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## A Model of Network Neutrality with Usage-based Prices<sup>\*</sup>

E. Altman · P. Bernhard · S. Caron · G. Kesidis · J. Rojas-Mora · S. Wong

**Abstract** Hahn and Wallsten [7] wrote that network neutrality “usually means that broadband service providers charge consumers only once for Internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users.” In this paper we study the implications of non-neutral behaviors under a simple model of linear demand-response to *usage-based* prices. We take into account advertising revenues for the content provider and consider both cooperative and non-cooperative scenarios. In particular, we model the impact of side-payments between service and content providers, consider an access provider that offers multiple service classes, and model leader-follower (Stackelberg game) dynamics. We finally study the additional

option for one provider to determine the amount of side payment from the other provider. We show that not only do the content provider and the internaut suffer, but also the Access Provider’s performance degrades.

### 1 Introduction

Network neutrality is an approach to providing network access without unfair discrimination among applications, content or traffic sources. Discrimination occurs when there are two applications, services or content providers that require the same network resources, but one is offered better quality of service (shorter delays, higher transmission capacity, etc.) than the other. How to define what is “fair” discrimination is still subject to controversy, and the underlying economic and policy issues have not been fully debated<sup>1</sup>. A preferential treatment of traffic is considered fair as long as the preference is left to the user<sup>2</sup>. Internet Service Providers

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<sup>1</sup> The recent decision on Comcast v. the FCC was expected to deal with the subject of “fair” traffic discrimination, as the FCC ordered Comcast to stop interfering with subscribers’ traffic generated by peer-to-peer networking applications. The Court of Appeals for the District of Columbia Circuit was asked to review this order by Comcast, arguing not only on the necessity of managing scarce network resources, but also on the non-existent jurisdiction of the FCC over network management practices. The Court decided that the FCC did not have express statutory authority over the subject, neither demonstrated that its action was “reasonably ancillary to the [...] effective performance of its statutorily mandated responsibilities”. The FCC was deemed, then, unable to sanction discriminatory practices on Internet’s traffic carried out by American ISPs, and the underlying case on the “fairness” of their discriminatory practices was not even discussed.

<sup>2</sup> Nonetheless, users are just one of many actors in the net neutrality debate, which has been enlivened throughout the world by several public consultations for new legislations on the subject. The first one, proposed in the USA, was looking for the best

(ISPs) may have interest in traffic discrimination either for technological or economic purposes. Traffic congestion, especially due to high-volume peer-to-peer traffic, has been a central argument for ISPs against the enforcement of net neutrality principles. However, it seems many ISPs have blocked or throttled such traffic even during periods without congestion.

ISPs recently claimed that net neutrality acts as a disincentive for capacity expansion of their networks. In [3], the authors studied the validity of this argument and came to the conclusion that, under net neutrality, ISPs invest to reach a social optimal level, while they tend to under/over-invest when neutrality is dropped. In their setting, ISPs stand as winners while content providers (CPs) are left in a worse position, and users who pay the ISPs for preferential treatment are better off while other consumers have a significantly worse service.

ISPs often justify charging CPs by quantifying the large amount of network resources “big” content providers use. On the other hand, the content and services the CPs offer contribute to the demand for Internet access, and thus benefit the ISPs. Shapley values may be used to obtain fair and Pareto optimal revenue-sharing between different types of players, *e.g.*, [10,11]. That is, Shapley values can prescribe whether and how (i) CPs should share third-party advertising revenue enabled by subscribers’ network access, or (ii) ISPs should share subscription revenue enabled by their customers’ demand for online content and services.

In this paper, we focus on violations of the neutrality principles defined in [7] where broadband service providers

- charge consumers more than “only once” through usage-based pricing, and
- charge content providers through side-payments.

Within a simple game-theoretic model, we examine how regulated<sup>3</sup> side payments, in either direction, and demand-dependent advertising revenues affect equilibrium usage-based prices. We also address equilibria in Stackelberg leader-follower dynamics.

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means of preserving a free and open Internet [15]. The second one, carried out in France, asks for different points of view over net neutrality [4]. A third one was presented by the EU during summer 2010, looking for a balance on the parties concerned as users are entitled to access the services they want, while ISPs and CPs should have the right incentives and opportunities to keep investing, competing and innovating [13].

<sup>3</sup> In the European Union, dominating positions in telecommunications markets (such as an ISP imposing side-payments to CPs at a price of his choice) are controlled by the article 14, paragraph 3 of the Directive 2009/140/EC, considering the application of remedies to prevent the leverage of a large market power over a secondary market closely related.

The rest of the paper is organized as follows. In section 2, we describe a basic model and derive Nash equilibria for competitive scenarios and optimal collaborative scenarios. We consider potentially non-neutral side-payments in section 3 and advertising revenues in section 4, analyzing in each case how they impact equilibrium revenues. We consider an ISP offering multiple service classes in section 5. In section 6, we study leader-follower dynamics. In Section 7 we consider the case where one provider can control the amount of side payments from the other one. We show that this results in a dramatic degradation of performance for all actors: at equilibrium, the demand is shown to be zero. We conclude in section 8 and discuss future work.

## 2 Basic model

Our model encompasses three actors:

- the internauts (users), collectively,
- a network access provider for the internauts, collectively called ISP1, and
- a content provider and its ISP, collectively called CP2.

The two providers play a game to settle on their (usage-based) prices. The internauts are modeled through their demand response.

Consumers are assumed willing to pay a usage-based fee (which can be \$0/byte) for service/content that requires both providers.

Denote by  $p_i \geq 0$  the usage-based price leveled by provider  $i$  (ISP1 being  $i = 1$  and CP2 being  $i = 2$ ). We assume that the demand-response of customers, which corresponds to the amount (in bytes) of content/bandwidth they are ready to consume given prices  $p_1$  and  $p_2$ , follows a simple linear model (*e.g.*, [5]):

$$D = D_0 - d(p_1 + p_2). \quad (1)$$

With such a profile, we are dealing with a set of homogeneous users sharing the same response coefficient  $d$  to price variations. Infrastructure and operating costs borne by the ISPs and CPs [12] are not considered here. The parameter  $D_0$  corresponds to the demand under pure *flat-rate* pricing ( $p_1 = 0 = p_2$ ).

Demand should be non-negative, *i.e.*,

$$p_1 + p_2 \leq \frac{D_0}{d} =: p_{\max}.$$

Provider  $i$ ’s usage-based revenue is given by

$$U_i = Dp_i. \quad (2)$$

## 2.1 Non-Cooperation

Suppose the providers do not cooperate. An interior Nash Equilibrium Point (NEP)  $(p_1^*, p_2^*)$  of this two-player can be obtained from the first order optimality conditions

$$\frac{\partial U_i}{\partial p_i}(p_1^*, p_2^*) = 0, \quad \text{for } i = 1, 2,$$

which lead to  $p_1^* = p_2^* = D_0/(3d)$ . The demand at equilibrium is thus  $D^* = D_0/3$  and the revenue of each provider is

$$U_i^* = \frac{D_0^2}{9d}. \quad (3)$$

## 2.2 Collaboration

Now suppose there is a coalition between ISP1 and CP2. Their overall utility is then  $U_{\text{total}} := U_1 + U_2 = Dp$ , and an optimal point  $(p_1^*, p_2^*)$  satisfies

$$\frac{\partial U_{\text{total}}}{\partial p_i}(p_1^*, p_2^*) = D^* - d(p_1^* + p_2^*) = 0 \quad \text{for } i = 1, 2,$$

which yields  $p^* := p_1^* + p_2^* = D_0/(2d)$ . The demand at equilibrium is then  $D^* = D_0/2$ , greater than in the non-cooperative setting. The overall utility  $U_{\text{total}}^* = D_0^2/(4d)$  is also greater than  $D_0^2/(4.5d)$  for the competitive case. Assuming both players share this revenue equally (trivially, the Shapley values are  $\{1/2, 1/2\}$  in this case), the utility per player becomes

$$U_i^* = \frac{D_0^2}{8d}, \quad (4)$$

which is greater than in the competitive case. So, both players benefit from this coalition.

## 3 Side-Payments under Non-Cooperation

Let us suppose now that there are *side payments* between ISP1 and CP2 at (usage-based) price  $p_s$ . The revenues of the providers become:

$$U_1 = D(p_1 + p_s) \quad (5)$$

$$U_2 = D(p_2 - p_s) \quad (6)$$

Note that  $p_s$  can be positive (ISP1 charges CP2 for “transit” costs) or negative (CP2 charges ISP1, *e.g.*, for copyright remuneration<sup>4</sup>). It is expected that  $p_s$  is *not* a decision variable of the players, since their utilities are monotonic in  $p_s$  and the player without control would

<sup>4</sup> In France, a new law has been proposed recently to allow download of unauthorized copyright content, and in return be charged *proportionally* to the volume of the download.

likely set (usage-priced) demand to zero to avoid negative utility. That is,  $p_s$  would normally be *regulated* and we will consider it as a fixed parameter in the following (with  $|p_s| \leq p_{\text{max}}$ ).

First, if  $|p_s| \leq \frac{1}{3}p_{\text{max}}$ , the equilibrium prices are given by

$$p_1^* = \frac{1}{3}p_{\text{max}} - p_s$$

$$p_2^* = \frac{1}{3}p_{\text{max}} + p_s$$

but demand  $D^* = D_0/3$  and utilities

$$U_i^* = \frac{D_0^2}{9d}$$

are exactly the same as (3) in the competitive setting with no side payment. Therefore, though setting  $p_s > 0$  at first seems to favor ISP1 over CP2, it turns out to have no effect on equilibrium revenues for both providers.

Alternatively, if  $p_s \geq \frac{1}{3}p_{\text{max}}$ , a boundary Nash equilibrium is reached when  $p_1^* = 0$  and  $p_2^* = \frac{1}{2}(p_{\text{max}} + p_s)$ , which means ISP1 does not charge usage-based fees to its consumers. Demand becomes  $D^* = \frac{1}{2}(D_0 - dp_s)$ , and utilities are

$$U_1^* = \frac{(D_0 - dp_s)dp_s}{2d}$$

$$U_2^* = \frac{(D_0 - dp_s)^2}{4d}$$

Though  $p_1^* = 0$ ,  $U_1^*$  is still strictly positive, with revenues for ISP1 coming from side-payments (and possibly from flat-rate monthly fees as well). Furthermore,  $p_s \geq \frac{1}{3}p_{\text{max}} \Leftrightarrow dp_s \geq \frac{1}{2}(D_0 - dp_s)$ , which means  $U_1^* \geq U_2^*$ : in this setting, ISP1’s best move is to set his usage-based price to zero (to increase demand), while he is sure to achieve better revenue than CP2 through side-payments.

Finally, if  $p_s < -\frac{1}{3}p_{\text{max}}$ , the situation is similar to the previous case (with  $-p_s$  instead of  $p_s$ ). So, here  $p_2^* = 0$  and  $p_1^* = \frac{1}{2}(p_{\text{max}} - p_s)$ , leading to  $U_2^* \geq U_1^*$ .

To remind, herein revenues  $U_i$  are assumed usage-based, which means there could also be flat-rate charges in play to generate revenue for either party. Studies of flat-rate compare to usage-based pricing schemes can be found in the literature, see, *e.g.*, [8].

## 4 Advertising revenues

We suppose now that CP2 has an additional source of (usage-based) revenue from advertising that amounts to  $Dp_a$ . Here  $p_a$  is not a decision variable but a fixed parameter.<sup>5</sup>

<sup>5</sup> One may see  $p_a$  as the result of an independent game between CP2 and his advertising sources, the details of which are out of the scope of this paper.

#### 4.1 Non-Cooperation

The utilities for ISP1 and CP2 are now:

$$U_1 = [D_0 - d \cdot (p_1 + p_2)](p_1 + p_s) \quad (7)$$

$$U_2 = [D_0 - d \cdot (p_1 + p_2)](p_2 - p_s + p_a) \quad (8)$$

Here, the Nash equilibrium prices are:

$$p_1^* = \frac{1}{3}p_{\max} - p_s + \frac{1}{3}p_a$$

$$p_2^* = \frac{1}{3}p_{\max} + p_s - \frac{2}{3}p_a$$

The cost to users is thus  $p^* = \frac{2}{3}p_{\max} - \frac{1}{3}p_a$  while demand is  $D^* = \frac{1}{3}(D_0 + dp_a)$ . Nash equilibrium utilities are given by

$$U_i^* = \frac{(D_0 + dp_a)^2}{9d} \quad \text{for } i = 1, 2, \quad (9)$$

which generalizes equation (3) and shows how advertising revenue quadratically raises players' utilities.

#### 4.2 Collaboration

The overall income for cooperating providers is

$$U_{\text{total}} = (D_0 - dp)(p + p_a). \quad (10)$$

So, solving the associated first-order optimality equation yields

$$p^* = \frac{p_{\max} - p_a}{2}. \quad (11)$$

The optimal demand is then  $D^* = (D_0 + dp_a)/2$ , and the maximal total revenue is  $U_{\text{total}}^* = (D_0 + dp_a)^2/(4d)$ . Assuming this revenue is split equally between the two providers, we get for each provider the equilibrium utility

$$U_i^* = \frac{(D_0 + dp_a)^2}{8d}, \quad (12)$$

which generalizes equation (4). As before, providers and users are better off when they cooperate.

Thus, we see that  $p_a > 0$  leads to lower prices, increased demand and more revenue for *both* providers (*i.e.*, including ISP1).

### 5 ISP Providing Multiple Service Classes

In this section, we suppose ISP1 is offering two types of network access service: a low-quality one  $l$  at price  $p_l$ , and a high-quality one  $h$  at price  $p_h \geq p_l$ . The role of multiple service classes in a neutral network, *i.e.*, as selected by the users, has previously been explored,

*e.g.*, [9]<sup>6</sup>. Here, we split the demand  $D$  into  $D_l$  and  $D_h$ :  $D = D_l + D_h$  (we will describe later how we implement the dichotomy between  $D_l$  and  $D_h$ ). For now, assume the overall demand still has a linear response profile, *i.e.*,

$$D = D_0 - d(\underbrace{p_l + p_h}_{\text{formerly } p_1} + p_2). \quad (13)$$

First, we make reasonable assumptions on  $D_l$ :

1. *Pricing incentives*: Define  $\Delta p := p_h - p_l$ .  $\Delta p$  is an incentive for consumers to chose between classes  $l$  and  $h$ : the higher  $\Delta p$  is, the more likely users are to select  $l$ . Thus, if we take  $x := 1/\Delta p$  and  $y := D_l/D$ , we may see  $y$  as a function of  $x$  and model this pricing response with the following properties:

$$y'(x) \leq 0 \quad (D_l \text{ increases with } \Delta p) \quad (14)$$

$$y(0) = 1 \quad (D_l \uparrow D \text{ as } \Delta p \uparrow \infty) \quad (15)$$

$$y(\infty) = 0 \quad (D_l \downarrow 0 \text{ as } \Delta p \downarrow 0) \quad (16)$$

2. *Congestion incentives*: As  $D_l$  approaches  $D$ , we assume congestion occurs in the low-quality network, further deterring users to chose it. This motivates the additional assumption that

$$|y'(x)| \downarrow 0 \text{ as } x \downarrow 0, \quad (17)$$

that is,  $D_l$  *decelerates* as it gets closer to  $D$ .

Define

$$\delta := \frac{\Delta p}{\gamma p_{\max}}, \quad (18)$$

where  $\gamma > 0$  is an additional users' price-sensitivity parameter. The following demand relation satisfies all conditions (14), (15), (16) and (17):

$$D_l := (1 - e^{-\delta}) D. \quad (19)$$

The providers' utilities are then:

$$U_1 = D_l p_l + D_h p_h = D(p_l + \Delta p e^{-\delta}) \quad (20)$$

$$U_2 = D p_2 \quad (21)$$

#### 5.1 Collaboration

If both players cooperate, their overall utility is

$$U_{\text{total}} = D(p_2 + p_l + \Delta p e^{-\delta}).$$

There is no NEP with strictly positive prices  $p_i \geq 0$  for  $i = 1, 2$ . To specify the boundary NEP (where at least one usage-based price is zero), define

$$\phi(x) := (1 - x)e^{-x}$$

and note that  $\phi$  is a bijection of  $[0, 1]$ .

<sup>6</sup> Non-neutral class assignment to applications, *i.e.*, not application-neutral networking, is discussed in [14].

– If  $p_2 = 0$ , NEP conditions imply

$$\begin{aligned}\delta^* &= \phi^{-1}(1/2) \\ p_l^* &= \frac{1}{3} \left( \frac{1}{2} - \gamma \delta e^{-\delta} \right) p_{\max}\end{aligned}$$

Utility at the NEP is therefore

$$U_{\text{total}}^* = \frac{D_0^2}{9d} \left[ \frac{1}{2} + 2\gamma \delta e^{-\delta} \right] \cdot [2 + (2e^{-\delta} - 3) \delta \gamma] \quad (22)$$

In this setting, the value of  $U_{\text{total}}$  is upper bounded by  $\approx 0.162 \frac{D_0^2}{d}$  which is achieved when  $\gamma \approx 1.53$  (recall that  $\gamma$  is not a decision variable).

– If  $p_l = 0$ , then  $p_h = 0$  and  $p_2 = \frac{1}{2} p_{\max}$ , yielding

$$U_{\text{total}} = \frac{D_0^2}{4d}. \quad (23)$$

Hence, irrespective of consumers' sensitivity  $\gamma$  to the price gap  $\Delta p$ , the best solution for the coalition is to set-up usage-based pricing for content only, at price  $p_2 = p_{\max}/2$ , while network access is subject to flat-rate pricing ( $p_l = p_h = 0$ ).

## 5.2 Splitting Demand-Response Coefficient

Now consider splitting the demand-response coefficient  $d$  into  $d_l$ ,  $d_h$  and  $d_2$ , that is:

$$D = D_0 - d_l p_l - d_h p_h - d_2 p_2. \quad (24)$$

If

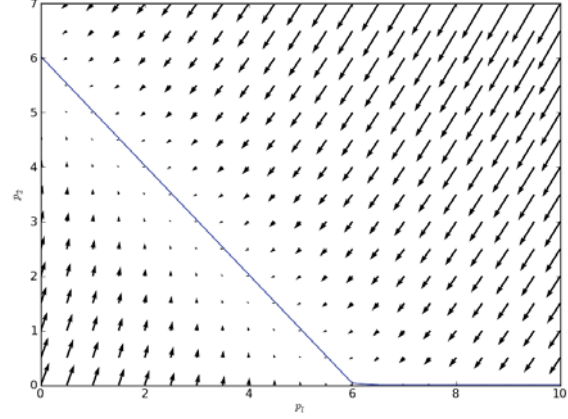
$$d_2 = d_l + d_h, \quad (25)$$

then the interior equilibrium conditions  $\nabla U_{\text{total}} = \underline{0}$  yield:

$$\begin{aligned}\delta &= \phi^{-1}(d_h/d_2) \\ p_l + p_2 &= \frac{D_0}{2d_2} - \frac{\delta \Delta p_0}{2} \left( \frac{d_h}{d_2} + e^{-\delta} \right)\end{aligned}$$

When the demand-response coefficients satisfy (25), we have an *equilibrium line*. Vector field plots of  $U_{\text{total}}$  suggest it is attractive (see Figure 5.2). In this particular setting, providers can thus reach  $U_{\text{total}}^*$  with non-flat rate pricing.

However, if  $d_2 \neq d_l + d_h$ , there exists a line of attraction, but with a non-null gradient on it driving players toward border equilibria. Hence, the conclusion of subsection 5.1 also holds in this more generalized setting.



**Fig. 1** Attraction of the equilibrium line.

## 5.3 Non-Cooperation

When ISP1 and CP2 compete, again there is no interior NEP (with all prices  $p_i$  strictly positive). In fact, the condition  $\nabla_{p_l, p_h} U_1 = \underline{0}$  implies  $p_l = 0 = p_h$  and  $D = 0$ , so ISP1 has to relax condition  $\frac{\partial U_1}{\partial p_l} = 0$  by setting  $p_l = 0$  (*i.e.*, flat-rate pricing for the best-effort service  $l$ ). The solution to the two remaining Nash equilibrium conditions is then:

$$p_2 = \frac{1}{4} \left[ \sqrt{9\gamma^2 + 2\gamma + 1} - 3\gamma + 1 \right] \cdot p_{\max} \quad (26)$$

$$p_h = \frac{\gamma}{2\sqrt{9\gamma^2 + 2\gamma + 1} - 3\gamma + 2} \cdot p_{\max} \quad (27)$$

By defining  $f_2(\gamma) := p_2/p_{\max}$  and  $f_h(\gamma) := p_h/p_{\max}$ , we then have

$$U_1^*(\gamma) = f_h(\gamma) \cdot (1 - f_h(\gamma) - f_2(\gamma)) D_0 p_{\max}$$

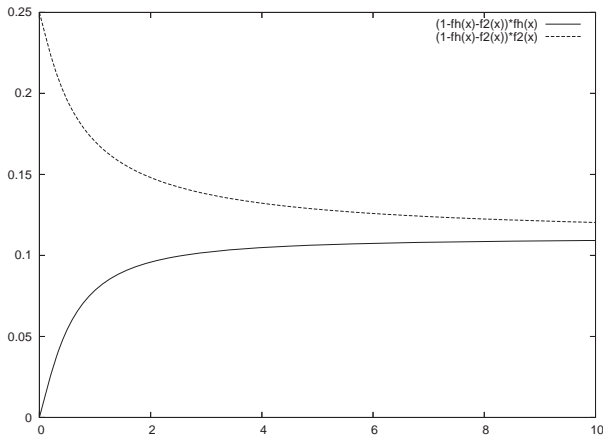
$$U_2^*(\gamma) = f_2(\gamma) \cdot (1 - f_h(\gamma) - f_2(\gamma)) D_0 p_{\max}$$

Figure 2 shows utilities at equilibrium (as fractions of  $D_0 p_{\max}$ ). We see that, in any case, CP2 has the advantage in this game:  $U_2^*$  is always greater to  $U_1^*$ , irrespective of consumers' sensitivity  $\gamma$  to usage-based prices.

Here,  $\gamma \rightarrow 0$  means users are so sensitive to any usage-based price that they will always choose the best-effort service (which is subject to flat-rate pricing). Users' price sensitivity decreases as  $\gamma$  increases, with the limit  $\gamma \rightarrow \infty$  corresponding to the setting of section 2 with  $\lim_{\gamma \rightarrow \infty} U_i^*(\gamma) = \frac{D_0^2}{9d}$ .

## 6 Stackelberg equilibrium

Stackelberg equilibrium corresponds to asymmetric game in which one player is the leader and the other a followers. That is, actions are no longer taken independently: the leader takes action first, and then the follower reacts.



**Fig. 2** Utilities as functions of users' sensitivity to usage-based pricing.

Though the dynamics of the games are different from the previous study, equations (7) and (8) still hold, with fixed  $p_a \geq 0$  and regulated  $p_s$ . In the following, we need to assume that

$$p_s \leq \frac{1}{2}p_{\max} + \frac{1}{2}p_a$$

$$p_a \leq \frac{1}{3}p_{\max} + \frac{1}{4}p_s$$

so that NEPs are reachable with positive prices.

If ISP1 sets  $p_1$ , then CP2's optimal move is to set

$$p_2 = \frac{1}{2}(-p_1 + p_{\max} + p_s - p_a).$$

This expression yields  $D = \frac{d}{2}(p_{\max} - p_1 - p_s + p_a)$  and  $U_1 = \frac{d}{2}(p_{\max} - p_1 - p_s + p_a)(p_1 + p_s)$ . Anticipating CP2's reaction in trying to optimize  $U_1$ , the best move for ISP1 is thus to set

$$p_1^* = \frac{1}{2}p_{\max} - p_s + \frac{1}{2}p_a,$$

which yields

$$p_2^* = \frac{1}{4}p_{\max} + p_s - \frac{3}{4}p_a.$$

Therefore, when ISP1 is the leader, at the NEP demand is  $D^* = \frac{1}{4}(D_0 + dp_a)$  and utilities are:

$$U_1^* = \frac{1}{8d}(D_0 + dp_a)^2, \quad (28)$$

$$U_2^* = \frac{1}{16d}(D_0 + dp_a)^2. \quad (29)$$

Suppose now that CP2 is the leader and sets  $p_2$  first. Similarly, we find:

$$p_2^* = \frac{1}{2}p_{\max} + p_s - \frac{1}{2}p_a$$

$$p_1^* = \frac{1}{4}p_{\max} - p_s + \frac{1}{4}p_a$$

These values yield the same cost  $p^*$  and demand  $D^*$  for the internauts at the NEP, while providers' utilities become:

$$U_1^* = \frac{1}{16d}(D_0 + dp_a)^2, \quad (30)$$

$$U_2^* = \frac{1}{8d}(D_0 + dp_a)^2. \quad (31)$$

Therefore, in either case of leader-follower dynamics, the leader obtains twice the utility of the follower at the NEP (yet, his revenue is not better than in the collaborative case).

## 7 Further abandoning neutrality

Throughout we assumed that the side payments between service and content providers were regulated. If  $p_s$  is allowed to be determined unilaterally by the service provider or by the content provider (as part of the game described in Section 3 or 4), then the worst possible performance is obtained at equilibrium. More precisely, the demand at equilibrium is zero, see [1]. The basic reason is that if the demand at equilibrium were not zero then a unilateral deviation of the provider that controls the side payment results in a strict improvement of its utility. (Note that the total demand is unchanged as it does not depend on  $p_s$ .)

More generally, assume that an ISP is given the authority to control  $p_s$  and that its utility can be expressed as  $U = f(D) \times (g(p_s) + h)$  where  $f$  is any function of the demand (and possibly also of prices other than  $p_s$ ),  $g$  is a monotone strictly increasing function of  $p_s$ , and  $h$  does not depend on  $p_s$ . Then at equilibrium, necessarily  $f(D) = 0$ , otherwise  $U$  can be further increased by the provider by (unilaterally) increasing  $p_s$ .

The same phenomenon holds also in case the CP is given full control over  $p_s$ .

## 8 Conclusions and on-going work

Using a simple model of linearly diminishing consumer demand as a function of usage-based price, we studied a game between a monopolistic ISP and a CP under a variety of scenarios including consideration of: non-neutral two-sided transit pricing (either CP2 participating in network costs or ISP1 paying for copyright remuneration), advertising revenue, cooperation and leadership.

In a basic model without side-payments and advertising revenues, both providers achieve the same utility at equilibrium, and all actors are better off when they cooperate (higher demand and providers' utility).

When regulated, usage-based side-payments  $p_s$  come into play, the outcome depends on the value of  $|p_s|$  compared to the maximum usage-based price  $p_{\max}$  consumers can tolerate:

- when  $|p_s| \leq \frac{1}{3}p_{\max}$ , providers shift their prices to fall back to the demand of the competitive setting with no side-payments;
- when  $|p_s| \geq \frac{1}{3}p_{\max}$ , the provider receiving side payments sets its usage-based price to zero to increase demand, while it is sure to be better off than his opponent.

When advertising revenues to the CP come into play, they increase the utilities of *both* providers by reducing the overall usage-based price applied to the users. ISP1 and CP2 still share the same utility at equilibrium, and the increase in revenue due to advertising is quadratic.

We considered in section 5 the implications of service differentiation from the ISP. In our model, when ISP1 and CP2 cooperate, the best solution for them is to set-up usage-based prices for content only and flat-rate pricing for network access. However, when providers do not cooperate, the ISP optimally offers its best-effort service for a flat rate (zero usage-based cost), resulting in more usage-based revenue for the CP.

Under leader-follower dynamics, the leader obtains twice the utility of his follower at equilibrium; yet, he does not achieve a better revenue than in the cooperative scenario.

In subsequent work [2], we explored the effects of content-specific (*i.e.*, not *application* neutral) pricing, including multiple CPs providing different types of content. Also, we considered competition among multiple providers of the same type, including different models consumer stickiness (inertia or loyalty). In on-going work, we are also considering providers' infrastructure and operating costs (as in, *e.g.*, [12]), more complex models of end-user demand and their collective social welfare, and the effects of different options for flat-rate pricing (*e.g.*, [16,8]).

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