Inverse problem in electrocardiography via factorization method of boundary problems : how reconstruct epicardial potential maps from measurements of the torso ?

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Inverse Problem in Electrocardiography via Factorization Method of Boundary Value Problems :

How reconstruct epicardial potential maps from measurements of the torso ?

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Motivation and goal

Motivation : Solve the inverse problem in electrocardiography from measurements of the torso.

Goal : Use factorization method to compute epicardial potential maps.

This work is a simplified presentation of the method by considering a cylinder as geometry of our problem.

Initial problem : electrical potential u in the domain Ω is governed by

$$(\mathcal{P}_0)\begin{cases}\Delta u = 0 \text{ in } \Omega\,, & \Omega : \text{cylinder}\\ u = 0 \text{ on } \Sigma\,, & \Sigma : \text{lateral surface}\\ u = T \text{ on } \Gamma_T\,, & T : \text{potential on the torso surface } \Gamma_T\\ \nabla u\cdot n = \Phi \text{ on } \Gamma_T\,, & \Phi : \text{normal derivative of the potential}\end{cases}$$

With T and Φ known, find potential t and his normal derivative ϕ on the heart surface Γ_H to complete this ill-posed Cauchy problem.

Optimal control problem

$(\mathcal{P}_0)$ is decomposed into two sub-problems, $\forall(\eta,\tau)$:

$$(1)\begin{cases}\Delta u^1 = 0 \text{ in } \Omega\\ u^1 = 0 \text{ on } \Sigma\\ u^1 = T \text{ on } \Gamma_T\\ \nabla u^1\cdot n = \eta \text{ on } \Gamma_H\end{cases} \quad\text{and}\quad (2)\begin{cases}\Delta u^2 = 0 \text{ in } \Omega\\ u^2 = 0 \text{ on } \Sigma\\ u^2 = \tau \text{ on } \Gamma_H\\ \nabla u^2\cdot n = \Phi \text{ on } \Gamma_T\end{cases}$$

Solve $(\mathcal{P}_0)$:

$\Longrightarrow$ Define the cost function $E(\eta,\tau) = \int_\Omega(\nabla u^1(\eta) - \nabla u^2(\tau))^2$

$\Longrightarrow$ Find (ϕ,t) : minimize $E(\eta,\tau)$

New approach : the factorization method by invariant embedding

Principle of invariant embedding

Principle : transport potential data from torso surface to heart surface

$\Longrightarrow$ Boundary value problems (1) and (2) are embedded into a family of similar problems on subdomains Ω_S

$\Longrightarrow$ Ω_S are bounded by a moving boundary Γ_S defined at $x = s$ for $x = 0 \longrightarrow x = a$

$\Longrightarrow$ At each position $x = s$, we impose a Neumann boundary condition $\frac{\partial u_1^S}{\partial x}_{|\Gamma_S} = \alpha$ for (1) and a Dirichlet boundary condition $(u_2^S)_{|\Gamma_S} = \beta$ for (2) :

$$(\mathcal{P}_S^1)\begin{cases}\Delta u_S^1 = 0 \text{ in } \Omega_S\\ u_S^1 = 0 \text{ on } \Sigma_S\\ u_S^1 = T \text{ on } \Gamma_T\\ \nabla u_S^1\cdot n = \alpha \text{ on } \Gamma_S\end{cases} \quad\text{and}\quad (\mathcal{P}_S^2)\begin{cases}\Delta u_S^2 = 0 \text{ in } \Omega_S\\ u_S^2 = 0 \text{ on } \Sigma_S\\ u_S^2 = \beta \text{ on } \Gamma_S\\ \nabla u_S^2\cdot n = \Phi \text{ on } \Gamma_T\end{cases}$$

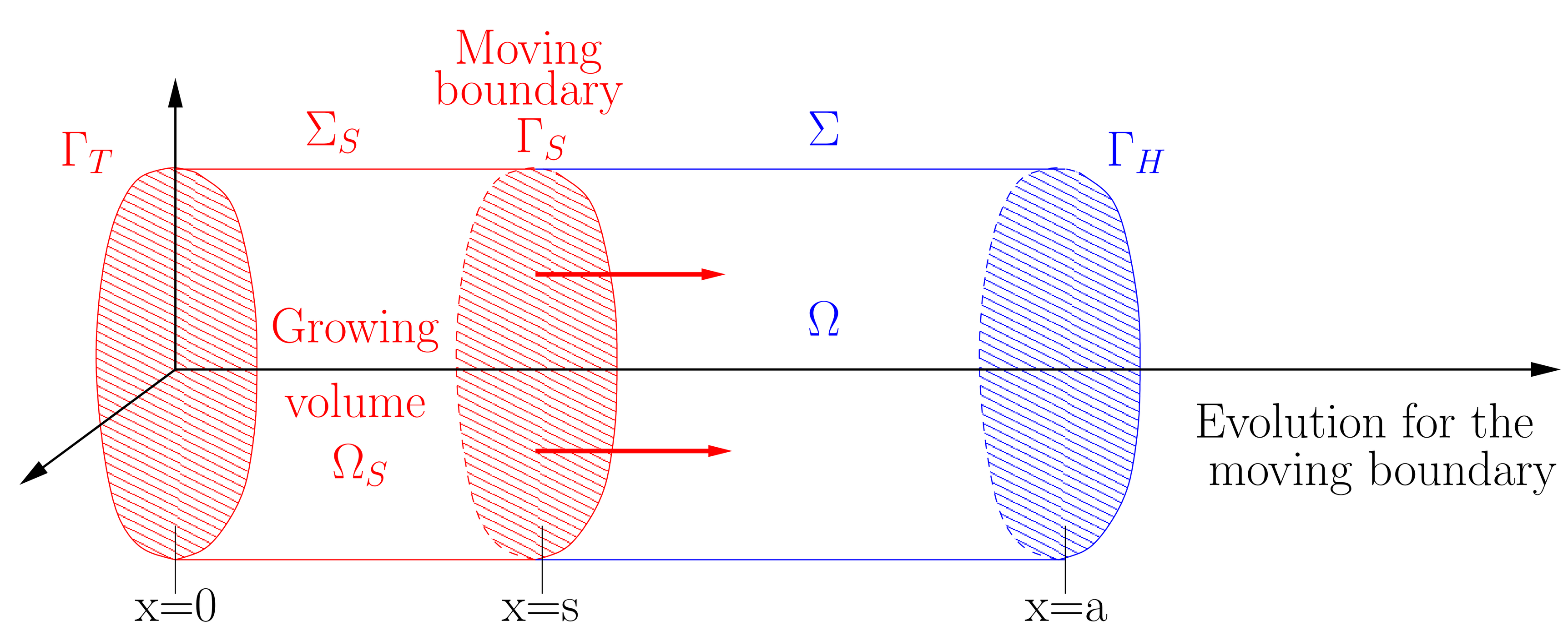

FIGURE 1: Illustration of the moving boundary. The domain Ω is a cylinder of a lenght a with a section $\mathcal{O}$ and a lateral surface Σ. The section $\mathcal{O} = \Gamma_T$ at $x = 0$ represents the torso surface and the section $\mathcal{O} = \Gamma_H$ at $x = a$ represents the heart surface. The moving boundary $\mathcal{O} = \Gamma_S$ at $x = s$ is defined between these two surfaces and moving along the axis of revolution for $x \in [0;a]$. For each position s, a cylinder with a section $\mathcal{O}$, a lateral surface Σ_S and a length s is defined and forms a new subdomain Ω_S.

Resolution method

At each position $x = s$ of Γ_S, we define two linear operators :

Neumann-to-Dirichlet application $Q(s) : \alpha \longmapsto u^1_{S|\Gamma_S}$, associated to $(\mathcal{P}_S^1)$

Dirichlet-to-Neumann application $P(s) : \beta \longmapsto \frac{\partial u_S^2}{\partial x}_{|\Gamma_S}$, associated to $(\mathcal{P}_S^2)$

$\Longrightarrow$ P and Q depend on s : variable that describes the axis of evolution

$\Longrightarrow$ P and Q act on functions defined on section $\mathcal{O}$

Let w_1 and w_2 : residual functions, defined on $\mathcal{O}$. $\forall x \in [0;a]$, we have :

$$(3)\begin{cases}u_S^1(x) = Q(x)\alpha + w_1(x)\\ \text{with } Q(0) = 0 \text{ and } w_1(0) = T\end{cases} \quad\text{and}\quad (4)\begin{cases}\frac{\partial u_S^2}{\partial x}(x) = P(x)\beta + w_2(x)\\ \text{with } P(0) = 0 \text{ and } w_2(0) = -\Phi\end{cases}$$

Let Δ_y : laplacian operator, defined on the section $\mathcal{O}$. $\forall x \in [0;a]$, we have :

$$(5)\begin{cases}\frac{dP}{dx} + P^2 = -\Delta_y,\ P(0) = 0\\ \frac{dw_2}{dx} + Pw_2 = 0, \quad w_2(0) = -\Phi\end{cases} \quad\text{and}\quad (6)\begin{cases}\frac{dQ}{dx} - Q\Delta_y Q = I, \quad Q(0) = 0\\ \frac{dw_1}{dx} - Q\Delta_y w_1 = 0,\ w_1(0) = T\end{cases}$$

$\Longrightarrow$ First solve Riccati equations for P and Q for $x = 0 \longrightarrow x = a$

$\Longrightarrow$ Then solve equations for w_1 and w_2 for $x = 0 \longrightarrow x = a$

$\Longrightarrow$ Compute operators and residuals at $x = a$.

Rename $P(a) = P$, $Q(a) = Q$, $w_1(a) = w_1$, $w_2(a) = w_2$

Define the matrix A as : $$A = \begin{pmatrix} Q & -QP\\ -PQ & P\end{pmatrix}$$

We can rewrite $E(\eta,\tau)$ as : $$E(\eta,\tau) = C + [\eta,\tau]A[\eta,\tau]' - 2\int_{\Gamma_H}(w_1P\tau + Qw_2\eta)$$

Finally :

$$(\phi,t) = \text{ arg min } E(\eta,\tau) \Longleftrightarrow A[\phi,t]' = [Qw_2, Pw_1]'$$

$\Longrightarrow$ Find (ϕ,t) : regularize previous system and inverse A

Conclusions and perspectives

Conclusions :

Direct optimal estimation of t and ϕ before using any discretisation :

$\Longrightarrow$ Analyse ill-posedness and propose a better regularization and discretization

Equations for P and Q depend only of the geometry :

$\Longrightarrow$ Not necessary to repeat resolution at every time step of cardiac cycle

Perspectives :

Apply the method to $3D$ case where the moving boundary Γ_S will be a deformed surface :

$\Longrightarrow$ First : model of spheres

$\Longrightarrow$ Then : realistic geometries : how compute $3D$ surfaces ? + numerical cost ?

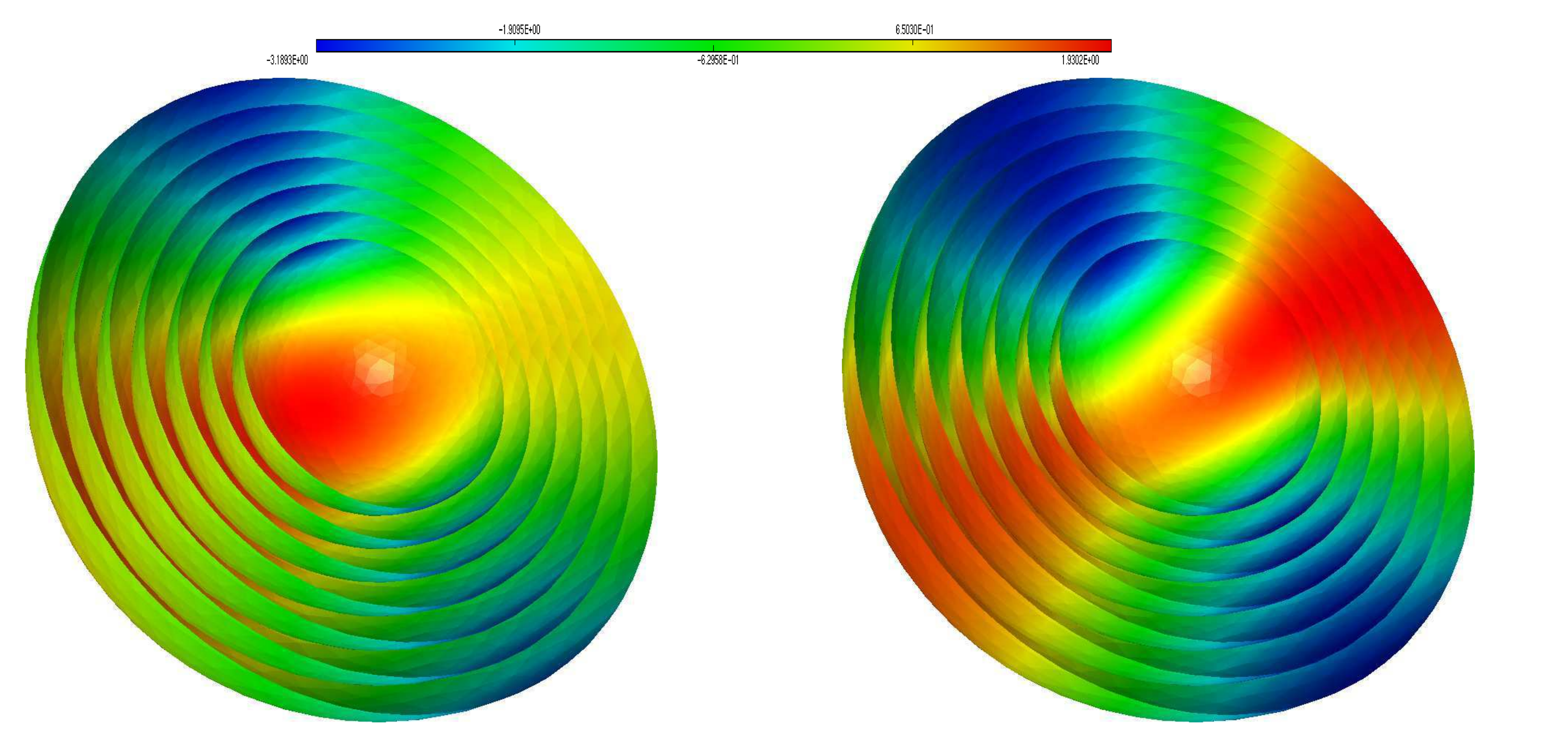

FIGURE 2: Illustration of Γ_S for the case of spheres, during the depolarization phase (left) and the repolarization phase (right). The smaller sphere represents the heart surface and the bigger one the torso surface. The heart was stimulated and direct problem was solved. We use computed potential data on the torso surface to apply factorization method and recover potential at heart surface. Spheres between torso and heart represent succesive positions of Γ_S during invariant embbeding. We also represent the potential recovered by the method.

References

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