3D Source location in optical mapping

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3D SOURCE LOCATION IN OPTICAL MAPPING

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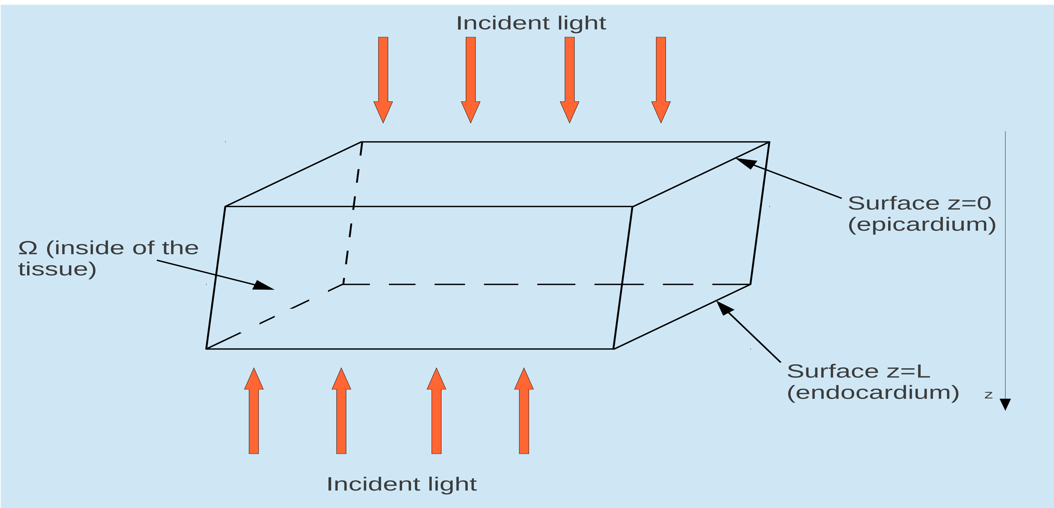
Problem statement and objectives

- Optical mapping enables to display optical potentials on the boundary of a slab of tissue.
- At a given time, we have 4 images: 2 from epi-illumination and 2 from endo-illumination.
- Exploit these images to reconstruct an optimal 3D depolarization wave front.

Equations

Domain:



finite elements discretization

Incident light:

$$\begin{cases} D_e \Delta \phi_e - \mu_e \phi_e = 0 & \text{in } \Omega \\ \phi_e = \dfrac{I_e \delta_e}{D_e} & \text{on illuminated surface} \\ \phi_e = d_e \dfrac{\partial \phi_e}{\partial \nu} & \text{elsewhere} \end{cases} \quad (1)$$

- λ: wavelength
- $I_e(\lambda)$: light intensity
- $D_e(\lambda), \delta_e(\lambda), \ldots$: material properties

Fluorescence:

source: $w = \beta(V_m - V_0)\phi_e$

$$\begin{cases} D\Delta\phi - \mu\phi + w = 0 & \text{in } \Omega \\ \phi = d\dfrac{\partial \phi}{\partial \nu} & \text{on } \partial\Omega \end{cases} \quad (2)$$

- V_m: transmembrane potential
- V_0: rest potential

Matrix relations

- discretization of (2) gives:

$$A\Phi = MW$$
$$L\Phi = \Phi_{\mathbb{S}}$$

$\Phi_{\mathbb{S}}$: projection on surface, observations

- reformulation: $B = A^{-1}M$

$$W(V_m) \xrightarrow{B} \Phi \xrightarrow{L} \Phi_{\mathbb{S}}$$

- under-determined problem because 19954 points in the whole domain including 1871 on the epicardium.

Wave front : restriction

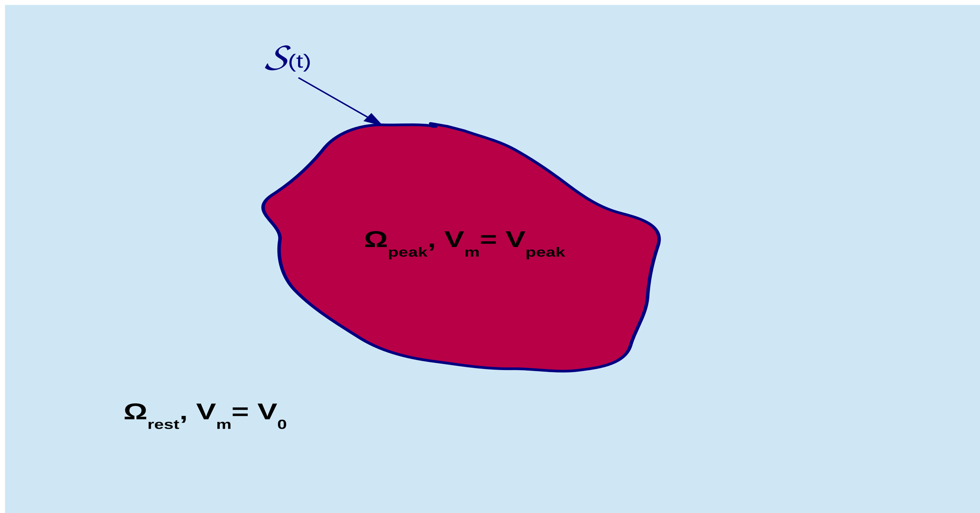
$$\mathcal{S}(t) \longrightarrow V_m \longrightarrow W \xrightarrow{B} \Phi \xrightarrow{L} \Phi_{\mathbb{S}}$$

We look for $V_m = \begin{cases} V_0 & \text{in } \Omega_{rest} \\ V_{peak} & \text{in } \Omega_{peak} \end{cases}$

where $\overline{\Omega_{rest}} \cap \overline{\Omega_{peak}} = \mathcal{S}(t)$

Wave front : restriction

2D representation:



choice: $\mathcal{S}(t) = \{ | X - X_0 | - c(t - t_0) = 0 \}$, expanding sphere

parameters to identify: X_0, t_0 and sometimes c.

Inverse problem

Minimize

$$e(X_0, t_0, \ldots) = \|\Phi_{\mathbb{S}} - \Phi^\star\|^2_{L^2(\mathbb{S})}$$

$\Phi^\star$: observation

Method:

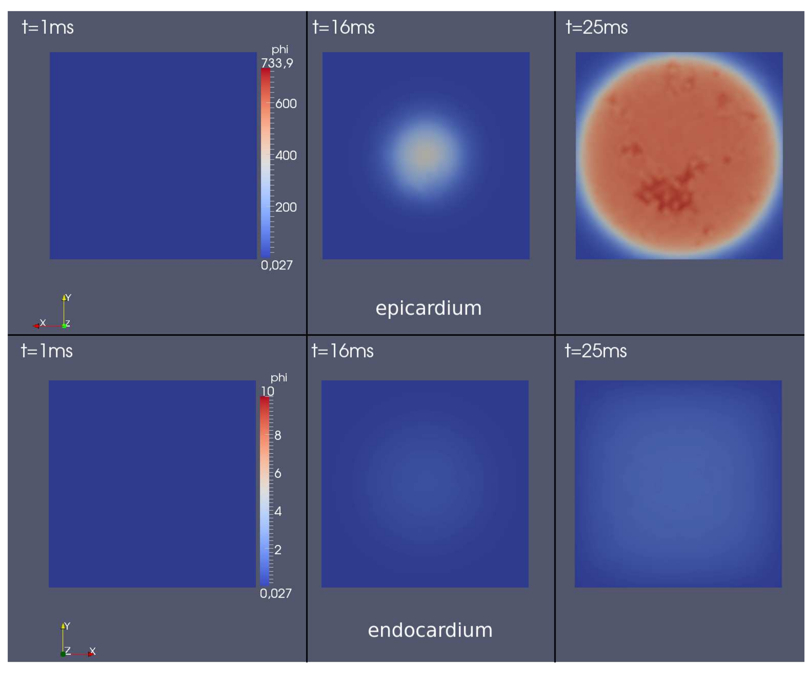
- BFGS method
- 1st case functional:
  $e(X_0, t_0, \ldots) = \|\Phi_{\mathbb{S}}(t_k) - \Phi^\star_k\|^2_{L^2(\mathbb{S})}$
  $\Phi^\star_k$: observation at time t_k
- 2nd case functional:
  $e(X_0, t_0, \ldots) = \left(\sum_k \|\Phi_{\mathbb{S}}(t_k) - \Phi^\star_k\|^2_{L^2(\mathbb{S})}\right)^{1/2}$

Results

In-silico example:

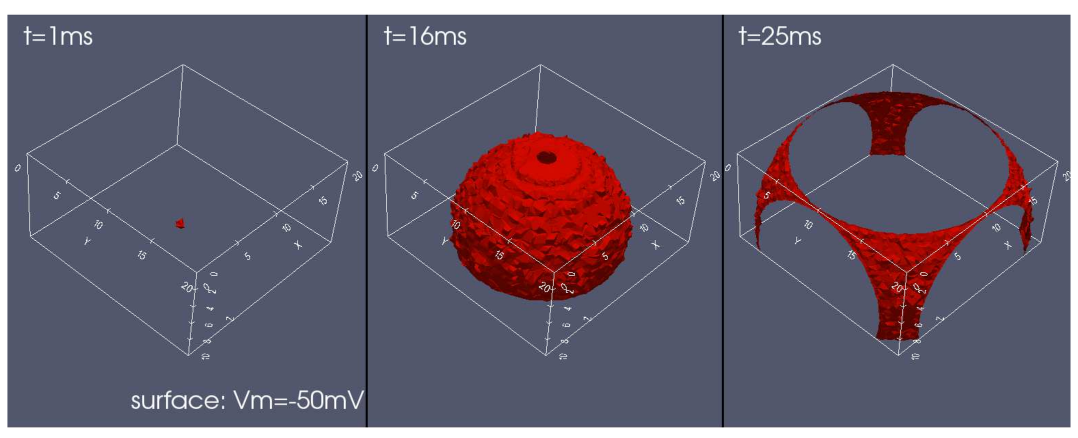
Direct simulation of a single source at $X^\star = (10, 10, 8)$ and $t^\star = 0$.

Simulated observations:



Reconstructed X_0: $X_0 = X^\star$

Reconstructed wavefront:



Results

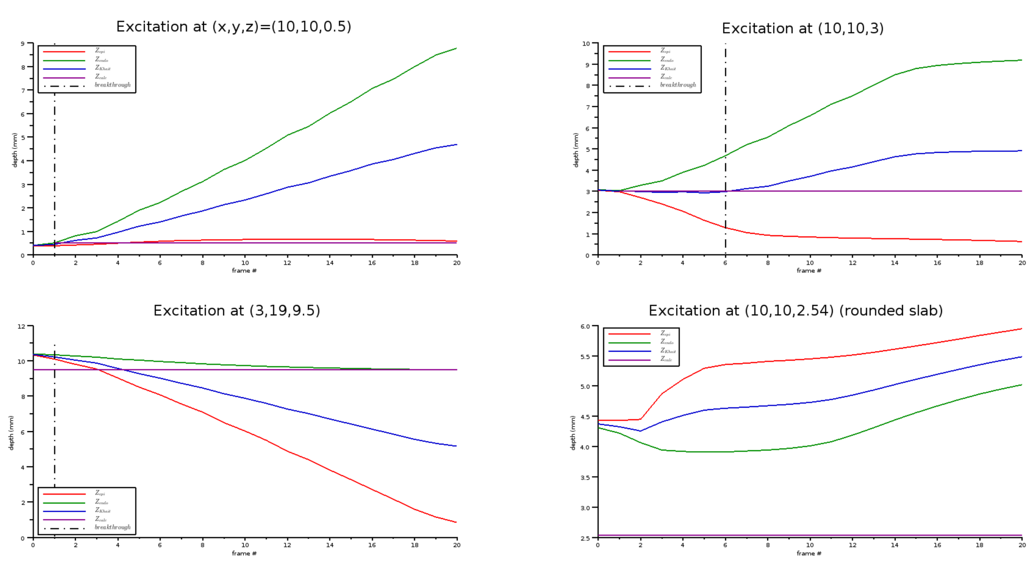
Our approach:

several situations tested:

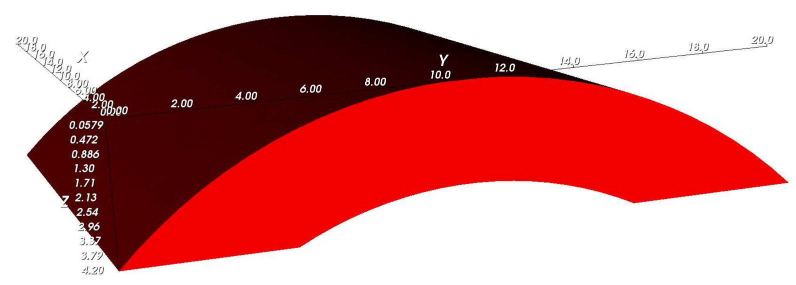
- reflexion in epi-illumination
- reflexion and transillumination in epi-illumination
- reflexion in epi- en endo-illumination
- unknown speed
- unknown excitation time

Any source could be reconstructed

Comparison with Khait



In [1], Khait calculates Z_{endo} and Z_{epi} and then defines the depth (Z_{Khait}) as the mean. The last curves were obtained with the following domain:



Discussion - Conclusion

Analysis:

- sometimes with not enough depolarised tissue, our method does not converge
- very good accuracy when it converges
- **convergence even after breakthrough**
- **convergence for sources close to boundaries**
- **independence of the domain**

Perspectives:

- validation with optical phantoms: find size and location of spherical fluorescent sources [2]
- generalize the wave front $\mathcal{S}(t)$:
  - Radial Basis functions
  - Eikonal or level sets equations

References

[1] Khait *et al.* "Method for 3-dimensional localization [...]" In: *JBO* (2006).

[2] Walton *et al.* "Experimental validation of alternating [...]" In: *IEEE* (2011).