^{n+1}-p(\mathbf{x})^{n+\frac{1}{2}}}{\Delta t} &= -\boldsymbol{\nabla}\cdot v(\mathbf{x})^{n+1} - \frac{\Delta t^2}{24}\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot v(\mathbf{x})^{n+1} \end{aligned}\right.$$

This method has already been applied to the second order formulation of the wave equation [1]. It is worth noting that (1) involves a third order operator in space, that we propose to discretize with DGM also.

2 Numerical Results

Our main goal is to limit the storage which is the bottleneck of the RTM. For the computational cost, it can be counterbalanced with new HPC techniques such that: MPI, OpenMP or GPU. We have performed a comparison between classical LF scheme using $\mathcal{P}^6$-elements in space and fourth order time schemes. The length of the domain is 6 m, the simulation time is 6.0 sec. The original space step is 0.2 m and we take different values of refinement. We consider periodic boundary conditions and the initial data is such that : $U(x,t) = (x-x_0-t)\,e^{-\left(\frac{(2\pi(x-x_0-t))^2}{r_0}\right)}$.

In Fig. 1, we represent the relative L^2-error as a function of the number of unknowns. As expected, for a given accuracy, HO-schemes require less degrees of freedom (dof) than LF. Besides, the Nabla-p scheme requires three times less dof than ADER. In Fig. 2 we represent the relative L^2-error as a function of the number of operations. For a given accuracy ADER and Nabla-p require approximatively the same number of operations. As a conclusion, Nabla-p scheme require less storage cost than ADER and the computional cost is similar. This indicates that Nabla-p scheme is well adapted for the RTM. We

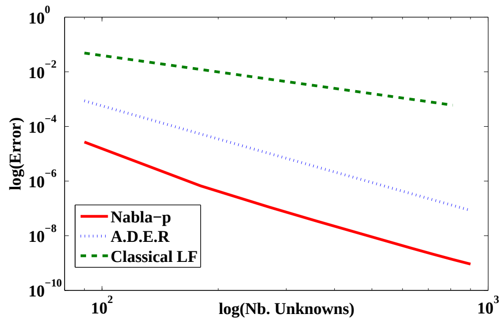

Figure 1: Number of Unknowns

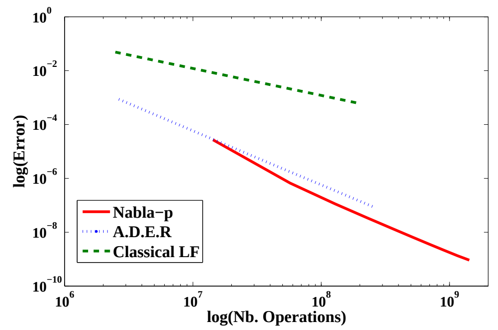

Figure 2: Number of operations

will present RTM results that will illustrate the performance of Nabla-p scheme in realistic 2D and 3D configurations.

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