COUPLING 3-D NAVIER STOKES AND 1-D SHALLOW WATER

Mehdi Pierre DAOU*
ARTELIA Eau & Environnement, 6, rue de Lorraine,38130 Echirolles, France
mehdi-pierre.daou@arteliagroup.com

Eric BLAYO[1], Antoine ROUSSEAU[2], Olivier BERTRAND[3], Manel TAYACHI[1],Christophe COULET[4], Nicole GOUTAL[5]
1: Laboratoire Jean Kuntzmann, Universitée Joseph Fourier, Grenoble, France,
2: Equipe LEMON, Inria - UM2, 95 rue de la Gal´era, 34090 Montpellier, France,
3: ARTELIA Eau & Environnement, 6, rue de Lorraine,38130 Echirolles, France,
4: ARTELIA Eau & Environnement-Bordeaux, Parc Sextant bâtiment D-6-8 Avenue des satellites-CS70048 33187 Le Haillan Cedex, France ,
5: EDF R&D and Laboratoire d'hydraulique Saint Venant, Chatou, France

KEY WORDS

Coupling, Schwarz iterative method, 3-D Navier Stokes model to a 1-D shallow water model, Mascaret-Telemac system.

ABSTRACT

The present work addresses the problem of coupling hydrodynamical models with different spatial dimensions, which can be used in order to reduce the computational cost of river numerical models. We show that this problem can be tackled quite efficiently by designing a simple algorithm using techniques borrowed from domain decomposition theory. This algorithm is non intrusive, i.e. allows using existing numerical models with very few modifications. The method is illustrated on an academic test-case, namely a free surface flow in a bend-shaped channel. A 3-D Navier-Stokes model is coupled with a 1-D shallow water model, and results are compared to those obtained in a fully 3-D case. It is shown that the coupling algorithm provides an accurate solution, which can be improved thanks to an iterative algorithm (Schwarz method). This study is performed using the Mascaret-Telemac system.

1. INTRODUCTION

Fluid dynamics studies, either for industrial or geophysical flows, become more and more complex, multidisciplinary and require numerical modeling tools which must be efficient and interoperable. For numerous applications (impact studies or flood prevention for instance), it may be necessary to design a modeling system that couples processes representing different parts of the physical system. These models may differ in several ways, related either to the physics and/or to the numerics. For instance it is rather frequent that the spatial flow heterogeneity is such that simplified versions of the equations may be valid in some parts of the studied domain. In this context, the present work addresses the question of coupling models with heterogeneous physics and dimensions, in the particular context of river and coastal flows.
Following the theoretical issues addressed by some of the authors [6,7], this work is the first numerical implementation of such algorithms with operational tools. We will first present the coupled problem that will be specifically addressed in this paper, and propose a simple coupling algorithm. The performances of this coupling strategy will be assessed in a simple operational test case using the Open TELEMAC-MASCARET software suite. A generalization of the algorithm will then be presented and discussed, based on the so-called Schwarz domain decomposition method.

*Corresponding author

2. THE COUPLING PROBLEM

We will deal all along this paper with the coupling of a 3-D Navier-Stokes model (NS3D) with a 1-D shallow water model (SW1D). Such an interaction occurs for instance in river modeling systems, when coupling areas where the flow is stable and laminar on the one hand, with areas where the flow is much more turbulent and requires a 3-D representation on the other hand.

2.1 Reference and reduced models

We consider the reference model as the complete 3-D Navier-Stokes equations, which read as follows:

$$\begin{cases} \nabla \cdot U = 0 \\ \dfrac{\partial u}{\partial t} + U \cdot \nabla u = \dfrac{-1}{\rho}\dfrac{\partial p}{\partial x} + S_x + \nu \Delta u \\ \dfrac{\partial v}{\partial t} + U \cdot \nabla v = \dfrac{-1}{\rho}\dfrac{\partial p}{\partial y} + S_y + \nu \Delta v \\ \dfrac{\partial w}{\partial t} + U \cdot \nabla w = \dfrac{-1}{\rho}\dfrac{\partial p}{\partial z} + g + S_z + \nu \Delta w \end{cases} , \tag{1}$$

where the unknowns are the velocity field $U=(u,v,w)$ and the pressure p . The source terms are denoted S_x, S_y, S_z, g is the gravity acceleration, ν the viscosity coefficient, and ρ the fluid density.
The pressure can be written as the sum of the hydrostatic pressure p_{hyd} and the dynamic pressure p_{dyn}:

$$p = p_{atm} + \rho g (Z_s - z) , \tag{2}$$

where p_{atm} is the atmospheric pressure and Z_s is the free surface elevation.
The external boundary conditions are not given at this stage, since they are not actually part of the coupling problem. They depend on the study case and shall be introduced in Section 2.3.2..
In areas where the flow is stable, drastic simplifications can be made. Considering that the curvature of the flow trajectory as well as the bottom slope are weak, accounting for the fact that the vertical acceleration is negligible and that the pressure is almost hydrostatic, the use of the following (SW1D) model can be justified:

$$\begin{cases} \dfrac{\partial S}{\partial t} + \dfrac{\partial Q}{\partial x} = 0 \\ \dfrac{\partial Q}{\partial t} + \dfrac{\partial}{\partial x}\left(\dfrac{Q^2}{S}\right) = -g S \dfrac{\partial Z_s}{\partial x} - g \dfrac{Q^2}{S K_s^2 R_h^{4/3}} \end{cases} , \tag{3}$$

where the unknowns Q and S stand respectively for the flow rate and the wet cross section $S = Z_s \cdot L_y$ with L_y the river width. The friction and viscosity forces are taken into account through the last term, where K_s is the Strickler coefficient and R_h is the hydraulic radius.

2.2 Interface conditions

A key point now is to express the physical conditions that must be satisfied by the coupled solution at the interface between the two models. Natural conditions consist in preserving the continuity of both the flow rate and the water height through the interface, *i.e.* to satisfy the following conditions at every time t :

$$Q_{1D}(.,t) = Q_{3D}(.,t) \quad \text{and} \quad Z_{s1D}(.,t) = \overline{Z_{s3D}}(.,t) , \tag{4}$$

where $\overline{Z_{s3D}}$ denotes the averaged value of Z_{s3D} over the 2-D interface. Unfortunately, the coupled system made of equations (1) and (3), together with boundary conditions (4), is ill-posed (the 3-D part is clearly under-determined). One must then rather consider interface conditions between $X1=(Q,S)$ and $X3=(U,Zs)$ such that the coupled problem is well-posed and that its solution satisfies conditions (4). This is precisely what we will do in the following.

2.3 Test case description

We now introduce the academic case on which the numerical experiments are performed.

2.3.1 Geometry

The computational domain is a cross-shaped channel with trapezoidal section (see Figure 1). The channel is $0.79\mathrm{m}$ wide everywhere, and more than $10\mathrm{m}$ long. Four different interface positions are indicated: they will be considered in the following.

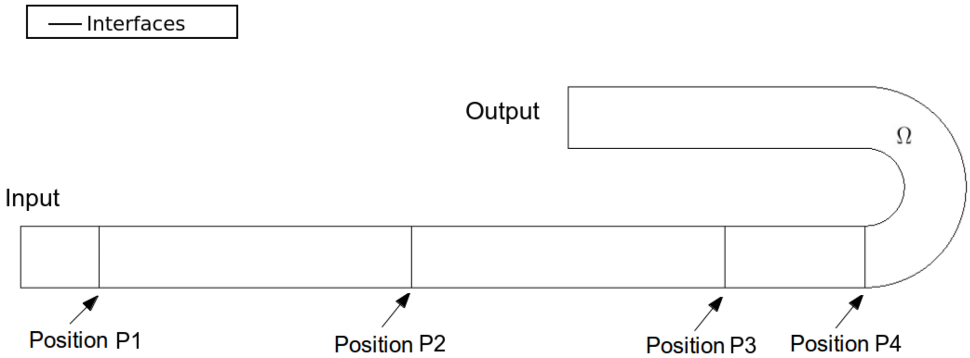


Figure 1: Schematic view of the channel, with indication of interface positions. The distances between the (left) input location and the 4 possible interface positions are respectively 1m (P1), 5m (P2), 9m (P3) and 10.8m (P4).

The reference simulation will be run in the whole 3-D domain Ω, where the 3-D Navier-Stokes (NS3D) equations are solved. The reduced model couples NS3D on a subdomain Ω_{3D} with a 1-D shallow water model (SW1D) on the 1-D restriction Ω_{1D} of $\Omega\backslash\Omega_{3D}$. The corresponding geometry is presented in Figure 2, and the choice for the location of the 1-D/3-D interface will be investigated below.

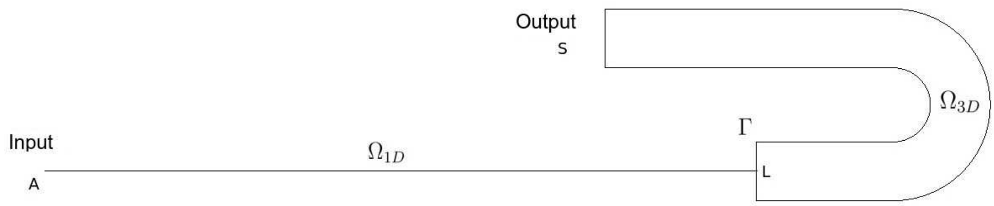


Figure 2: Schematic view of the computational domain for the coupled problem. Positions A, S and L correspond respectively to the input, output and interface locations.

Numerical computations are done thanks to modules of the open TELEMAC-MASCARET[1] integrated suite, dedicated to the simulation of free-surface flows. More specifically, we shall couple Telemac-3D (for the 3-D Navier-Stokes equations) and Mascaret (for the 1-D shallow water equations). The coupling algorithm will be designed thanks to Open-Palm[2]. Additional information on the equation implementation can be found in [1, 3] for Telemac-3D and in [2] for Mascaret.

2.3.2 Initial and boundary conditions

We describe here the initial and boundary conditions for the reference and coupled cases. In the sequel, we will consider a small viscosity parameter $\nu=10^{-6}$ (which is consistent with the inviscid shallow water approximation), and we assume that we have no bottom friction, $Ks\rightarrow+\infty$.

[1] Open TELEMAC-MASCARET is developed by "Laboratoire National d'Hydraulique et Environnement" (LNHE) d'EDF R&D.

[2] Open-Palm is a platform developed by CERFACS, thanks to the library CWIPI developed by ONERA DSNA/ELCI.

The **initial conditions** are $U=0$ (fluid at rest) and constant height $Z_s^0=0.114\text{m}$. Both reference and reduced models were run in two configurations: with a constant inflow or with a time-oscillating inflow rate. We only present the numerical results obtained in the latter case, which is numerically more challenging. The corresponding inflow rate is $Q_d=0.01372[1-0.2\cos(0.01\pi t)]$.

The boundary conditions for the full 3-D reference case read:

$$\begin{cases} U\cdot\vec{n}=0 \quad \text{no flow through solid boundaries (bottom and lateral)} \quad , & (5a)\\ U=U_d(Q_d) \quad \text{at the input} \quad , & (5b)\\ Z_s=Z_{sd} \quad \text{at the output} \quad , & (5c)\\ \dfrac{\partial Z_s}{\partial t}+U_h\cdot\nabla_h Z_s-w=0 \quad \text{at the free surface } z=Z_s(x,y,t) \quad , & (5d)\end{cases}$$

where $Z_{sd}=0.114\text{m}$ is the imposed outflow water depth, and U_d is a velocity field computed by TELEMAC from a given flow rate Q_d. The bottom friction terms are removed in order to have an almost one-directional flow in the channel straight part (no dependency wrt y and z variables). Finally, $U_h=(u,v)$ is the horizontal velocity and Δh is the horizontal gradient.
Regarding the 1-D/3-D coupled system, we have two sub-domains Ω_{1D} and Ω_{3D} (see Figure 2). The boundary conditions that supplement the 3-D part of the model are similar to equations (5), except (5b) which is removed and replaced by an inflow condition on the 1-D part of the domain Ω_{1D}, namely;

$$Q(A,t)=Q_d \quad \text{at} \quad x=A \quad \text{(see Figure 2)} \quad , \tag{6}$$

and Q_d is the imposed inflow rate (same as in Equation (5b)).

3. A FIRST EFFICIENT COUPLING METHOD

In this section, we consider a very simple method where the flow rate and the water height are exchanged at each time step.

3.1 Coupling algorithm

The corresponding algorithm is the following:

- **At** $t=0$: initialization
- <u>**Time loop**</u> While $t=n\Delta t\le t_{max}$:

$$\begin{cases}\mathcal{L}_{3D}(U_{3D}^{n+1})=F \text{ in } \Omega_{3D}\\ Z_{s3D}^{n+1}=Z_{sd} \text{ on S}\\ U_{3D}^{n+1}(L,y,z)=\mathcal{P}(Q_{1D}^n(L)) \text{ on L}\end{cases} \qquad \begin{cases}\mathcal{L}_{1D}(u_{1D}^{n+1})=\bar{F} \text{ in } \Omega_{1D}\\ Q_{1D}(A)^{n+1}=Q_d \text{ on A}\\ Z_{s1D}^{n+1}=\overline{Z_{s3D}}^{n+1} \text{ on L}\end{cases}$$

$n\leftarrow n+1$

End of time loop

Algorithm 1: Simple coupling algorithm

The interface condition provided by the 1-D model to the 3-D is a Dirichlet condition on the 3-D velocity, computed from the flow rate Q_{1D} with the assumption that the velocity is equally distributed over the interface. This is the role of the $\mathcal{P}$ operator. Note also that the time step index is not identical in the two interface conditions: the 3-D solution is computed using the 1-D solution at previous time step, while the 1-D solution is then computed using the 3-D solution at the same time step.

3.2. Numerical results

We now compare the numerical results obtained with the reference 3-D model (Figure 3) and the coupled model using Algorithm 1, in the case of a time-oscillating inflow rate. Even if both water height and velocities were investigated, for the sake of clarity we choose to present numerical results related to the water height only: indeed, velocity snapshots are really similar and do not bring any additional information.

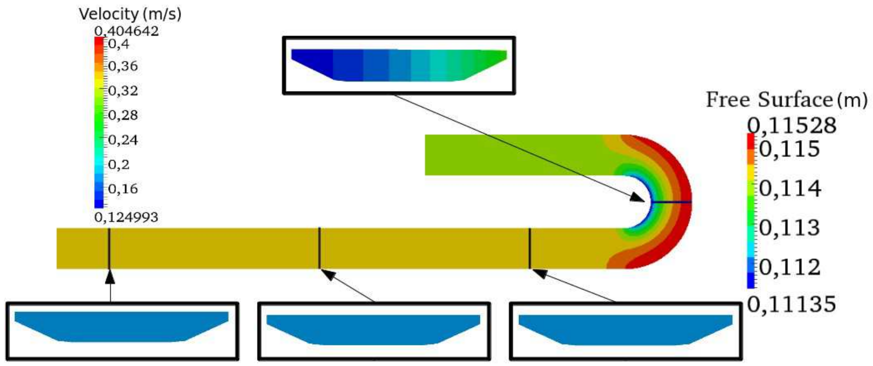
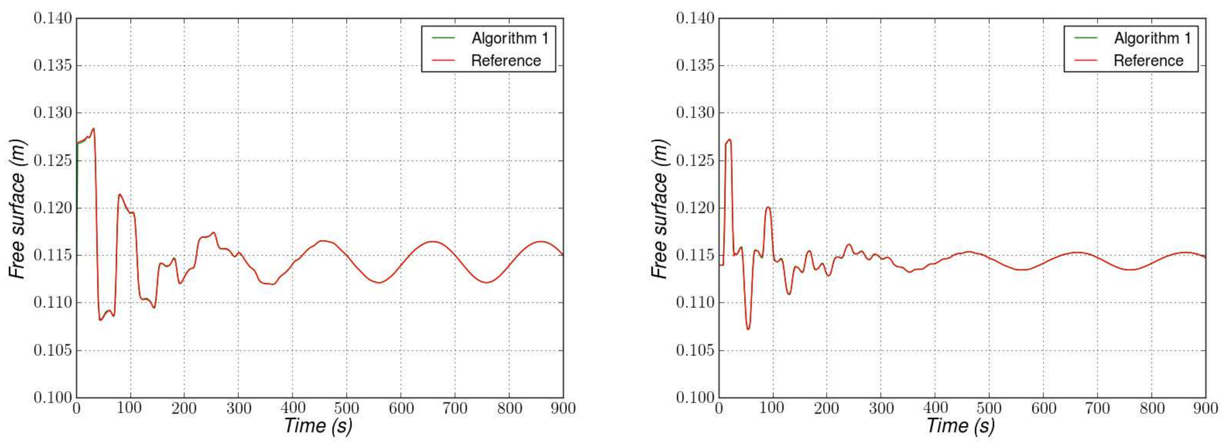


Figure 3: Reference solution: snapshot of the free surface elevation and velocity cross-sections

We present in Figure 4 the comparison between a water height computed thanks to the 3-D reference model (red curve) and to the 1-D/3-D reduced model (green curve). The numerical results are in very good accordance, even if some tiny differences can be observed locally, particularly during the transient state (t < 300s).



(a) Free surface evolution at position P 1. **(b)** Free surface evolution at position P 4.

Figure 4. Free surface evolution at two different locations. The 1-D/3-D interface is located at P 3 (see Figure 2). Both figures compare reference and reduced models.

4. ITERATIVE ALGORITHM (SCHWARZ METHOD)

When considering carefully the numerical solutions computed with the reference and reduced models, one can notice a slight difference. In particular, condition (4) is not exactly satisfied, and this could lead to numerical issues in long-time and/or stiff configurations. It is well known in the DDM[3] that the intuitive Algorithm 1 can be seen as the first iteration of the so-called Schwarz iterative algorithm, which improves the modeling accuracy and possibly reduces the prediction uncertainty (e.g. [4]). In this section, we will introduce this algorithm, and compare it to Algorithm 1 on the same test case .

4.1 . Coupling algorithm

The Schwarz method [5] was first proposed in 1870, but became popular only quite recently with the development of parallel computing. The main advantages of such a coupling method is its easy implementation, and its non-intrusiveness in existing codes (only boundary conditions routines are to be changed). Its main drawback can be its computational cost, in particular when too little attention is paid to interface conditions. We adapt this method to Algorithm 1 to obtain Algorithm 2, in which ε is a tolerance parameter for the convergence criteria, and kmax is the maximum number of iterations.

4.2 . Numerical results

As can be seen in Figure 5, allowing iterations thanks to Algorithm 2 brings some improvements, in particular in the first 300 seconds. However, it is clear from this same picture that Algorithm 1 is widely sufficient (and computationally preferable) in the permanent regime. Regarding the error evolution in one single time-step with respect to the number of Schwarz iterations (Figure 6), it is interesting to mention that the error decreases rapidly within the 5 first Schwarz iterations. It is no longer the case for $k > 10$, and the distance between two consecutive Schwarz iterates converges to the TELEMAC precision, specified in our case to 10^{-5} .

- **At** $t = 0$, initialization phase
- **Time loop** While $t = n\Delta t \le t_{max}$:

  $k = 0$; $err = +\infty$

  Schwarz loop While $err \ge \varepsilon$ and $k \le k_{max}$:

$$\begin{cases} \mathcal{L}_{3D}(U_{3D}^{n,k+1}) = F \text{ in } \Omega_{3D} \\ Z_{s3D}^{n,k+1} = Z_{sd} \text{ on S} \\ Z_{s3D}^{n,k+1}(L,y,z) = \mathcal{P}(Q_{1D}(L))^{n,k} \text{ on L} \end{cases} \qquad \begin{cases} \mathcal{L}_{1D}(U_{1D}^{n,k+1}) = \bar{F} \text{ in } \Omega_{1D} \\ Q_{1D}(A)^{n,k+1} = Q_d \text{ on A} \\ Z_{s1D}^{n,k+1} = (\overline{Z_{s3D}})^{n,k+1} \text{ on L} \end{cases}$$

  compute the convergence error err and iterate: $k \leftarrow k+1$

  End of Schwarz loop

  update variables: $X_{1D}^{n+1} := X_{1D}^{n,k}$, $X_{3D}^{n+1} := X_{3D}^{n,k}$

  $n \leftarrow n+1$

  End of time loop

Algorithm 2 : Schwarz based coupling algorithm

[3]Domain Decomposition Methods, see http://www.ddm.org.

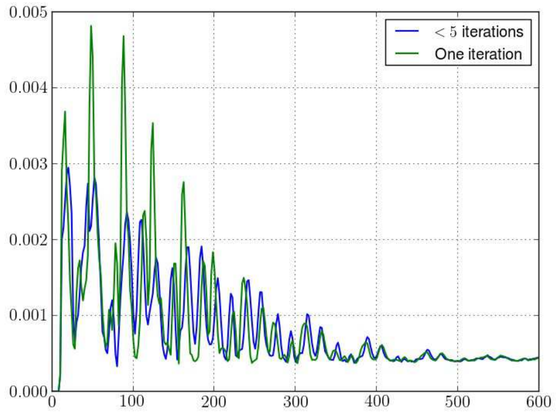


Figure 5 : Norm of the error on the free surface between the coupled solution and the reference solution in the 3D area, as a function of time .

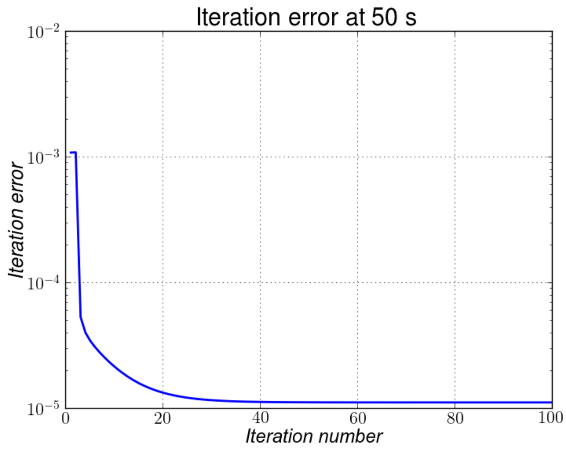


Figure 6 : $\left\|Z_s^{n,k+1} - Z_s^{n,k}\right\|$ as a function of k, the number of Schwarz iterations, at time $t = n\,\Delta t = 50$s

5 . CONCLUSION

The main objective of this paper was to propose algorithms to couple a 3-D Navier-Stokes model to a 1-D shallow water model in the context of river modeling. The performance of a simple intuitive coupling strategy was assessed in an idealized test case, using the operational Open TELEMAC-MASCARET software suite. In this very simple test case, this coupling algorithm leads to almost the same results than the reference full 3-D simulation. However it is shown that an even can be obtained by introducing an iterative coupling algorithm, based on the so-called Schwarz domain decomposition method. This approach is really of interest in more demanding applications, where the intuitive algorithm leads to less accurate results (e.g. [4]).

It is important to notice that those coupling strategies make sense only if the reference flow is almost one-dimensional in some part of the domain, and if the interface between the 1-D and the 3-D models is located within this area. We studied this aspect in the test case described previously, and obtained consistent results (the coupled solution corresponds to the reference solution if the interface L is located left from position P4).

The next step for this work will be to validate this iterative algorithm on a more complex test case, in the context of the EU CRISMA project. Since the coupled solution must of course be cheaper to obtain than the reference one (which means that only very few iterations must be performed), we will have then probably to use more complex interface conditions, in order to ensure a fast convergence. The OpenFoam software should also be used in this model coupling project.

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