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# Comments on Finite-time Stability of Time-Delay Systems

Efimov D.<sup>1,2,4</sup>, Polyakov A.<sup>1,2</sup>, Fridman E.<sup>3</sup>,  
Perruquetti W.<sup>1,2</sup>, Richard J.-P.<sup>1,2</sup>

<sup>1</sup>*Non-A team @ INRIA - LNE, Parc Scientifique de la Haute Borne, 40 avenue Halley, 59650 Villeneuve d'Ascq, France*

<sup>2</sup>*LAGIS (UMR-CNRS 8146), Ecole Centrale de Lille, BP 48, Cité Scientifique, 59651 Villeneuve-d'Ascq, France*

<sup>3</sup>*School of Electrical Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel*

<sup>4</sup>*Department of Control Systems and Informatics, University ITMO, 49 av. Kronverkskiy, 197101 Saint Petersburg, Russia*

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## Abstract

Recently proposed conditions on finite-time stability in time-delay systems are revisited and it is shown that they are incorrect. General comments on possibility of finite-time convergence in time-delay systems and a necessary condition are given.

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## 1. Introduction

The time-delay dynamical systems attract a lot of attention in different areas of practice Chiasson and Loiseau [3], Erneux [6]. Analysis of stability in these systems is complicated Richard [16], the most of stability conditions deal with linear time-delay models, where the necessary and sufficient conditions have been obtained for some special cases Gu et al. [7], Hale [8], Kolmanovskiy and Nosov [11]. Prior to N.N. Krasovskii's papers on Lyapunov functionals and B.S. Razumikhin's papers on Lyapunov functions, L.E. El'sgol'ts (see El'sgol'ts and Norkin [5] and references therein) considered the stability problem of the solution  $x(t) \equiv 0$  of time-delay systems by proving that the function  $t \rightarrow V(x(t))$  is decreasing. Here  $V$  is some Lyapunov function. He showed that it is only possible in some rare special cases. Therefore, there are two generic methods for stability analysis in time-delay systems: Lyapunov-Krasovskii approach and Lyapunov-Razumikhin method. The former one is based on analysis of derivative for a functional, and it provides *qualitative* and *quantitative* estimates on

the system convergence. The latter approach is based on derivative analysis of a function and, from the point of view of the convergence rate, it gives mainly a *qualitative* conclusion (stability/instability of a time-delay system can be detected without estimation of the convergence rate, see Theorem 1 below).

It is frequently important to quantify the rate of convergence in the system: exponential, asymptotic, finite-time or fixed-time (see the results obtained for ordinary differential equations in Roxin [17], Dorato [4], Moulay and Perruquetti [13], Nersesov et al. [14], Polyakov [15]). Frequently, the homogeneity theory is used to evaluate finite-time or fixed-time stability in the delay-free case Bhat and Bernstein [2], Andrieu et al. [1], Polyakov [15]: for example, if a system is globally attractive and homogeneous of negative degree, then it is finite-time stable. There is a recent interest to analysis of finite-time stability behavior for time-delay systems Karafyllis [9], Moulay et al. [12], Yang and Wang [18, 19]. The paper Karafyllis [9] proposes design of a control, which implicitly contains some prediction mechanisms and time-varying gains in order to compensate the delay influence on the system dynamics and guarantee a kind of finite-time stability for the closed-loop system. The main result of Moulay et al. [12] is given in Proposition 2 below: in order to establish finite-time stability for a functional differential equation it is necessary to find a Lyapunov-Krasovskii functional  $V(\phi)$  whose derivative is upper bounded by a certain negative function of the functional  $V(\phi)$  itself (it is a more restrictive condition than in the conventional Lyapunov-Krasovskii approach, where a function of  $|\phi(0)|$  is required). Yang and Wang [18, 19] base their study on the Lyapunov-Razumikhin approach, they claim improvements over earlier results.

In this note, we argue that some key results in Yang and Wang [18, 19] are incorrect as stated, and we provide new insight on the features of finite-time stability for time-delay systems. The El'sgol'ts' arguments are recalled.

The outline of this work is as follows. The preliminary definitions and finite-time stability conditions for time-delay systems are given in Section 2. A counterexample to a key result in Yang and Wang [18, 19] is presented and discussed in Section 3. A necessary condition for finite-time stability for a class of time-delay systems and some supplementary comments are provided in Section 4. The result of Moulay et al. [12] is quoted in concluding Section 5.

## 2. Preliminaries

Consider an autonomous functional differential equation of the retarded type Kolmanovskiy and Nosov [11]:

$$dx(t)/dt = f(x_t), \quad t \geq 0, \quad (1)$$

where  $x \in \mathbb{R}^n$  and  $x_t \in C_{[-\tau,0]}$  is the state function,  $x_t(s) = x(t+s)$ ,  $-\tau \leq s \leq 0$  (we denote by  $C_{[-\tau,0]}$ ,  $0 < \tau < +\infty$  the Banach space of continuous functions  $\phi : [-\tau, 0] \rightarrow \mathbb{R}^n$  with the uniform norm  $\|\phi\| = \sup_{-\tau \leq \varsigma \leq 0} |\phi(\varsigma)|$ , where  $|\cdot|$  is the standard Euclidean norm);  $f : C_{[-\tau,0]} \rightarrow \mathbb{R}^n$  is a continuous function,  $f(0) = 0$ . The representation (1) includes pointwise or distributed retarded systems. We assume that (1) has a solution  $x(t, x_0)$  satisfying the initial condition  $x_0 \in C_{[-\tau,0]}$ , which is defined on some finite time interval  $[-\tau, T)$  (we will use the notation  $x(t)$  to reference  $x(t, x_0)$  if the origin of  $x_0$  is clear from the context).

The upper right-hand Dini derivative of a locally Lipschitz continuous functional  $V : C_{[-\tau,0]} \rightarrow \mathbb{R}_+$  along the system (1) solutions is defined as follows for any  $\phi \in C_{[-\tau,0]}$ :

$$D^+V(\phi) = \lim_{h \rightarrow 0^+} \sup \frac{1}{h} [V(\phi_h) - V(\phi)],$$

where  $\phi_h \in C_{[-\tau,0]}$  for  $0 < h < \tau$  is given by

$$\phi_h = \begin{cases} \phi(\theta + h), & \theta \in [-\tau, -h) \\ \phi(0) + f(\phi)(\theta + h), & \theta \in [-h, 0]. \end{cases}$$

For a locally Lipschitz continuous function  $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$  the upper directional Dini derivative is defined as follows:

$$D^+V[x_t(0)]f(x_t) = \lim_{h \rightarrow 0^+} \sup \frac{V[x_t(0) + hf(x_t)] - V[x_t(0)]}{h}.$$

A continuous function  $\sigma : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  belongs to the class  $\mathcal{K}$  if it is strictly increasing and  $\sigma(0) = 0$ ; it belongs to the class  $\mathcal{K}_\infty$  if it is also radially unbounded.

### 2.1. Stability definitions

Let  $\Omega$  be an open subset of  $C_{[-\tau,0]}$  containing 0.

**Definition 1.** Moulay et al. [12] The equilibrium  $x = 0$  of (1) is said to be

(a) stable if there is  $\sigma \in \mathcal{K}$  such that for any  $x_0 \in \Omega$ ,  $|x(t, x_0)| \leq \sigma(\|x_0\|)$  for all  $t \geq 0$ ;

(b) asymptotically stable if it is stable and  $\lim_{t \rightarrow +\infty} |x(t, x_0)| = 0$  for any  $x_0 \in \Omega$ ;

(c) finite-time stable if it is stable and for any  $x_0 \in \Omega$  there exists  $0 \leq T^{x_0} < +\infty$  such that  $x(t, x_0) = 0$  for all  $t \geq T^{x_0}$ . The functional  $T_0(x_0) = \inf\{T^{x_0} \geq 0 : x(t, x_0) = 0 \forall t \geq T^{x_0}\}$  is called the settling time of the system (1).

If  $\Omega = C_{[-\tau, 0]}$ , then the origin is called *globally* stable/asymptotically stable/finite-time stable.

For the forthcoming analysis we will need Lyapunov-Razumikhin theorem, which is given below (we have adapted to our case the formulation of Gu et al. [7], where time-dependent functional differential equations are considered).

**Theorem 1.** *Gu et al. [7] Let  $\alpha_1, \alpha_2 \in \mathcal{K}$  and  $\eta : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  be a continuous nondecreasing function. If there exists a Lipschitz continuous function  $V : \mathbb{R}^n \rightarrow \mathbb{R}$  such that*

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \quad \forall x \in \mathbb{R}^n$$

and the derivative of  $V$  along a solution  $x(t)$  of (1) satisfies

$$D^+V[x(t)]f(x_t) \leq -\eta(|x(t)|) \text{ if } V[x(t+\theta)] \leq V[x(t)] \quad (2)$$

for  $\theta \in [-\tau, 0]$ , then (1) is stable at the origin.

If, in addition,  $\eta \in \mathcal{K}$  and there exists a continuous nondecreasing function  $p(s) > s$  for  $s > 0$  such that the condition (2) is strengthened to

$$D^+V[x(t)]f(x_t) \leq -\eta(|x(t)|) \text{ if } V[x(t+\theta)] \leq p\{V[x(t)]\}$$

for  $\theta \in [-\tau, 0]$ , then (1) is asymptotically stable at the origin.

If in addition  $\alpha_1 \in \mathcal{K}_\infty$ , then (1) is globally asymptotically stable.

## 2.2. Sufficient conditions of finite-time stability in time-delay systems

The following is stated in Yang and Wang [19] as Lemma 1 (an extension of the Lyapunov-Razumikhin method for analysis of finite-time stability in (1)):

“Consider the system (1) with  $f(\phi) = F(\phi(0), \phi(-\tau))$ ,  $\phi \in \Omega$ ,  $F(0, 0) = 0$  and uniqueness of the solution in forward time. If there exist real numbers  $\beta > 1$ ,  $k > 0$ , a Class- $\mathcal{K}$  function  $\sigma$  and a differentiable Lyapunov function,  $V(x)$ , of the system (1) such that

$$\begin{aligned} \sigma(|x|) &\leq V(x), \\ \dot{V} &\leq -kV^{\beta-1}(x), \quad x \in \Omega \end{aligned} \quad (3)$$

hold along the trajectory of the system whenever  $V[x(t+\theta)] \leq V[x(t)]$  for  $\theta \in [-\tau, 0]$ , then the system (1) is finite-time stable. If  $\Omega = \mathbb{R}^n$  and  $\sigma$  is a Class- $\mathcal{K}_\infty$  function, then the origin is a globally finite-time stable equilibrium of the system

(1). Furthermore, the settling time of the system (1) with respect to the initial condition  $\phi \in C_\delta$  satisfies

$$T_0(\phi) \leq \frac{\beta}{k(\beta-1)} V^{\frac{\beta-1}{\beta}}[\phi]$$

for all  $t \geq 0$ .”

For completeness we are going to give the “proof” of this lemma from Yang and Wang [19]:

“Since  $V(x)$  is a Lyapunov function for the system (1), applying Razumikhin Theorem Gu et al. [7], it is easy to know that the system (1) is asymptotically stable under the conditions of the lemma. Next, we need to prove that  $T_0(\phi) < +\infty$ . Based on Condition for  $\dot{V}$ , one can obtain  $\int_{V(\phi)}^0 \frac{dz}{z^{1/\beta}} \leq -k \int_0^t d\tau$ , from which it follows that  $T_0(\phi) \leq \frac{\beta}{k(\beta-1)} V^{\frac{\beta-1}{\beta}}[\phi]$  for all  $t \geq 0$ . Thus, the proof is completed.”

In Yang and Wang [18] the same conclusion is obtained for the time-varying system (1) using a similar argumentation.

### 3. Comments on Yang and Wang [19]

We claim that Lemma 1 in Yang and Wang [19] is incorrect as stated. Indeed, as we can conclude from the result of Theorem 1, Lemma 1 in Yang and Wang [19] is based on the condition (2), which allows only stability (non asymptotic) to be concluded. However, even if we would ask the inequality (3) to be satisfied whenever  $V[\phi(\theta)] \leq p\{V[\phi(0)]\}$  for all  $\theta \in [-\tau, 0]$ , where  $p : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is a continuous nondecreasing function such that  $p(s) > s$  for all  $s > 0$ , then only asymptotic stability can be proven following Theorem 1. The claim about a finite-time convergence stays wrong since for a solution  $x(t, x_0)$  with initial condition  $x_0 \in \Omega$  the Lyapunov-Razumikhin condition (3) defines the rate of convergence of  $V$  only for the set of time instants

$$\mathbb{T}_{x_0}^+ = \left\{ t \in \mathbb{R}_+ : \sup_{\theta \in [-\tau, 0]} V[x(t+\theta, x_0)] \leq p\{V[x(t, x_0)]\} \right\}$$

while for  $t \in \mathbb{T}_{x_0}^-$  with

$$\mathbb{T}_{x_0}^- = \left\{ t \in \mathbb{R}_+ : \sup_{\theta \in [-\tau, 0]} V[x(t+\theta, x_0)] > p\{V[x(t, x_0)]\} \right\}$$

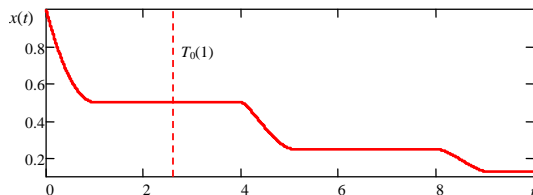


Figure 1: A trajectory of the counterexample (4)

there is no restriction on convergence rate of  $V$ . It is exactly the arguments used in the “proof” of Lemma 1 from Yang and Wang [19] given above: a finite-time rate of convergence is established for  $t \in \mathbb{T}_{x_0}^+$  only, which is clearly not sufficient since, for an illustration, if  $V[x(t)]$  is strictly decreasing then  $\mathbb{T}_{x_0}^- = [0, +\infty)$ .

For another illustration consider the following counterexample:

$$\dot{x}(t) = -|2x(t) - x(t - \tau)|^{0.5} \text{sign}[2x(t) - x(t - \tau)]. \quad (4)$$

Using  $V(x) = 0.5x^2$  we obtain

$$\dot{V} \leq -\sqrt{2 - \sqrt{2}}|x|^{1.5} = -kV^{\beta-1} \text{ if } V[x(t - \tau)] \leq 2V[x(t)]$$

for  $\beta = 4/3$  and  $k = \sqrt{2 - \sqrt{2}}2^{3/4}$ . Thus, by Theorem 1 the system is globally asymptotically stable, and from Yang and Wang [18, 19] since all conditions of Lemma 1 Yang and Wang [19] are also satisfied, one would conclude that the system is finite-time stable. Consequently, for the initial conditions  $x_0 \in C_{[-\tau, 0]}$  such that  $|x_0(0)| \leq 1$  the settling time function would possess the estimate:

$$T_0(x_0) \leq 3.$$

Take  $\tau > 3$  and for any  $\delta \in [-1, 1] \setminus \{0\}$  select initial conditions

$$x_0(s) = \begin{cases} 2\delta, & s \in [-\tau, -\tau + 3] \\ \frac{\delta}{3-\tau}s + \delta, & s \in (-\tau + 3, 0] \end{cases}.$$

Obviously,  $\dot{x}(t, x_0) = 0$  and  $x(t, x_0) = x_0(0) = \delta \neq 0$  for all  $t \in [0, T_0(x_0)]$ , therefore *the given settling time estimate is invalid*. In addition, the results of the system simulation for  $x_0(s) = 1$  for  $s \in [-\tau, 0]$  are shown in Fig. 1 for  $\tau = 4$  and the simulation step  $h = 10^{-5}$ . Clearly the system is not finite-time stable and the settling time estimate is wrong.

A peculiarity of the Lyapunov-Razumikhin conditions of stability is that they

are delay-independent (see Theorem 1), thus the estimate on the settling time  $T_0(x_0)$  obtained in Lemma 1 of Yang and Wang [19] is also delay independent. Consequently, it is possible to select delay value  $\tau$  for a given initial conditions  $x_0 \in C_{[-\tau,0]}$  such that  $T_0(x_0) < \tau$ , as it has been performed in the counterexample above. Obviously, convergence to zero independently on the part of initial conditions  $x_0(t)$  with  $t \in [T_0(x_0) - \tau, 0]$  is possible only under special restrictions on  $f$  in (1), which we are going to analyze in the next section.

#### 4. About necessary conditions for finite-time stability in time-delay systems

Assume that the system (1) is finite-time stable for some  $\tau > 0$  and the settling-time functional is continuous and  $T_0(0) = 0$ . Then there is  $x_0 \in C_{[-\tau,0]}$  such that  $T_0(x_0) \leq \tau$  and at the instant  $T_0(x_0)$  the right-hand side of (1) is still dependent on the initial conditions  $x_0$ . Thus, without additional assumptions on the right-hand side  $f(x_t)$  and its dependence on  $x_t(0)$ , or without skipping the continuity requirement of  $T_0(x_0)$ , an existence of finite-time stability phenomenon for time-delay systems is questionable.

In the remainder of this section we will consider

$$f(\phi) = F(\phi(0), \phi(-\tau))$$

for all  $\phi \in \Omega$ , where  $F : \mathbb{R}^{2n} \rightarrow \mathbb{R}^n$  is a continuous function. Our conjecture is that the condition  $F(0, z) = 0$  for any  $z \in \mathbb{R}^n$  is “necessary” for a finite-time convergence of (1) from  $\Omega$  to the origin.

*Remark 1.* We would like to stress that requirement on continuity of  $F$  is crucial for the consideration below. Indeed in discontinuous time-delay systems the finite-time stability may be easily observed without such a restriction on  $F$ . For example, it is a simple exercise to check that the system

$$\dot{x}(t) = -(1 + |x(t - \tau)|)\text{sign}[x(t)] + x(t - \tau)$$

is globally finite-time stable (take  $V(x) = 0.5x^2$ ,  $\dot{V} \leq -\sqrt{2V}$  and apply a variant of Proposition 2). In this discontinuous case  $0 \in F(0, z)$  for any  $z \in \mathbb{R}^n$ .

Note that by the definition of finite-time convergence  $x(t, x_0) = 0$  for all  $t \geq T_0(x_0)$  and by the definition of  $T_0(x_0)$  there is a non-empty set of time instants

$$\mathcal{T}_{x_0} = \{t \in [T_0(x_0) - \tau, T_0(x_0)] : x(t, x_0) \neq 0\}.$$



**Proposition 1.** *Let (1) be finite-time convergent in  $\Omega$ , then*

$$\forall t \in \mathcal{T}_{x_0} : F[0, x(t, x_0)] = 0 \quad (5)$$

for any  $x_0 \in \Omega$ .

*Proof.* Take  $x_0 \in \Omega$  and the corresponding settling time  $T_0(x_0)$ . Assume that the necessary condition (5) is not satisfied, its negation implies that for some  $x_0 \in \Omega$  there exists  $t' \in \mathcal{T}_{x_0}$  such that  $F(0, x(t', x_0)) \neq 0$  (the measure of  $\mathcal{T}_{x_0}$  is not zero since  $x(t)$  and  $F$  both are continuous), then  $\dot{x}(t' + \tau) \neq 0$  and  $x(t, x_0) \neq 0$  for some  $t \geq T_0(x_0)$  that is a contradiction.  $\square$

A simple, but not equivalent, way to check this condition in practice is to verify that

$$F(0, z) = 0 \quad (6)$$

for any  $z \in \mathbb{R}^n$ . In Kolmanovskii and Myshkis [10], a similar “necessary” condition has been indicated for the El’sgol’ts approach El’sgol’ts and Norkin [5]. Indeed, consider the case  $n = 1$ , take a Lyapunov function  $V(x) = 0.5x^2$ , which is a reasonable choice for the scalar case. We have  $\dot{V}(t) = x(t)F[x(t), x(t-\tau)]$ , if the inequality  $\dot{V}(t) \leq 0$  is satisfied around the origin, then we necessarily obtain (6).

In Yang and Wang [18, 19] the restriction  $F(0, 0) = 0$  has been imposed that is not sufficient.

## 5. Discussion

Finite-time stability can be presented in time-delay systems, but only under rather strong restrictions on the right-hand side of the system. The most general sufficient conditions are given in Moulay et al. [12]:

**Proposition 2.** *Moulay et al. [12] Let system (1) have unique solutions in forward time. If there exist a continuous functional  $V : \Omega \rightarrow \mathbb{R}_+$ ,  $\epsilon > 0$  and two functions  $\alpha, r$  of class  $\mathcal{K}$  such that  $\dot{z} = -r(z)$  has a flow,  $\int_0^\epsilon \frac{dz}{r(z)} < +\infty$  and for all  $\phi \in \Omega$*

$$\alpha(|\phi(0)|) \leq V(\phi), \quad D^+V(\phi) \leq -r[V(\phi)],$$

*then the system (1) is finite-time stable with a settling time functional  $T_0(\phi)$  satisfying the inequality:*

$$T_0(\phi) \leq \int_0^{V(\phi)} \frac{dz}{r(z)}.$$

The following example has been given in Moulay et al. [12] for any  $0 < \alpha < 1$ :

$$\dot{x}(t) = -|x(t)|^\alpha \text{sign}[x(t)]\{1 + x(t-\tau)^2\}. \quad (7)$$

Using the Lyapunov functional  $V(\phi) = 0.5\phi(0)^2$  with  $\dot{V}(\phi) \leq -2^{\frac{1+\alpha}{2}} V^{\frac{1+\alpha}{2}}(\phi)$  the finite-time stability has been established. Note that for this example the “necessary” condition (6) is satisfied.

The system (7) is an example, where the El’sgol’ts’ arguments can be applied ( $V(\phi) = 0.5\phi(0)^2$  is in fact a Lyapunov function). Note that the result of Lemma 1 in Yang and Wang [19] without the Razumikhin condition (“whenever...”) is correct and in this case it is also a special case of Proposition 2. In such a reformulation the result extends El’sgol’ts and Norkin [5] to finite-time stability, as well as the finite-time stability results of Roxin [17] to time-delay systems.

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