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# Tag Second-preimage Attack against $\pi$ -cipher

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**Abstract.** The  $\pi$ -cipher is one of the candidates of the CAESAR competition. One of the advertised features of the  $\pi$ -cipher is tag second-preimage resistance: it should be hard to generate a message with a given tag, even for the legitimate key holder (insider attack).

In this note, we show that the generalized birthday attack of Wagner gives a practical tag second-preimage attack against the  $\pi$ -cipher.

## 1 Introduction

The  $\pi$ -cipher [2] is an authenticated encryption algorithm submitted to the CAESAR competition. One of the extra features advertised by the designers is tag second-preimage resistance: it should be hard to produce second-preimages of a given tag, even for an adversary who knows the secret key (most authenticated encryption algorithm do not have this feature, and an insider can easily generate tag second-preimages).

As written in [2, 4.1], the tag generation of an  $m$ -block message with the  $\pi$ -cipher can be written as:

$$T = T'' \boxplus_8 e(1, M_1) \boxplus_8 e(2, M_2) \boxplus_8 \cdots \boxplus_8 e(m, M_m)$$

where  $e$  denotes a keyed function known to the key holder (the e-triplex),  $\boxplus_8$  is a component-wise addition of vectors of 8 elements in  $\mathbb{Z}_{2^\omega}$ , and  $T''$  is the associated data tag (known to the insider). The word-size  $\omega$  is 16, 32, or 64, depending on the security level. In a tag second-preimage attack, an insider wants to build a message  $M$  reaching a fixed tag  $\bar{T}$ . Without loss of generality, we assume  $T'' = 0$  and  $\bar{T} = 0$ .

In the submission document of  $\pi$ -cipher, the tag second-preimage problem is seen as a knapsack problem, and the main attack considered is a variant of an attack by Camion and Patarin [1]. However, the generalization of this attack due to Wagner [3] can break the problem more efficiently.

## 2 Wagner's Generalized Birthday Attack

The generalized birthday attack of Wagner is an attack against the  $m$ -sum problem: given  $m$  lists  $L_1, L_2, \dots, L_m$  of  $n$ -bit words, one find values  $l_1 \in L_1, \dots, l_m \in L_m$  such that  $\bigoplus_{i=1}^m l_i = 0$ . If each list contains at least  $2^{n/m}$  elements there is a good probability that a solution exists, but the best known algorithm is a simple birthday attack in time and memory  $\tilde{O}(2^{n/2})$ . One would first build two lists  $L_A$  and  $L_B$  with all the sums of elements in  $L_1, \dots, L_{m/2}$  and  $L_{m/2+1}, \dots, L_m$  respectively, then sort  $L_A$  and  $L_B$ , and look for a match between the two lists ( $L_A$  and  $L_B$  contain  $2^{n/2}$  elements each).

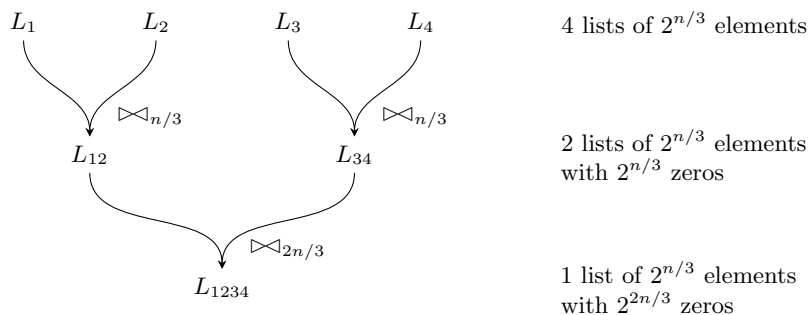
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Wagner's algorithm has a lower complexity, but it requires more elements in the lists. For instance, with  $m = 4$ , it uses lists of size  $2^{n/3}$  in order to find one solution using  $\tilde{O}(2^{n/3})$  time and memory. The basic operation of the algorithm is the general join  $\bowtie_{\tau}: L \bowtie_{\tau} L'$  consists of all the elements of  $L \times L'$  that agree on their  $\tau$  least significant bits. More precisely, the operation can be defined over list of values with associated data:

$$L \bowtie_{\tau} L' = \{(l \oplus l', (a, a')) \mid (l, a) \in L, (l', a') \in L', \text{low}_{\tau}(l \oplus l') = 0\}.$$

The join operation is computed efficiently by sorting the lists  $L$  and  $L'$  according to the lower  $\tau$  bits, and stepping through the lists simultaneously in order to find values that agree on their low bits. Moreover, the sorting can be done in linear time using a hash table, or a radix sort.



**Fig. 1.** Wagner's algorithm for  $m = 4$

The generalized birthday algorithm for  $m = 4$  is described by Figure 1. We first build the lists  $L_{12} = L_1 \bowtie_{n/3} L_2$  and  $L_{34} = L_3 \bowtie_{n/3} L_4$ , containing about  $2^{n/3}$  elements. Next, we build  $L_{1234} = L_{12} \bowtie_{2n/3} L_{34}$ . Since the elements of  $L_{12}$  and  $L_{34}$  already agree on their  $n/3$  lower bits, we are only matching bits  $n/3$  to  $2n/3$ , so we still expect to find  $2^{n/3}$  elements. Finally, we expect one of the elements of  $L_{1234}$  to be zero. This can be generalized to any  $m$  that is a power of two, using a binary tree: if  $m = 2^a$ , we need  $m$  lists of  $2^{n/(a+1)}$  elements and the time and memory used by the algorithm is  $2^a \cdot r 2^{n/(a+1)}$ . The algorithm for  $m = 8$  is shown by Figure 2.

### 3 Application to the $\pi$ -cipher

In order to apply this attack to the  $\pi$ -cipher, we need to solve the  $m$ -sum problem for the word-wise modular addition  $\boxplus_8$ , instead of the exclusive-or  $\oplus$ . Wagner showed how to solve the generalized birthday problem with a modular addition, and his trick also works for the word-wise modular addition. More precisely, we have to modify the join operator to:

$$L \blacktriangleright_{\tau} L' = \{(l \boxplus_8 l', (a, a')) \mid (l, a) \in L, (l', a') \in L', \text{low}_{\tau}(l \boxplus_8 l') = 0\}.$$

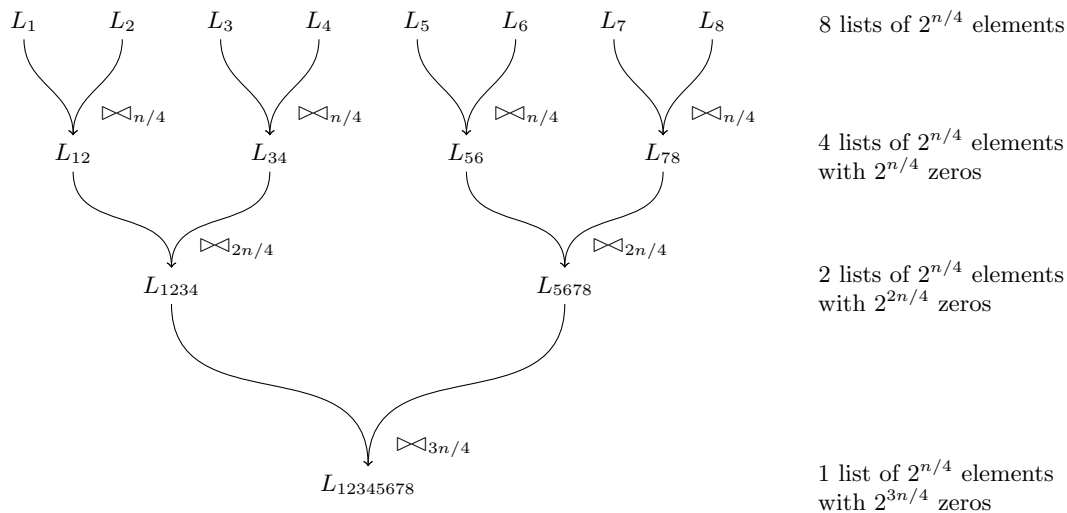
Since the word-wise modular addition  $\boxplus_8$  only has carries from the low order bits to the high order bits, when  $x$  and  $y$  have their  $\tau$  low-order bits set to zero,  $x \boxplus_8 y$

also has  $\tau$  low-order bits set to zero. Moreover, the join  $\bowtie$  can still be computed efficiently. We first negate the list  $L$  and define  $-L = \{(-l, a) \mid (l, a) \in L\}$ , where  $-l$  is the additive inverse with regard to the word-wise addition, *i.e.*  $l \boxplus_8 (-l) = 0$ . Then we sort  $-L$  and  $L'$  according to their lower  $\tau$  bits, and step through the lists in parallel. When an element of  $-L$  and an element of  $L'$  agree on their low bit, the corresponding sum will have its low bits equal to zero. Therefore, this variant of Wagner's algorithm is suitable for a tag second-preimage attack on the  $\pi$ -cipher.

We give a full description of an attack with  $\omega = 16$  in Algorithm 1; this attack uses 8 lists of size  $2^{32}$  (illustrated by Figure 2), *i.e.* we consider an 8-block message, with  $2^{32}$  possibilities for each block. This gives a complexity of  $2^{35}$ . More generally, we can apply Wagner's attack to different versions of  $\pi$ -cipher (*i.e.* with different values of  $\omega$ ), and several trade-offs between the message length and the attack complexity are possible. We give some parameters in Table 1.

**Table 1.** Attack parameters

$\omega$	Optimal parameters			Short messages		
	$m$	$ L $	Complexity	$m$	$ L $	Complexity
16	$2^{11}$	$2^{11}$	$2^{22}$	$2^3$	$2^{32}$	$2^{35}$
32	$2^{16}$	$2^{15}$	$2^{31}$	$2^7$	$2^{32}$	$2^{39}$
64	$2^{22}$	$2^{23}$	$2^{45}$	$2^{15}$	$2^{32}$	$2^{47}$



**Fig. 2.** Wagner's algorithm for  $m = 8$

## References

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2. Gligoroski, D., Mihajloska, H., Samardjiska, S., Jacobsen, H., El-Hadedy, M., Jensen, R.E.:  $\pi$ -Cipher. Submission to CAESAR. Available from: <http://competitions.cr.yu.to/round1/picipherv1.pdf> (v1) (March 2014)
3. Wagner, D.: A generalized birthday problem. In: Yung, M. (ed.) CRYPTO. Lecture Notes in Computer Science, vol. 2442, pp. 288–303. Springer (2002)

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**Algorithm 1** Short message attack with  $\omega = 16$  and  $m = 8$ .

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for  $0 \leq i < 8$  do
  for  $0 \leq j < 2^{32}$  do
     $L[i][j] \leftarrow (e(i, [j]), j)$ 
  end for
end for
 $L[8] \leftarrow \text{MERGE}(L[0], L[1], 32)$ 
 $L[9] \leftarrow \text{MERGE}(L[2], L[3], 32)$ 
 $L[10] \leftarrow \text{MERGE}(L[4], L[5], 32)$ 
 $L[11] \leftarrow \text{MERGE}(L[6], L[7], 32)$ 
 $L[12] \leftarrow \text{MERGE}(L[8], L[9], 64)$ 
 $L[13] \leftarrow \text{MERGE}(L[10], L[11], 64)$ 
 $L[14] \leftarrow \text{MERGE}(L[12], L[13], 96)$ 
for all  $(l, ((a_1, a_2), (a_3, a_4), ((a_5, a_6), (a_7, a_8)))) \in L[14]$  do
  if  $l = 0$  then
    return  $[a_1] \parallel [a_2] \parallel [a_3] \parallel [a_4] \parallel [a_5] \parallel [a_6] \parallel [a_7] \parallel [a_8]$ 
  end if
end for

function MERGE( $L, L', \tau$ )
  SORT( $L, -\text{low}_\tau$ )
  SORT( $L', \text{low}_\tau$ )
   $i \leftarrow 0$ 
   $j \leftarrow 0$ 
   $M \leftarrow \emptyset$ 
  while  $i < |L|$  and  $j < |L'|$  do
     $(l, a) \leftarrow L[i]$ 
     $(l', a') \leftarrow L'[j]$ 
    if  $\text{low}_\tau(-l) = \text{low}_\tau(l')$  then
       $M \leftarrow M \cup \{(l \boxplus_8 l', (a, a'))\}$ 
    else if  $\text{low}_\tau(-l) < \text{low}_\tau(l')$  then
       $i \leftarrow i + 1$ 
    else
       $j \leftarrow j + 1$ 
    end if
  end while
  return  $M$ 
end function
```

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