Multi-lane Vehicle-to-Vehicle Networks with Time-Varying Radio Ranges: Information Propagation Speed Properties

Emmanuel Baccelli, Philippe Jacquet, Bernard Mans, Georgios Rodolakis

Multi-lane Vehicle-to-Vehicle Networks with Time-Varying Radio Ranges: Information Propagation Speed Properties

Emmanuel Baccelli*, Philippe Jacquet†, Bernard Mans‡, Georgios Rodolakis§
*INRIA, France, {firstname.lastname}@inria.fr
†Alcatel Lucent Bell Labs, Villarceaux, France, {firstname.lastname}@alcatel-lucent.fr
‡Macquarie University, Australia, {firstname.lastname}@mq.edu.au
§ITI, Information Technologies Institute, CERTH, Centre for Research and Technology, Greece, {lastname}@iti.gr

***Abstract*—We study the information propagation speed in multi-lane vehicle-to-vehicle networks such as roads or highways. We focus on the impact of time-varying radio ranges and of multiple lanes of vehicles, varying in speed and in density. We assess the existence of a vehicle density threshold under which information propagates on average at the fastest vehicle speed and above which information propagates dramatically faster. We first prove that no such phase transition occurs if there is only one lane, regardless of the density of vehicles, when one takes into account real-time radio communication range variations at the MAC layer. We then prove that, on the other hand, a phase transition exists as soon as there are multiple lanes with different vehicle speeds and appropriate densities. We characterize conditions under which the phase transition occurs and we derive bounds on the corresponding threshold as a simple relationship between the vehicle density on the fastest lane and the sum of densities on the other lanes. Our results intrinsically encompass a wide range of vehicular network scenarios, including one-way and two-way roads, as well as special cases such as road side units and/or parked cars being used as relays. We confirm our analytical results using simulations.**

I. Introduction

We investigate the information propagation speed in vehicle-to-vehicle networks such as multi-lane roads or highways. Large countries typically cover a great number of miles with multi-lane roads or highways, in part without telecommunication infrastructure. Our analysis shows the conditions under which a phase transition occurs in this context, concerning information propagation speed, with respect to the vehicle density. We prove that under a certain threshold, information propagates on average at the fastest vehicle speed, while above this threshold, information propagates significantly faster.

With applications such as safety on the road, ad hoc vehicular networks and vehicle-to-vehicle are receiving increasing attention (see recent survey in [5]). Vehicular DTNs (Delay Tolerant Networks, *i.e.*, networks in which end-to-end paths may not exist and communication routes may only be available through mobility and accumulation over time) have thus been considered in recent studies, and various analytical models have been proposed. The papers [1], [2] focus on information propagation speed in one-dimensional vehicle-to-vehicle with constant speeds in each direction. In [1] the authors introduce a model based on space discretization to derive upper and lower bounds in the highway model under the assumption that the radio propagation speed is finite. Their bounds indicate the existence of a phase transition phenomenon for the information propagation speed. Comparatively in [2], the authors introduce a model based on Poisson point process on continuous space, that allows both infinite and finite radio propagation speed, and derive more fine-grained results above and below the threshold. Using the model, the authors prove and explicitly characterize the phase transition. In [7], the authors derive equations for the expected value and variance of propagation distance through inter-vehicle communication when the distance between vehicle follows a general distribution.

Studies such as [8], [9] are the closest related work. In [9], the authors consider a one-dimensional vehicle-to-vehicle network with unit-disk radio range where all vehicle travel in the same direction, with random speeds following a Gaussian distribution. The paper derives the impact of various parameters on information propagation speed. The authors extend their results in [8] to a multi-stream case where vehicles in different streams have different speed distributions and derived analytical formulas. Comparatively, we here consider a model accommodating real time variation of radio ranges for vehicle-to-vehicle communications on multi-lane roads, where vehicles on a particular lane all have the same speed. Since we consider the speed of vehicles only relatively to the speed of vehicles on the fastest lane, our model naturally encompasses both cases of one-way and two-way roads, as well as special cases such as road side units and/or parked cars being used as relays The goal of this paper is to increase our understanding of the fundamental performance limits of mobile vehicle-to-vehicle networks with time-varying radio ranges and to thus complement the above-cited works.

In this context, our contributions are as follows: (1) in Section II we present a time-varying radio range model for information propagation in multi-lane vehicle-to-vehicle networks that takes into account real-time radio communication range variations at the MAC layer and list our corresponding results for all cases of number of lanes; (2) we prove that no such phase transition occurs concerning the information propagation speed with respect to the vehicle densities if there is only one lane, regardless of the car densities in Section III, and (3) we prove that a phase transition exists as soon as there is a second lane with different vehicle speed and sufficient density in Section IV; (4) in Section V, we focus on information propagation speed when there are three lanes or more. We show that under certain conditions a phase transition exists and we derive bounds on the threshold as a simple relationship between the vehicle density on the fastest lane and the sum of the vehicle densities on the other lanes. We also prove that there are still cases where no such phase transition occurs; (5) Finally we validate the provided analysis with simulations in Section VI.

II. Model and Results

We consider a highway with multiple lanes of vehicles at fixed speed, numbered 0 to k: on lane i, the vehicle speed is v_i. The vehicle density on lane i is Poisson of intensity λ_i. Vehicle speeds are in decreasing order: lane 0 is called the *fast lane*, determined as the lane of vehicles with the fastest eastbound speed. Lanes from 1 to k are conversely called the *slow lanes*. In the following, we consider vehicle speeds in the referential of the vehicles of the fast lane.

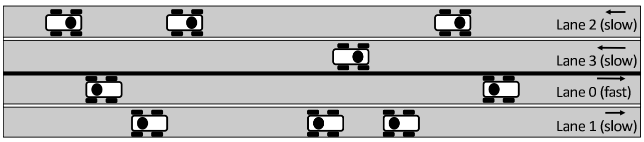

Fig. 1. Vehicular network on a bidirectional highway with four lanes.

For instance, in Figure 1, we depict an example of a four-lane highway, with two eastbound and two westbound lanes. We number lanes in decreasing eastbound speed: 0. fast eastbound lane, 1. slow eastbound lane, 2. slow westbound lane, and 3. fast westbound lane. We note that, once we have defined the lane numbering, the actual directions of the lane speeds do not matter anymore in our model. Note that this also allows to account for static lanes where cars are parked along the road. As we consider the speeds in the referential of the fast lane, the speed of lane 0 will be 0, and the speeds of the remaining lanes will be negative.

The radio propagation, as well as the store-and-forward time are considered instantaneous. For radio transmissions, we will consider alternatively (i) a fixed radio range, or, more realistically, (ii) a time-varying radio range.

In the former case (i), we will assume that the radio range of each transmission is of length R in each direction.

In the latter case (ii), we suppose that the radio range R varies in time in a random way and is unbounded (*i.e.*, for any distance x, $P(R > x) \neq 0$). We suppose that each vehicle transmits according to a time Poisson process of intensity $\nu > 0$, with a random range R. We assume the following properties concerning the random variable R. Let ζ be a complex quantity with the dimension of inverse distance. For all ζ,

1) $f(\zeta) = E(e^{-\zeta R})$ is defined and is not infinite;
2) for all x, $E(e^{-(R-x)\zeta} | R > x)$ is defined and bounded by some function $g(\zeta)$.

The first condition implies that, $\lim_{x\to\infty} e^{\alpha x} P(R > x) = 0$, for all $\alpha > 0$. It is satisfied, *e.g.*, when there exist $D > 0$ and $\delta > 1$, such that $P(R > x) \sim \exp(-Dx^\delta)$. The second condition is satisfied when, for all x, the random variable $R - x$, under the condition that $R > x$, is smaller or equal in probability to a certain random variable $\bar{R}$, such that $E(e^{-\zeta \bar{R}})$ is defined; in this case, we can take $g(\zeta) = E(e^{-\zeta \bar{R}})$.

Assuming such conditions makes sense in relevant contexts, such as slotted Aloha in the Poisson shot noise model [4]. For example, in a one-dimensional highway network, when the power attenuation is of path-loss exponent 2, $P(R > x)$ is of the form e^{-x^2}.

Without loss of generality, we focus on the propagation of information in the eastbound direction. An information packet propagates in the following manner: it moves toward the east jumping from car to car until it stops because the next car is beyond radio range; the information packet waits on the right-most car until either a transmission of large enough radio range occurs to move forward, or the gap is filled by cars of other lanes (in the case of fixed range, this is the only way for the information to propagate further). We consider the distance $L(t)$, traveled by the information during time t, with respect to the referential of the fast-lane cars. We define the average information propagation speed v_p as: $v_p = \lim_{t\to\infty} \frac{\mathbf{E}(L(t))}{t}$.

A. Results

The main results presented in this paper are the proofs of the following five theorems, corresponding to various cases of number of lanes: one (Theorem 1), two (Theorems 2 and 3), and three or more lanes (Theorems 4 and 5). For all five theorems, we analyze the asymptotic average information propagation speed v_p (without loss of generality, in the referential of the vehicles of the fastest eastbound lane).

For the remainder of the paper, for $x > 0$, we denote x^* the conjugate of x with respect to the function xe^{-x}: x^* is the alternate solution of the equation $x^* e^{-x^*} = xe^{-x}$. Notice that $x^{**} = x$ and $1^* = 1$.

Theorem 1. *In a vehicular network with a single (one-way) lane where the radio range varies, the average asymptotic information propagation speed v_p with respect to the referential of the cars is null, regardless of the vehicle density λ_0.*

Theorem 2. *Consider a vehicular network with two lanes, where the radio range R varies, with respective car densities λ_0 and λ_1, and respective speeds v_0 and v_1. Let a distance $r_0 > 0$ such that $P(R \geq r_0) \neq 0$. For all $0 < a < 1/2$, and for $\rho = -\log\left(1 - \exp(-P(R \geq r_0)\frac{\nu a}{v_0 - v_1})\right)$, the average asymptotic information propagation speed v_p with respect to the referential of the fastest cars is strictly larger than 0, when $r_0\lambda_0 > \frac{1}{1-2a}(2\rho + (r_0\lambda_1)^*)$.*

Theorem 3. *Consider a vehicular network with two lanes, where the radio range R varies, with respective car densities*

λ_0 *and* λ_1*, and respective speeds* v_0 *and* v_1*. There exists* $\rho_0 > 0$*, which is a function of* $(v_0 - v_1)$ *and* λ_1*, such that, when* $\lambda_0 < \rho_0$*, the average asymptotic information propagation speed* v_p *with respect to the referential of the fastest cars is null.*

Theorem 4. *Consider a vehicular network with three or more lanes, numbered 0 to k, where the radio range varies. For all* $0 < a < 1/2$*, there exists a distance* $r_0 > 0$ *and a scalar* $\rho > 0$*, such that the average asymptotic information propagation speed* v_p *with respect to the referential of the fastest cars (lane 0) is strictly larger than 0, when* $r_0\lambda_0 \geq \frac{1}{1-2a}\left(2\rho + (r_0 \sum_{i=1}^{i=k} \lambda_i)^*\right)$.

Theorem 5. *Consider a vehicular network with three or more lanes, numbered 0 to k. There exists a tuple of car densities, such that* $\forall i$*:* $\lambda_i \neq 0$*, for which the average information propagation speed* v_p *is null.*

III. Single Lane: Proof of Theorem 1

As the information propagation speed is considered in the referential of the vehicles (say, in eastbound direction), we can equivalently assume that the cars are fixed, and consider the information propagation speed v_p toward the east, solely based on the transmissions. We recall that the emission process of the vehicles is a time Poisson process of intensity $\nu > 0$, with a random radio range R.

We call the rightmost car that has received the information at a given time t, the *current car* at time t. Without loss of generality, we consider that only the current car transmits the information; and, similarly, we can assume that the random radio range process includes the overlap from the ranges of previous cars to the left, if they participate in the transmission.

In the following, we describe a process to provide an upper bound on the information propagation speed $\bar{v_p}$. If the gap from the current car to the next car to the right is x, then one has to wait that the radio range exceeds x, before the information reaches the next car. When the radio range R exceeds x, all the cars that are located between x and R receive the information. The speed upper-bound process consists in assuming that the next current car is located exactly at the range limit, *i.e.* at distance R from the previous current car.

Therefore, under the upper bound model, the gaps x between the current cars and the following cars are *i.i.d.* random exponential variables, with rate λ_0. Given that the gap is of length x, the time $T(x)$ needed to move the message to the next current car satisfies: $E(T(x)) = \frac{1}{\nu P(R>x)}$, and the distance $D(x)$ traveled by the information at this time satisfies: $E(D(x)) = E(R|R > x)$. Since this is a memoryless process, the average information propagation speed upper-bound $\bar{v_p}$ is:

$$\bar{v_p} = \frac{\int_0^\infty E(D(x))\lambda_0 e^{-\lambda_0 x} dx}{\int_0^\infty E(T(x))\lambda_0 e^{-\lambda_0 x} dx} .$$

From the condition $\lim_{x\to\infty} e^{\lambda_0 x} P(R > x) = 0$, we get $\int_0^\infty E(T(x))\lambda_0 e^{-\lambda_0 x} dx = \infty$. From the condition $E(R|R > x) \leq x + E(\bar{R})$, we have $\int_0^\infty E(D(x))\lambda_0 e^{-\lambda_0 x} dx \leq \frac{1}{\lambda_0} + E(\bar{R}) < \infty$. Thus, the information propagation speed $\bar{v_p}$ is 0.

IV. Two Lanes

A. Proof of Theorem 2

For the purpose of the proof, we define a lower bound process on the information propagation speed. Let $r_0 > 0$ be such that $P(R \geq r_0) \neq 0$. We bound the radio range R with a new range R', as follows:

- when $R \geq r_0$, then $R' = r_0$;
- when $R < r_0$, then $R' = 0$.

We consider the information propagation on the fast (say eastbound) lane. In the lower bound model, cars must be separated by less than r_0, in order to be able to communicate. We call r_0*-cluster* a set of consecutive cars such that two consecutive cars are separated by distance less than r_0. We call *length of a cluster* the distance between the first and the last car of a cluster. A recursive definition of an r_0-cluster, spreading eastwards from position y, namely $\mathcal{C}_{r_0}(y)$, is:

$$\mathcal{C}_{r_0}(y) = \mathcal{I}_{r_0}(y) \cup \mathcal{C}_{r_0}(z),$$

where $\mathcal{I}_{r_0}(y)$ is the set of cars located in the segment $[y, y + r_0]$, and z is the position of the rightmost car in $\mathcal{I}_{r_0}(y)$.

Therefore, if the current car on lane 0 (the fast lane) is separated from the next car on lane 0 by a gap of length $r_0(1 + x)$, for $x > 0$, then it needs to wait for a slow lane cluster of length at least $r_0 x$ to bridge the gap.

The number of retransmissions of radio range r_0, to cover a cluster of length $r_0 x$, cannot exceed $\lceil 2x \rceil$. Let $a > 0$ be a real number, and assume that the time between two retransmissions is smaller than $a\frac{r_0}{v_0 - v_1}$. The latter occurs with probability $e^{-\rho} = 1 - \exp(-P(R \geq r_0)\nu a \frac{r_0}{v_0 - v_1})$, for some $\rho > 0$ (since, we recall that the node emission process is a Poisson process of intensity $\nu > 0$). This also implies that the cluster of length $x r_0$ travels a distance smaller than $\lceil 2x \rceil a r_0$. Assuming now that $a < 1/2$, a cluster of length $\frac{x r_0}{1-2a}$ would suffice to bridge a gap of length $r_0(1 + x)$, provided that successive retransmissions are all separated by less than $a\frac{r_0}{v_0 - v_1}$ time units (which occurs with probability greater than $e^{-\rho 2x/(1-2a)}$). From [2, equation (10)], with a lane density normalized by the radio range, we know that the average time to get an r_0-cluster of length $y r_0$ is $\Theta(e^{(r_0\lambda_1)^* y})$, as expressed in the following lemma.

Lemma 1. *The average distance to a car on the slow lane, which begins an* r_0*-cluster of slow lane cars of length larger than* $r_0 x$*, is* $\Theta(e^{(r_0\lambda_1)^* x})$*, when* $x \to \infty$*.*

The time needed to get an r_0-cluster of length $r_0 y$, where all successive retransmissions are separated by less than $a\frac{r_0}{v_0 - v_1}$ time units, is $\Theta(e^{(2\rho + (r_0\lambda_1)^*)y})$. Using the methodology in [2, Section III-A, eq. (9)], the average distance to such a cluster on the slow lane is $\Theta(\exp\left(\frac{2\rho + (r_0\lambda_1)^*}{1-2a} x\right))$, and thus the time $T'(x)$ needed to bridge a gap of length $r_0(1 + x)$ is:

$$E(T'(x)) = \Theta(\exp\left(\frac{2\rho + (r_0\lambda_1)^*}{1 - 2a} x\right)).$$

The distance $D'(x)$ traveled by the information (in the referential of fast-lane cars) satisfies: $E(D'(x)) \geq r_0(1 + x)$.

Therefore, the information propagation speed v_p is:

$$\begin{aligned} v_p &\geq \frac{\int_0^x E(D'(x))\lambda_0 r_0 e^{-\lambda_0 r_0 x} dx}{\int_0^x E(T'(x))\lambda_0 e^{-\lambda_0 r_0 x} dx} \\ &\geq \frac{r_0 + 1/\lambda_0}{\int_0^x E(T'(x))\lambda_0 r_0 e^{-\lambda_0 r_0 x} dx} . \end{aligned}$$

It turns out that the integral in the denominator converges when $r_0\lambda_0 > \frac{2\rho+(r_0\lambda_1)^*}{1-2a}$ and, with this condition, $v_p > 0$.

B. Proof of Theorem 3

See technical report [3, Section 7.1].

V. Three or more Lanes

A. Proof of Theorem 4

We take the framework of the proof of Theorem 2, with r_0 such that $P(R > r_0) \neq 0$. We will first consider the case where $\nu \to \infty$, *i.e.*, when the channel access delay $\frac{1}{\nu}$ tends to 0. In this case, we have $\rho = a = 0$, and we are in the framework of [2] with a unit disk graph model, but with the difference that instead of 1, the range here is r_0.

To simplify the notation, we denote $\Lambda_s = \sum_{i=1}^{i=k} \lambda_i$. We consider that the information is blocked by a gap of length $r_0(1+x)$ on the fast lane. The aim is to give an upper bound of the time needed to have an r_0-cluster of length at least $r_0 x$, made of cars on slow lanes, which covers the gap. But, since clusters are not permanent in time, and experience continuous changes due to the differences of car motions, we will consider here *instantaneous* r_0-clusters. A recursive definition of an instantaneous r_0-cluster, spreading eastwards from position y and at time t, namely $\mathcal{C}_{r_0}(y,t)$, is:

$$\mathcal{C}_{r_0}(y,t) = \mathcal{I}_{r_0}(y,t) \cup \mathcal{C}_{r_0}(z,t),$$

where $\mathcal{I}_{r_0}(y,t)$ is the set of cars located in the segment $[y, y+r_0]$ at time t, and z is the position of the rightmost car in $\mathcal{I}_{r_0}(y,t)$ at time t. Thus, we use a version of lemma 1 adapted to the case where the cars on slow lanes have relative motions. The following is directly derived from [2, Lemma 2].

Lemma 2. *The probability that a given car, at a given time, begins an instantaneous r_0-cluster, of slow lane cars of length longer than $r_0 x$, is $\Theta(e^{-(r_0\Lambda_s)^* x})$, when $x \to \infty$.*

The main point now is to track the correlation between cars and time, since cars move relatively to one another. To this end, we consider the process of cluster formation that starts with a car on lane 1, the fastest among the slow lanes. Without loss of generality, we assume that $\lambda_1 \neq 0$.

We place ourselves in the referential of cars on lane 1. We consider a road segment $[A, B)$ in this referential, for some $A < B$. Since the cars on the other slow lanes have a relative westbound motion, and since Poisson processes are memoryless, the events in the interval $[A, B)$, at a given time t, and the future events in the interval $[B, \infty)$ are independent.

To make use of this property for instantaneous cluster formations, we split the slowest lane k into segments of length $(1 + x)r_0$, numbered from 1 and onwards.

We concentrate on the first segment. Let us consider the first car in lane 1. The probability that this lane 1 car starts a slow lane r_0-cluster, of length smaller than xr_0, at the time when this lane 1 car arrives in range of the current car on lane 0, is smaller than $1 - Ce^{-(r_0\Lambda_s)^* x}$ (with $C > 0$, as deduced from Lemma 2). Similarly, if we include the probability that no car of lane 1 is present in this segment, (*i.e.*, $e^{-(1+x)r_0\lambda_1}$), then the probability that there is no instantaneous r_0-cluster, starting in the first segment, with a length greater than $r_0 x$ when in range with the current car, is smaller than $1 - C(1 - e^{-(1+x)r_0\lambda_1})e^{-(r_0\Lambda_s)^* x}$.

Since this event only depends on the car positions in the first and second segments when the first car arrives in range of the current car, this event is independent of what will happen in the next segments. Thus, the occurrences of further r_0-clusters of length smaller than $r_0 x$ on odd segments are *i.i.d.* The average number of segments, before the formation of an instantaneous r_0-cluster of length greater than $r_0 x$, is smaller than:

$$\sum_{n\geq 0} 2(1-(1-e^{-(1+x)r_0\lambda_1})Ce^{-(r_0\Lambda_s)^* x})^n = 2\frac{\exp((r_0\Lambda_s)^* x)}{C(1 - e^{-r_0\lambda_1})} .$$

Therefore, the time $T'(x)$, before the gap of length $(1+x)r_0$ is bridged by an instantaneous cluster made of slow lane clusters, satisfies the inequality:

$$E(T'(x)) \leq \frac{2(x+1)r_0}{v_0 - v_1} \frac{\exp((r_0\Lambda_s)^* x)}{(1 - e^{-r_0\lambda_1})C} .$$

Thus, this average time $\int_0^\infty E(T'(x))\lambda_0 r_0 e^{-x r_0 \lambda_0} dx$ is finite as soon as $r_0\lambda_0 < (r_0\Lambda_s)^*$. This terminates the proof for $\nu \to \infty$. Notice again that, in this case, $\rho = a = 0$.

When $\nu < \infty$, we cannot use the same approach: indeed, in this case, the time between retransmissions does not allow us to work with instantaneous clusters. In this case, we must consider a refined version of Lemma 2, where instantaneous r_0-clusters are replaced by τ-timed r_0-clusters, *i.e.*, r_0-clusters obtained with retransmissions delayed each by τ seconds exactly. A recursive definition of an instantaneous (r_0, τ)-cluster, spreading eastbound from position y and at time t, namely $\mathcal{C}_{r_0,\tau}(y,t)$, is:

$$\mathcal{C}_{r_0,\tau}(y,t) = \mathcal{I}_{r_0}(y,t) \cup \mathcal{C}_{r_0,\tau}(z, t+\tau),$$

where z is the position of the rightmost car in $\mathcal{I}_{r_0}(y)$ at time $t + \tau$. We have the following lemma, equivalent to lemma 2:

Lemma 3. *Let $\tau > 0$, the probability that a given car, at a given time, begins an instantaneous τ-timed r_0-cluster of slow lane cars of length larger than $r_0 x$ is $\Theta(e^{-(r_0\Lambda_s)^* x})$, when $x \to \infty$.*

The end of the proof follows an approach similar to Theorem 2. We define $\tau = a\frac{r_0}{v_0 - v_1}$ for some $a > 0$ small enough, and we consider timed clusters larger than $(1 + x)r_0\frac{1}{1-2a}$, ensuring they can actually bridge a gap of length $(1 + x)r_0$. We then consider $(r_0 - \varepsilon)$-clusters instead of r_0-clusters, in order to cope with the small local modification of the clusters, since the retransmissions are not exactly after a time τ, but within a time τ (we take $\varepsilon = \tau(v_1 - v_k)$).

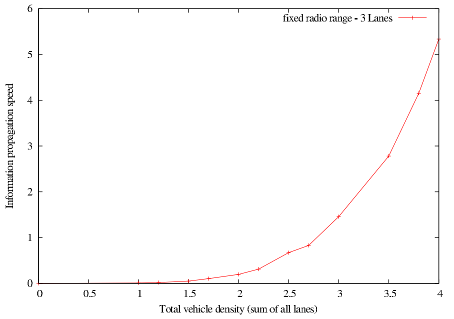

Fig. 2. 3 lanes, fixed radio range. Information propagation speed v_p for $\lambda_0 = \lambda_1 = \lambda_2$, versus the total vehicle density.

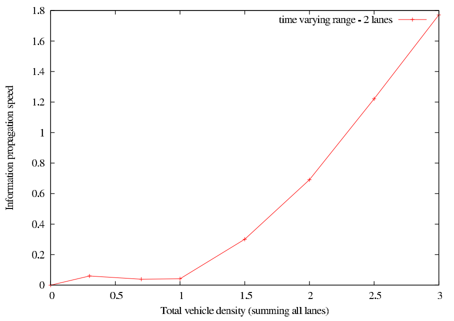

Fig. 3. 2 lanes, varying radio range ($P(R > x) = e^{-x^2}$)). Information propagation speed v_p for $\lambda_0 = \lambda_1$, versus the total vehicle density.

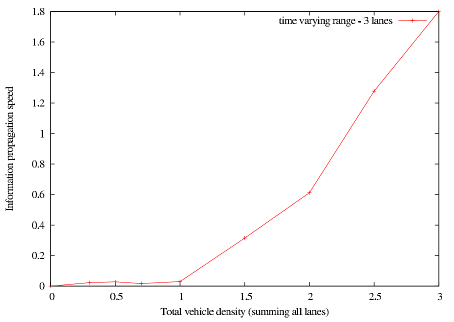

Fig. 4. 3 lanes, varying radio range ($P(R > x) = e^{-x^2}$)). Information propagation speed v_p for $\lambda_0 = \lambda_1 = \lambda_2$, versus the total vehicle density.

B. Proof of Theorem 5

See technical report [3, Section 7.2].

VI. Simulations

We present simulation results obtained with the Opportunistic Network Environment (ONE [6]) simulator. Vehicles are distributed uniformly on the length of two or more lanes of a road, and move at a constant speed. The total number of vehicles varies from 300 to 4000. We measure the fastest possible information propagation speed using epidemic broadcast, assuming that radio transmissions are instantaneous and that there are no buffering or congestion delays. We vary the vehicle densities λ_i (vehicles per radio range), and we perform several simulation iterations of randomly generated traffic.

Figure 2 plots the information propagation speed versus the total vehicle density for three lanes, when $\lambda_0 = \lambda_1 = \lambda_2$, with a fixed radio range $R = 1$. The fast lane speed is 0 (as measured in the referential of the fast cars), the slow lane speeds are 0.4 radio range per second and 0.6 radio range per second. We observe the existence of a threshold near $\sum_{i=0}^{i=k} \lambda_i = 2$, which confirms the analysis: below the threshold the information propagation speed is null, while above the threshold, it increases rapidly. We then simulate the case of time-varying radio range. We consider that $P(R > x) = e^{-x^2}$, corresponding to the Poisson shot noise model of a one dimensional highway, with path-loss exponent 2. Vehicles transmit packets of information with Poisson intensity $\nu = 5$ transmissions per second. Figures 3 and 4 plot the information propagation speed versus the total vehicle density (with $\lambda_i = \lambda_0$, for all i, and vehicle speeds as above), for two and three lanes respectively. Again, we notice the existence of a threshold, where a phase transition occurs.

VII. Conclusion

In this paper, we provided a detailed analysis of information propagation speed in multi-lane vehicle-to-vehicle networks. We considered both the case of constant radio transmission range, and the more realistic case of time-varying radio transmission range (due to environment versatility and interferences). We have shown the existence of a vehicle density threshold, under which information propagates on average at the speed of the fastest vehicle, and above which information propagates significantly faster. Our model covers scenarios including one-way and two-way roads, and cases where road side units or parked cars are used as relays. Our methodology is also extendable to cases where car speeds may take an infinite number of values within a given speed range, which corresponds to an infinite number of lanes in our model.

Acknowledgment

The work of G. Rodolakis was supported in part by the EU grants INFLOW (FP7-294236) and STAMINA (FP7-265496).

References

[1] A. Agarwal, D. Starobinski and T. Little, "Phase Transition of Message Propagation Speed in Delay-Tolerant Vehicular Networks", IEEE Transactions on Intelligent Transportation Systems 13(1): 249-263 (2012).
[2] E. Baccelli, P. Jacquet, B. Mans and G. Rodolakis, "Highway Vehicular Delay Tolerant Networks: Information Propagation Speed Properties". IEEE Transactions on Information Theory 58(3): 1743-1756 (2012).
[3] E. Baccelli, P. Jacquet, B. Mans and G. Rodolakis, "Multi-lane Vehicle-to-Vehicle Networks with Time-Varying Radio Ranges: Information Propagation Speed Properties". INRIA research report RR-8037, http://hal.inria.fr/hal-00722384 (2012).
[4] F. Baccelli, B. Blaszczyszyn and P. Mühlethaler, "An Aloha protocol for multihop mobile wireless networks". IEEE Transactions on Information Theory 52(2): 421-436 (2006).
[5] F. Dressler, F. Kargl, J. Ott, O. Tonguz and L. Wischhof, "Research challenges in intervehicular communication: lessons of the 2010 Dagstuhl Seminar". IEEE Communications Magazine 49(5): 158-164 (2011).
[6] A. Keranen, J. Ott and T. Karkkainen, "The ONE Simulator for DTN Protocol Evaluation". SIMUTools'09, Rome, 2009.
[7] B.X. Wang, T.M. Adams, W. Jin and Q. Meng, "The process of information propagation in a traffic stream with a general vehicle headway: A revisit", Transportation Research Part C, 18 (2010) pp. 367–375.
[8] Z. Zhang, G. Mao and B.D.O. Anderson, "On the Information Propagation Process in Multi-lane Vehicular Ad-hoc Networks", In ICC 2012.
[9] Z. Zhang, G. Mao and B.D.O. Anderson, "On the Information Propagation Speed in Mobile Vehicular Ad Hoc Networks", In Globecom 2010.