Theoretical and experimental investigations of harp's sympathetic modes

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THEORETICAL AND EXPERIMENTAL INVESTIGATIONS OF HARP'S SYMPATHETIC MODES

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ABSTRACT

The harp is composed of a soundboard, a cavity with soundholes and 47 strings. When one string is plucked, other strings are excited via the soundboard. This phenomenon, called sympathetic vibrations, was investigated in a recent paper [1], both theoretically and experimentally using an ersatz of string instrument, composed of 2 strings connected to a beam clamped at both ends. In order to extend the analysis to the case of the harp, an analytical model of a set of 35 strings coupled to a beam, whose characteristics are designed in order to be equivalent to the harp's soundboard, has been developed. The identification of sympathetic modes from the analytical modal basis of the beam-strings assembly is performed using a criterion, the Kinetic Energy Ratio. For some particular tuning conditions, these modes are present in the sound measured on a Camac concert harp. In a same partial, several components, whose frequencies can be really very close one to another are present but cannot be separated using classical Fourier transform. The identification of these components associated to sympathetic modes is achieved using specific signal processing techniques, called High Resolution methods. Identified experimental sympathetic modes are found in good agreement with those theoretically obtained.

INTRODUCTION

The harp is a complex vibroacoustic system, composed of a soundboard, a cavity with five soundholes and 47 strings. When one string is plucked, other strings are excited via the soundboard of the instrument. This phenomenon, called sympathetic vibrations, involves many couplings between the soundboard and the strings. Because of these couplings, the assembly composed by the strings and the soundboard has several modes whose eigenfrequencies are very close one to another. As a consequence, each partial of the musical sound contains multiple components which cannot generally be separated by standard Fourier analysis. The existence of such multiple components in one partial leads to beatings in the musical sound or to non-uniform decrease of the partials [2]. The sensation caused by this phenomenon of sympathetic vibrations is sometimes described by the harp player as unpleasant and therefore, the harp maker would like to limit them.

The aim of this paper is to investigate this phenomenon both experimentally and theoretically. In the first part, an adequate experimental setup and an analysis method are carried out to identify frequency components present in one partial of the musical sound when one string is plucked. In the second part, the sympathetic vibrations are investigated by the use of a theoretical model of the concert harp. In the final part, a comparison between theoretical and experimental results allows us to explain the origin of the sympathetic phenomenon in the instrument.

EXPERIMENTAL INVESTIGATION OF THE HARP'S SYMPATHETIC MODES

Experimental setup

In order to experimentally highlight the presence of sympathetic modes in the concert harp, the following experimental protocol has been set up: the harp is plucked whereas an accelerometer measures the vibratory signal on the soundboard between the D_3-string and the C_3-string, respectively labeled 30 and 31. The played string is string 31 whereas the other strings are either damped or stopped during oscillations. The 4 studied experimental configurations are the following: (1) all strings free to vibrate, (2) all strings damped except string 31, (3) strings 31 and 38 stopped during oscillations and (4) string 31 and 42 stopped during oscillations. All measured signals are sampled to 4096 Hz and last 8 seconds. The string is stopped a few seconds after the plucking of string 31 by the harp player.

Fourier analysis

The analysis of the spectral content of the signal's first partial is performed using an appropriate filtering. The magnitude of its Fourier transform is presented in Figure 1, showing that several sinusoidal components are present in the signal. Indeed, since the system is in free oscillations, the response of the instrument to the plucking action, measured by the accelerometer, corresponds to the superposition of several modes whose frequencies are very close one to another. In the spectrum zoomed in the vicinity of the first partial, peaks of every component cannot be clearly separated. This shows the limit of the Fourier analysis. The presence of several vibration modes having close frequencies is a common characteristic for numerous free stringed instruments.

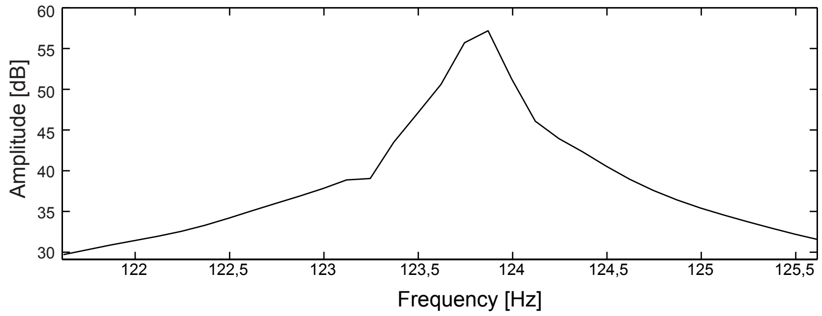

Figure 1.- Spectrum of the first partial of the vibratory signal measured on the soundboard when string 31 is plucked. All strings are free to vibrate.

Method for extracting parameters

Various methods, applied to musical instruments, allow the identification of spectral components, like methods based on the Hilbert transform [3] or high resolution methods [3],[4],[5]. For the identification of modes with close frequencies in the first partial, we choose to use a high resolution method: the ESPRIT method (Estimation of Signal Parameters via Rotational Invariance Techniques) [6]. In this method, the signal model $x[t]$ is supposed to be the sum of complex exponentials:

$$x[t]=\sum_{k=1}^{K} a_k e^{\delta_k t} e^{i(2\pi f_k t+\phi_k)}, \quad t\in[1,N] \qquad \text{(Eq. 1)}$$

where each frequency $\left(f_k \in \left[-\frac{1}{2},\frac{1}{2}\right]\right)$ is associated to a real magnitude $(a_k > 0)$, a phase $(\phi_k \in [-\pi,\pi])$ and a damping or amplification factor $(\delta_k \in \mathbb{R})$. The whole number K is the number of complex exponentials, also called model order, and N is the number of the signal's samples. The ESPRIT method consists in analyzing the signal measured by using a K-dimensional signal subspace and n-length data vectors with $n > K$. It can be shown that parameter n is ideal when $n = N/3$ or $n = 2N/3$ [5]. The different steps for determining all parameters of the signal model (Eq. 1) are detailed in [5] and [7].

The main difficulty of the method consists in the evaluation of the number K of components present in the signal. The technique commonly used is the over-estimation of this number and the discrimination of spurious results by mean of an indicator such as the components energy or the error between the measured signal and the model. Other more effective methods exist for estimating the model order K such as the ESTER method (ESTimation Error) [5],[8] also used in the study, consisting in the computation of an inverse error function.

In order to minimize the computation time and increase the results accuracy [5], the ESPRIT method implementation is carried out according to the following procedure: after the centering of the studied partial around the null frequency, a finite impulse response (FIR) filter selects the frequency range containing the partial to analyze. The filter is chosen with a linear-phase to keep the signal form. The filter is known to have a finite transitory response which corresponds to the length of its impulse response. We can avoid this problem by cancelling the filtered signal points belonging to its transitory phase [3]. The filtered and centered signal is highly decimated to limit the computational time of the ESPRIT method. After the estimation of the model order by the ESTER method, the ESPRIT algorithm is then applied. The final validation of the model order is performed using a comparison between measured and synthesized signals.

Results

The implemented ESPRIT method, as previously explained, is applied to the signal measured on the concert harp in its final part, between 4s and 8s. The estimated components found in the first partial are gathered in Table I. The model order stretches from 1, for configurations (3) and (4), to 4, for configuration (1). In Table I, the components are classified according to the energy type, from top to bottom, and are aligned in order to facilitate the reading of the missing components.

Thanks to the developed experimental protocol and the identification method, the components present in the first partial are obtained. The results show that the estimated parameters are stable following the four experiences. These results are compared in the following section to the modal basis of the simplified harp.

Table I.- Frequencies of identified components in the first partial of the accelerometer signals in the 4 configurations: (1) all free strings, (2) all strings damped except string 31, (3) strings 31 and 38 stopped during oscillations, (4) strings 31 and 42 stopped during oscillations.

	Configurations			
	(1)	(2)	(3)	(4)
Spectral Components [Hz]	123.59	123.53		
	123.35		123.28	
	123.08			123.08
	123.78	123.66		

THEORETICAL STUDY OF THE HARP'S SYMPATHETIC MODES

Model of the concert harp

The modal behavior of the concert harp in low frequencies has been studied in a previous paper [9]. The vibrations of the soundboard correspond to bending motions which are symmetric according to the strings. The soundboard vibrations can then be described using an equivalent beam clamped at both ends, one corresponding to the pillar and the other one corresponding to the location of string 11. The vibratory model of the concert harp is thus constituted of an equivalent beam connected to 35 strings as shown in Figure 2.

The mechanical properties of the equivalent beam and of each string have to be determined: for the strings, most parameters are directly measured on the harp [7] except for the Young modulus and the density which are supposed to equal the data given in [10] and [11]. Values of the tension are computed from the fundamental frequency of the tones, by considering strings fixed at both ends. For the equivalent beam, the geometrical parameters are directly measured on an isolated soundboard, allowing us to evaluate the density (ρ=553 Kg/m^3). The area of the cross section A and the second moment of area J are measured at different positions along the axis. The average value of these two parameters is thus retained to characterize the equivalent beam (A_{eq}=38.3 cm^2 and J_{eq}=38.9 cm^4). Finally, the Young modulus E of the equivalent beam is determined so that the first bending mode of the beam in the beam-35 strings assembly equals 150 Hz (E= 5.9 Gpa).

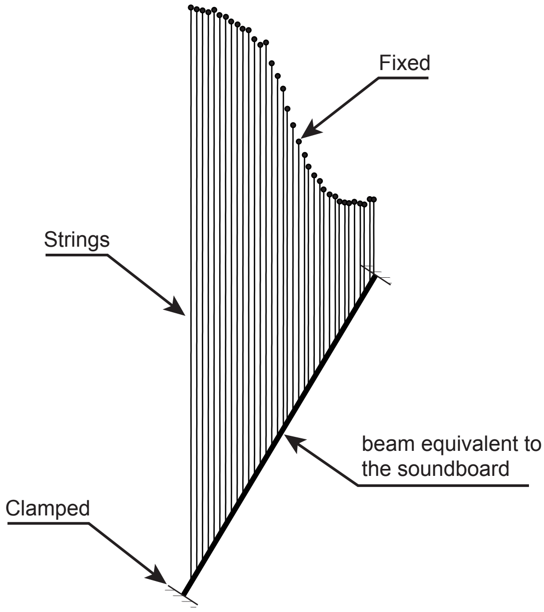

Figure 2.-Model of the concert harp: beam-35 strings assembly.

Modal basis

The modal basis of the simplified harp is computed using the transfer matrix method detailed in [1]. This computation is based on a wave guide model for each sub-structure of the beam-strings assembly. In the frequency range [0Hz-500Hz], 151 modes are obtained for the coupled system. Examples of modal shapes, associated to modes 26, 27 and 28 are presented in Figure 3. These modes are selected because their eigenfrequencies are close to the fundamental frequency of string 31, studied in the experimental section. In order to classify the modes of the assembly, a criterion called Kinetic Energy Ratio (KER) (see [1] for an exact definition) has been defined. It corresponds to the ratio of kinetic energy of one sub-structure divided by the total kinetic energy of the structure. Its value is a percentage and allows us to identify the relative importance of each sub-structure displacement field.

Table II.- Eigenfrequencies and Kinetic Energy Ratio for each sub-structure (strings 42, 38, 31 and the beam). KER expressed in % and rounded to the nearest whole number.

Mode label	Eigenfrequency [Hz]	KER for each string [%]			KER [%]
		String 42	String 38	String 31	Beam
26	122,95	0	99	1	0
27	123,29	97	1	2	0
28	123,48	1	2	97	0

In Table II, the eigenfrequencies and KER of each sub-structure of modes 26, 27 and 28 are reported. For these modes, the KER is significant only for three strings: 31, 38 and 42. String 31 (Cb_3 note of fundamental frequency 123.5 Hz) corresponds to the upper octave of string 38 (Cb_2 note at fundamental frequency 61.7 Hz) which is the upper fifth of string 42 (Fb_1 at fundamental frequency 41.2 Hz). The beam's KER equals zero, showing that mode shapes 26, 27 and 28 are dominated by the string's motion since the KER is distributed according to two or three strings, allowing the definition of these modes as string-string modes or sympathetic modes [1], because they are responsible for the sympathetic vibrations. Actually, by the modal superposition principle, if string 31 is plucked, modes 26 to 28 are set into vibration and so are strings 38 and 42, generating the phenomenon of sympathetic vibrations.

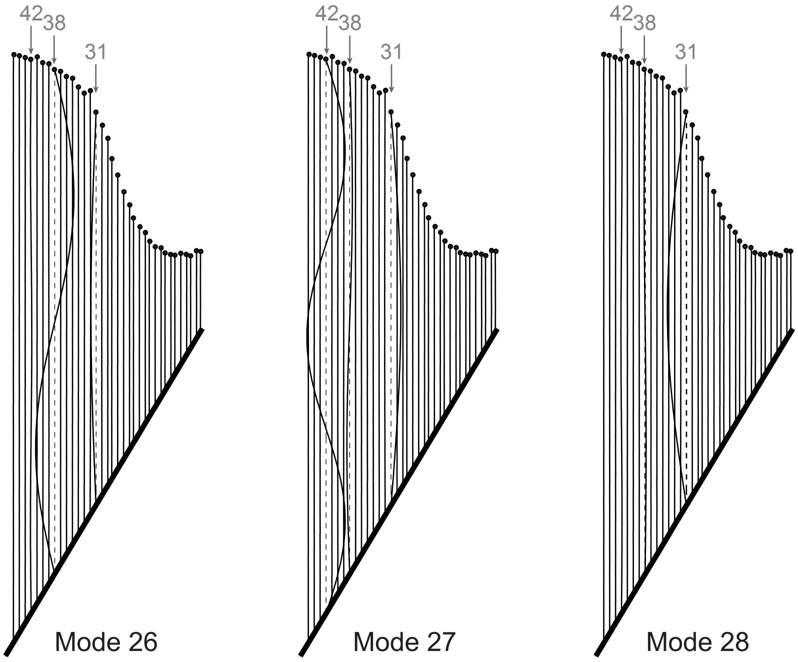

Figure 3.-Modal shape associated to modes 26, 27 and 28. The number indicated below the harp's arm points out the string number.

ANALYSIS OF THE SPECTRAL COMPONENTS OF THE FIRST PARTIAL

When string 31 of the concert harp is free to oscillate (configuration (1) in Table I), two sinusoidal components are identified in the first partial. When this string is stopped (configurations (2), (3) and (4)), these two components disappear. In playing configuration, these two sinusoids have close frequencies, separated by 0.2 Hz. This result thus points out that the string vibrates following its two polarizations. These two polarizations are excited by the harp player in a different way depending on the string plucking, involving different initial amplitudes (showed by the difference of energy of each component). Moreover, as for the piano, one of the polarizations seems to be favored for transmitting its energy to the soundboard implying a rapidly decaying component at the plucking moment [12]. This result shows that for a good modeling of the strings modal behavior, the two polarizations of string vibrations have to be taken into account. In the model of simplified harp, only one polarization is considered, which constitutes a limitation.

When the string is stopped during the oscillations, modes present in the instrument response are selected. By stopping strings 31 and 38, one more vibrating component disappears and by stopping strings 31 and 42, another one is absent from the instrument's response. With configurations (3) and (4), we can deduce that strings 38 and 42 participate in the sound radiated by the instrument. When the harpist plucks string 31, four modes are set into vibrations at the same time: two modes involving string 31 (one mode per polarization), one mode involving strings 31-38 and one mode involving strings 31-42. These last two modes are thus sympathetic modes. The stopping of string 31 does not necessary lead to the weakening of these modes, proving that the kinetic energy present in strings 42 and 38 is definitely more important than in string 31. Moreover, these results show that the sympathetic modes coming from each polarization of string 31 are not visible in the response. This fact can be explained by the weakening energies of these modal components, not allowing them to emerge from the noise.

These experimental results can be compared to those obtained from the theory. Experimentally, we found two modes which mostly involve string 31 at 123.59 Hz and at 123.78 Hz. With the vibratory model, the string mode associated to string 31 is found at 123.48 Hz. Note that the model takes only one string polarization into account. The agreement between model and measurement is thus very good. For sympathetic modes 31-42 and 31-38, their eigenfrequencies are measured at 123.28 Hz and at 123.08 Hz. It should be noticed that the theoretical eigenfrequencies of these modes are found at 123.29 Hz and at 122.95 Hz, coinciding almost perfectly with experimental results. Apart from the fact that the vibratory model of the concert harp does not take the two polarizations of the string into account, results obtained from the model are in very good agreement with those obtained during experiments, thus validating our approach.

CONCLUSION

In order to analyze the sympathetic vibrations phenomenon present in the concert harp, experimental and theoretical investigations have been carried out. It has been shown that sympathetic modes, responsible of the sympathetic vibrations, are present in the instrument response. For the identification of these modes, a high resolution method, called ESPRIT, has been used. It allows the analysis of the spectral components present in the first partial of the plucked string. The components are very close one to another and cannot be identified with a classical Fourier analysis. With a simplified model of the vibratory behavior of the concert harp, composed of a beam connected to several strings, these modes are also found. The comparison between experimental and theoretical results is very concordant showing that our approach for modeling the harp is adequate.

The principal restriction of the model is that only one polarization of each string is taken into account. An extension of that study will be to add a supplementary polarization per string. From an experimental point of view, a study of the repeatability of the results will be investigated. The analysis of partials of higher order, where other phenomena such as octave vibrations can be present, will also be performed.

References:

[1] J-L. Le Carrou, F. Gautier, N. Dauchez, J. Gilbert: Modelling of sympathetic string vibrations. Acta Acustica united with Acustica, **91** (2005) 277-288

[2] J-L. Le Carrou, F. Gautier, R. Badeau: Analyse des modes de cordes couplées d'une harpe par une méthode à haute resolution (Analysis of coupled modes of a harp by a high resolution method). In proceedings of Congrès Français d'Acoustique, Tours, France (2006) [In French]

[3] B. David: Acoustical characterization of vibrating structures in near vacuum. Estimation methods for eigenfrequencies and damping factors. Applications in musical acoustics. PhD thesis, Université de Paris 6, France (1999) [In French]

[4] J. Laroche: The use of the matrix pencil method for the spectrum analysis of musical signals. Journal of the Acoustical Society of America **94, No. 4** (1993) 1958–1965

[5] R. Badeau. High resolution methods for estimating and tracking modulated sinusoids. Application to music signals. PhD Thesis, GET-Télécom Paris (ENST), France (2005) [In French]

[6] R. Roy, A. Paulraj, T. Lailath: ESPRIT-A subspace rotation approach to estimation of parameters of cisoids in noise. IEEE Transactions on Acoustics, Speech and Signal processing **34, No. 4** (1986) 1340-1344

[7] J-L. Le Carrou: Vibroacoustics of the concert harp. PhD thesis, Université du Maine, France (2006)

[8] R. Badeau, B. David, G. Richard: A new perturbation analysis for signal enumeration in rotational invariance techniques. IEEE Transactions on Signal Processing **54, No. 2** (2006) 450-458

[9] J-L. Le Carrou, F. Gautier, E. Foltête: Experimental study of A0 and T1 modes of the concert harp. Journal of the Acoustical Society of America **121, No. 1** (2007) 559–567

[10] A.J. Bell, I.M. Firth: The physical properties of gut musical instrument strings. Acustica **60** (1986) 87-89

[11] A.J. Bell: An acoustical investigation of the concert harp. PhD thesis, University of St Andrews, United Kingdom (1987)

[12] G. Weinreich: Coupled piano strings. Journal of the Acoustical Society of America **162, No. 6** (1977) 1474–1484