Structural Health Monitoring of the Pitt River Bridge in British Columbia, Canada

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Abstract – **Vibration based damage detection of engineering structures has become an important issue for maintenance operations on transport infrastructure. Research in vibration based structural damage detection has been rapidly expanding from classic modal parameter estimation to modern operational monitoring. Methodologies from control engineering have been adopted and converted for the application on Civil Engineering structures. In this paper a statistical null space based damage detection algorithm is presented. This technique compares a current structural (possibly damaged) state to a reference state by a chi-squared (χ^2) test, once the test parameters are established in the reference state. This is an efficient way to detect changes in the modal parameters (natural frequencies, damping ratios, mode shapes) of a structure without actually computing them. The χ^2 test needs the ambient output-only vibration data of the structural state to be tested, a null space matrix determining the structural behavior in the reference state and the residual covariance, also determined in the reference state, and is a fully automated algorithm. The proposed method is tested on the Pitt River Bridge in Canada. The bridge opened on October 4, 2009. Since 2011 the bridge has been monitored as part of the British Columbia Smart Infrastructure Monitoring System (BCSIMS). The system has been performing daily measurements and the methods proposed in this paper have been tested using this data.**

I. INTRODUCTION

The British Columbia Ministry of Transportation (MoT) in Canada has been instrumenting structures in collaboration with the Earthquake Engineering Research Facility (EERF) at the University of British Columbia (UBC) since the late 1990's. The initial purpose of the program was to monitor the effects of seismic activity on instrumented bridges. In the last five years the instrumentation program has been accelerated to incorporate Structural Health Monitoring (SHM). Two design-build bridges have included instrumentation; one existing bridge has also been instrumented, and up to eight more bridges will be added by the end of 2014. Building on this collaboration, in 2009 the MoT and UBC embarked on a program called the British Columbia Smart Infrastructure Monitoring System (BCSIMS). The goals of BCSIMS are to: 1) Provide a real-time seismic structural response system to enable rapid deployment and prioritized inspections of the Ministry's structures; and 2) Develop and implement a health monitoring program to address the need for safe and cost-effective operation of structures in BC. Further details about the BCSIMS project are available in reference [1] and at www.bcsims.ca .

In order to achieve the second goal, UBC has been collaborating with engineers and researchers from Structural Vibration Solutions (SVS) in Aalborg, Denmark, and the Institut Nationale de Recherche en Informatique et en Automatique (Inria) in Rennes, France on the development and implementation of vibration-based techniques for system identification and damage detection of instrumented bridges. As part of this collaboration, the group was able to secure funding from the Marie Curie Action Industry-Academia Partnership and Pathways – within the Specific Programme People – of the EU Seventh Framework Programme for Research and Technological Development in 2009 to develop and implement a project called Internet-Based Structural Health Monitoring (ISMS), which allowed the group to develop advanced techniques for SHM. The objectives of the ISMS project are:

1. Address the significant commercial opportunity and rapidly emerging technological potential of improved Damage Detection or Structural Health Monitoring (SHM) technologies for large-scale civil infrastructure.
2. Design and development of a fully automated internet based damage detection procedure robust to environmental changes with application to fully instrumented large-scale civil infrastructures, primarily bridges.
3. Create a progressive damage monitoring system for damage assessment enabling the development of an internet-based SHM system to monitor hundreds of bridges autonomously.

This paper presents one of the outcomes of this project related to damage detection using vibration-based approaches and shows how the proposed method has been implemented as part of the monitoring activities of one of the bridges in the BCSIMS project. The theory behind the proposed damage detection method is presented first in a very succinct form. After

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this, a description of the bridge used as a case study is presented. This is followed by the presentation and discussion of results obtained from the implementation of the proposed approach.

II. SUBSPACE-BASED DAMAGE DETECTION

Damage detection can be performed by detecting changes in the modal parameters between a reference state and the current state of a structure from measured output-only vibration data [2, 3, and 4]. A possible way of doing this is by implementing a subspace-based damage detection test to detect changes in the modal parameters, but without the need to actually estimate the modal parameters themselves. The test can run in an automated way directly on the vibration measurements without "human intervention" so it is very suitable for near-real-time SHM. In this approach, a data-driven model obtained in the reference state is compared to data from the possibly damaged state using a subspace-based residual function and a χ^2-test built on it for a hypothesis test, without actually estimating the modal parameters in the tested, possibly damaged states. A χ^2-test is any statistical hypothesis test in which the sampling distribution of the test statistic is a chi-squared distribution when the null hypothesis is true, so this type of test is a very suitable way to determine if changes have happened to the structure being monitored, or to the monitoring system itself.

One major advantage of this approach is that the computations in the tested (possibly damaged) state are fully automated. The computed χ^2-value is compared to a threshold to decide if damage (or a change to the monitoring system) has occurred or not. The subspace-based damage detection method developed in [5-6] has been recently implemented for the SHM of one of the bridges in the BCSIMS program. A brief review of the background behind this method is presented below.

A. Linear Time Invariant Systems:

The dynamic behavior of a mechanical system assumed to be linear and stationary can be described by the following system of differential equations:

$$\mathcal{M}\ddot{X}(t)+\mathcal{C}\dot{X}(t)+\mathcal{K}X(t)=\upsilon(t) \tag{1}$$

in which t denotes a continuous time, $M, C, K \in \mathbb{R}^{d\times d}$ are the mass, damping and stiffness matrices, respectively, and $X \in \mathbb{R}^{d}$ is the vector of displacements of the d degrees of freedom (DOF) of the system. The external and non-measured force $\upsilon(t)$ is assumed to have white noise characteristics.

The discrete-time state space formulation for this system at discrete times is given by:

$$\begin{cases} x_{k+1} = Ax_k + v_k \\ y_k = Cx_k + w_k \end{cases} \tag{2}$$

In this well-known equation, at the k-th data sample: $x_k \in \mathbb{R}^{n}$ is the state vector, $y_k \in \mathbb{R}^{r}$ is the outputs vector, $A \in \mathbb{R}^{nxn}$ is the state transition matrix, and $C \in \mathbb{R}^{rxn}$ is the observation matrix. The number of sensors is denoted by the variable r and n is the order of the system. The excitation ν_k is an unmeasured Gaussian white noise vector with zero mean and constant covariance matrix, and w_k is the measurement noise.

B. Identifying System Matrices:

The A and C matrices can be found from the multiple measured response data by the construction of the following block Hankel matrix populated with the cross-covariance of the states and the outputs:

$$\mathcal{H}_{p+1,q} \stackrel{\text{def}}{=} \begin{bmatrix} R_1 & R_2 & \dots & R_q \\ R_2 & R_3 & \dots & R_{q+1} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p+1} & R_{p+2} & \dots & R_{p+q} \end{bmatrix} \stackrel{\text{def}}{=} \text{Hank}\,(R_i) \tag{3}$$

The parameters p and q are chosen such that $min\{pr,qr\} \geq n$ so the size of the matrix above is $(p+1)r$ x qr. Often, $p+1 = q$. Furthermore, this matrix has the well-known factorization property

$$\mathcal{H}_{p+1,q} = \mathcal{O}_{p+1}\,\mathcal{C}_q \tag{4}$$

in which the obserevability and controllability matrices are given as

$$O_{p+1} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^p \end{bmatrix}, \quad \mathcal{C}_q = \begin{bmatrix} G & AG & \dots & A^{q-1}G \end{bmatrix} \tag{5}$$

From the observability matrix O_{p+1}, the matrices C and A can be recovered and subsequently the modal properties of the system. But recognizing that damage to the system results in changes to A and C and, subsequently to the Hankel matrix in (3) through the properties (4) and (5), one could then detect this change by a statistical test, rather than performing a system identification first in order to determine this change. And this is the basis for the proposed damage identification method discussed here.

C. Identifying Damage

By making use of the N measured data values, y_k, a consistent estimate of the Hankel matrix in (3) can be obtained from the covariance of the output as

$$\widehat{R}_i = \frac{1}{N} \sum_{k=1}^{N} y_k y_{k-i}^T, \quad \widehat{\mathcal{H}}_{p+1,q} = \text{Hank}\left(\widehat{R}_i\right). \tag{6}$$

Let $\widehat{\mathcal{H}}^{\text{ref}}_{p+1,q}$ be the averaged Hankel matrix in the reference state. The Singular Value Decomposition (SVD) can be used to compute its left null space $S = \widehat{U}_0$

$$\widehat{\mathcal{H}}^{\text{ref}}_{p+1,q} = \begin{bmatrix} \widehat{U}_1 & \widehat{U}_0 \end{bmatrix} \begin{bmatrix} \widehat{\Delta}_1 & 0 \\ 0 & \widehat{\Delta}_0 \end{bmatrix} \begin{bmatrix} \widehat{V}_1^T \\ \widehat{V}_0^T \end{bmatrix} \tag{7}$$

where Δ_1 is of size n x n and where $\widehat{\Delta}_0 \approx 0$

The characteristic property of the reference state is expressed as the product of the null space S and the averaged Hankel matrix in (7) such that

$$S^T \widehat{\mathcal{H}}_{p+1,q} \approx 0 \tag{8}$$

But note that in a damage state, this product deviates from 0. So, in order to determine whether or not the measured data is within the domain of the reference state, a residual vector can be computed and used to perform this evaluation. The residual vector, denoted as ζ, is computed as:

$$\zeta = \sqrt{N} \text{vec}\left(S^T \widehat{\mathcal{H}}_{p+1,q}\right) \tag{9}$$

The χ^2 test is used to determine how different from zero is the value of ζ as computed using (9). That is,

$$\chi^2_\zeta = \zeta^T \Sigma_\zeta^{-1} \zeta \tag{10}$$

In this equation, the empirical residual covariance, $\sum_\zeta = cov(\zeta)$, is computed using several measurements from the reference state. In order to determine if damage (or a change to the system) has occurred or not, the test value χ^2 is compared to a threshold, which can also obtained from χ^2 test values from several measurements corresponding to the reference state. This approach allows to process the measured data in a much more efficient and direct way.

A variant of this approach that is robust with respect to significant changes in the ambient excitation levels has been recently presented in [7]. This variation makes use of the relationship between the Hankel matrix in (3) and the matrix of its principal left singular vectors in (7) share the same left null space S. Thus, another characteristic property of the reference state can be written in a similar form as in (8) as

$$S^T \widehat{U}_1 \approx 0, \tag{11}$$

It should be noted that while the Hankel matrix in (3) is dependent on the ambient excitation characteristics, the matrix of singular vectors $\hat{U}_1$ is a matrix with orthonormal columns and thus it can be regarded as independent of the excitation properties. Since $\hat{U}_1$ is independent of the excitation, a robust residual vector is defined as

$$\xi = \sqrt{N} \text{vec}\left(S^T \widehat{U}_1\right) \tag{12}$$

The χ^2 test computed as per (10) is then used to determine how different from zero is the value of ζ as computed using (12). In the remainder of the paper it is shown has this method been implemented for the SHM of an existing bridge in Canada.

III. PITT RIVER BRIDGE

The Pitt River Bridge is a cable-stayed bridge connecting Pitt Meadows and Maple Ridge in British Columbia, Canada. The bridge opened to traffic in October, 2009. The bridge has a single span, with three 60m concrete towers on either side of the 190 m main span. An overview of the bridge and an elevation schematic are shown in Fig 1. A photo showing the east towers is shown in this figure.

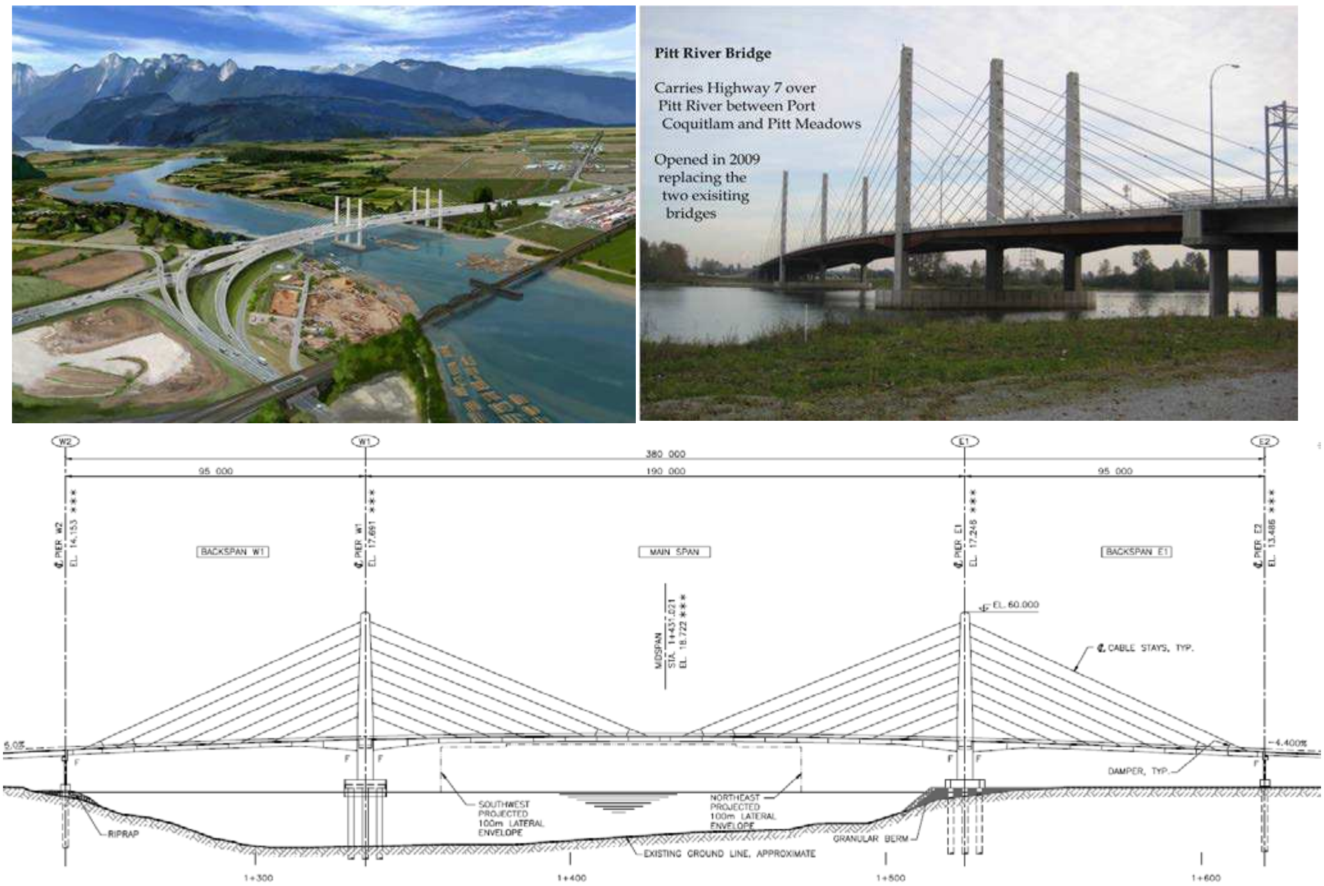

Figure 1. Overview and Schematic elevation of Pitt River Bridge[4]

The structural health monitoring system for Pitt River Bridge is part of the Gateway Program which was established by the Province of British Columbia in 2003 to improve the transportation throughout Metro Vancouver. The Pitt River Bridge Project includes a new bridge to replace the older swing bridges. A permanent vibration monitoring system has been installed on the bridge, with accelerometers mounted on selected locations on cables, towers and deck. The Pitt River Bridge SHM system collects data from 18 accelerometers mounted on critical positions of the structure and 1 wind sensor. In total 46 channels of data are collected by two data recorders and passed to a server for triggering, notification and providing data access. A diagram showing the locations and orientation of the accelerometers installed at selected locations of the deck and piers of the bridge, and a sample of an accelerometer attached to a stay cable are shown in Fig. 2 below. The accelerometers mounted on six selected stay cables are not shown in the diagram, as well as a triaxial accelerometer installed in a 10 m downhole at the east end of the bridge.

The data acquisition system was installed by Weir-Jones & Terrascience Systems Ltd of Vancouver in Canada. The daily monitoring activities, data processing and notifications are the responsibility of UBC as part of the BCSIMS program. The system dumps raw measurement files in a network folder on a daily basis. An example of a typical 30 minute record from three of the accelerometers is shown in Fig. 3 below. The sampling rate is 250 samples per second.

[4] Left photo courtesy of the Government of British Columbia, Gateway Program

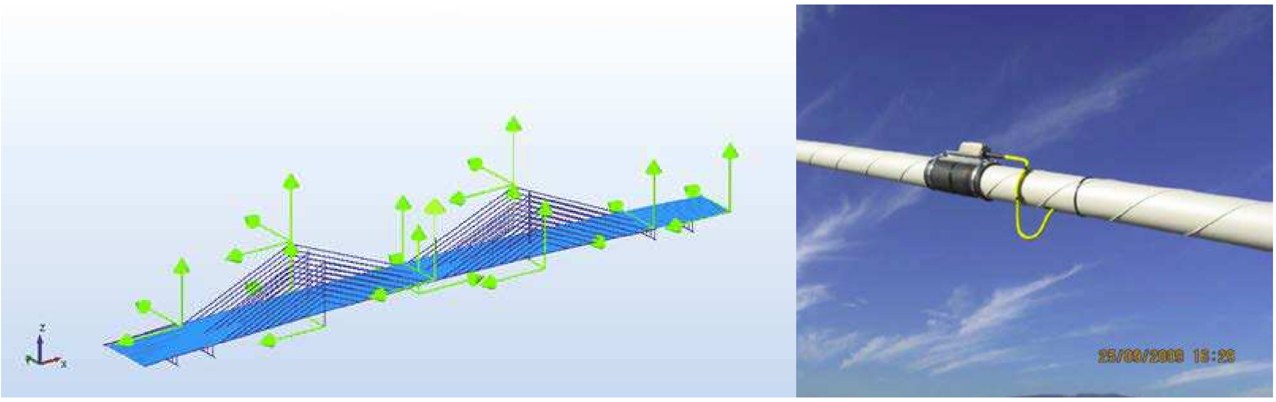

Figure 2. View of the Pitt River Bridge and Schematics of the Sensor Locations

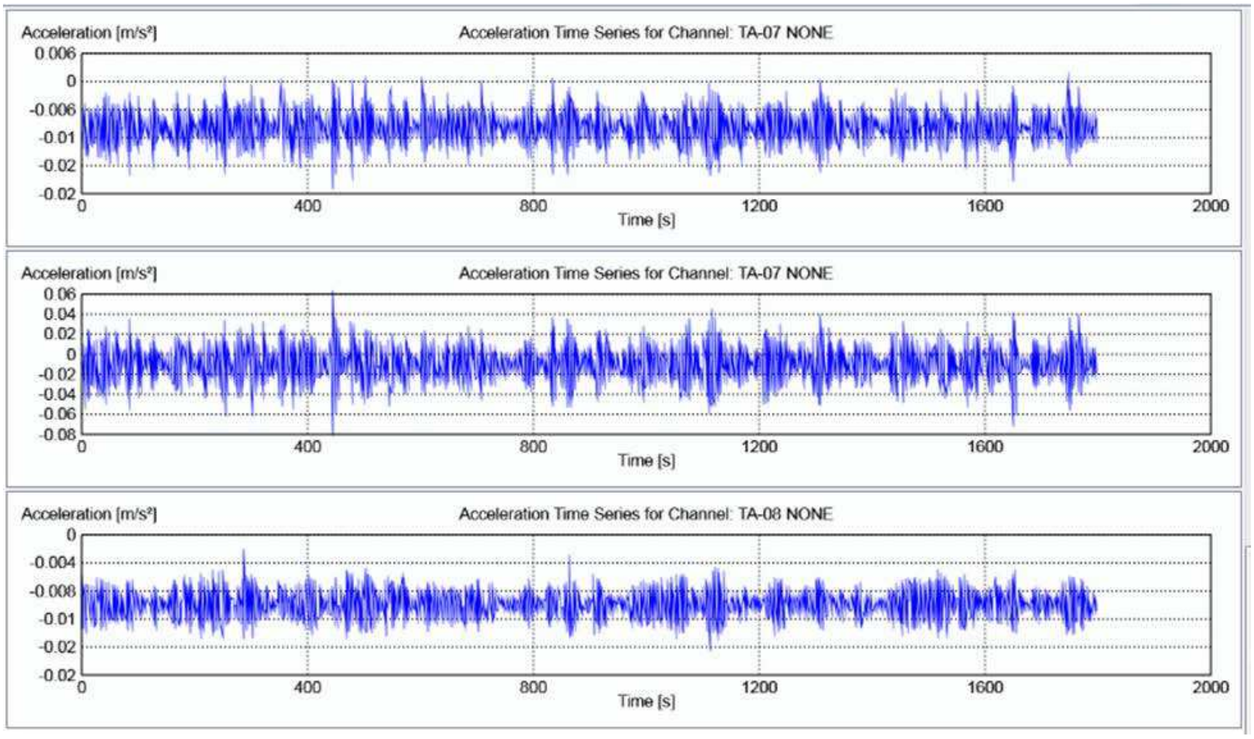

Figure 3. Example of a 30 minute long record from 3 selected channels of the Pitt River Bridge instrumentation.

Analysis of the recorded motions at the different locations of the bridge allows determination of the natural frequencies that dominate each of the main components of the bridge. For instance, the dominant frequencies of the cables with instrumentation varies between 0.7 and 0.9 Hz, the frequency of the towers is around 1.2 Hz and the vertical frequency of the deck is around 1.2 Hz. Vibration tests conducted at selected cables (see [8]) showed that the fundamental vibration frequency of the cables varied from 0.9 Hz for the long cables to 3.2 Hz for the short cables. The frequency range of interest for this bridge is from DC to about 25 Hz as illustrated in the spectral density plots of the significant singular values of this bridge, as shown in Fig. 4. So all the analyses being conducted are concentrated in this frequency range.

A detailed finite element model (FEM) of the bridge using a commercially available program is also available as part of this project, and this model has been used for complementary studies to better understand the dynamic behavior of the bridge. Of significant concern in British Columbia is the seismic activity of the Vancouver region, so the model has also been used estimate the seismic response of the bridge under different levels of ground shaking. A detailed ambient vibration test of the bridge is going to be conducted in the summer of 2014 in order to use the results of these tests to update the FEM and gain more confidence on the reliability of the FEM to estimate the actual bridge response to strong ground shaking or sever wind excitation.

As part of the SHM of the Pitt River Bridge, the authors having been using the recorded data at the bridge since 2011 to evaluate various damage detection algorithms, including the one described above in Section II. The following section describes the results obtained so far and the lessons learned from the analysis and interpretation of the results.

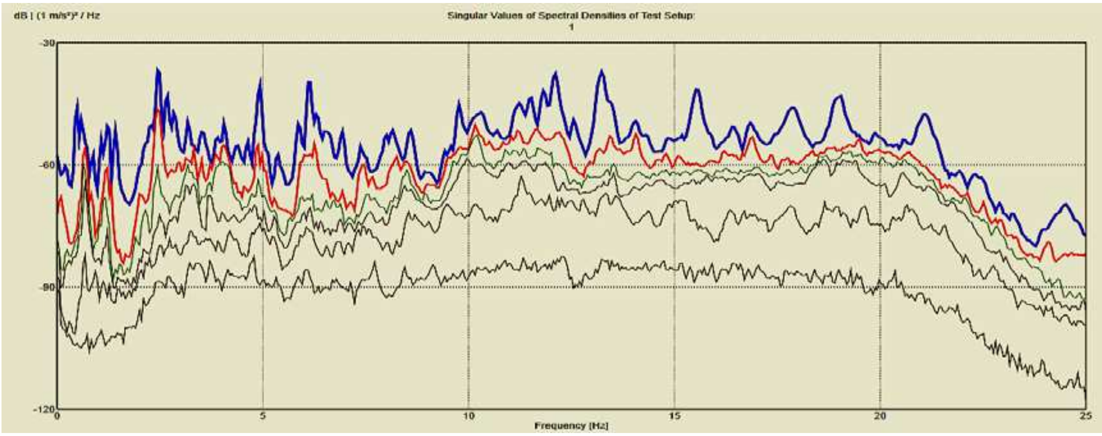

Figure 4. Singular values of the spectral densities of all the sensors installed at the Pitt River Bridge.

IV. DATA ANALYSIS AND DAMAGE DETECTION

The computer program ARTeMIS Modal Pro [8] is used for processing the data and implementation of the damage detection algorithm described above. The Data Acquisition plugin of ARTeMIS is used to automatically upload the measurement files obtained at a predefined daily schedule. The uploaded measurements are then inserted as a new Analysis Session and the Damage Detection is performed automatically as soon as the data has become available. The testing of a new measurement takes about a minute, and the result is broadcasted via calls to a web service that updates the www.bcsims.ca web page. The notification is an integrated part of the Damage Detection plugin. The detection procedure that has been implemented consists of two parts: a) setting up a reference state; and b) testing for potential damage every time a new dataset is obtained.

A. Setting up a reference state

This is accomplished by implementing the following steps:

1. Create a new ARTeMIS project from using the information typically used to analyze ambient vibration data (e.g. a CFG file used by ARTeMIS), and upload a dataset of “undamaged” conditions of the bridge.
2. Upload additional sets of measurement files acquired in a reference "undamaged" state of the structure.
3. Estimate a Reference State Model as an average of the dynamic behavior of the uploaded measurement files.
4. Test all individual measurement files of the reference state with the Reference State Model using the Chi-Squares test approach described in Section II above. The outcome is a single Chi-Square value per measurement, as per equation (10).
5. Use these values to establish thresholds for Chi-Square values being inside the predefined undamaged Safe Zone or in the Critical Zone.

B. Testing for potential damage

This is accomplished by implementing the following steps:

1. Upload measurement files of the potentially damaged structure and test against the Reference State Model using equation (10).
2. If the estimated Chi-Square values exceed the thresholds, it indicates a significant change of the dynamics of the structure being measured, and as such a potential damage (or an anomalous situation detected by the monitoring system).
3. Review the data in detail and determine if the need of a detailed inspection of the structure is warranted.

A typical output of the analysis data collected over time that may indicate damage to a structure looks as the information presented by the Control Chart in Fig. 5. Although the results in this figure are for a different structure, they serve to illustrate how damage, or a significant change in the normal operation of the monitoring system, can be reported. Each bar in this figure represents the results of the analysis of a dataset from a number of sensors. The amplitude of each bar is nothing more than the numerical value obtained from equation (10). The set of green bars on the left side are the results of implementing the reference state. The red horizontal bar indicates the threshold level that has been deemed to be an acceptable transition between a safe state of the structure and an unsafe state of the structure. A 99% confidence level has been used in the case of the Pitt River Bridge to determine this threshold level. The set of red bars in the figure below illustrate the case when subsequent set of measurements consistently result in values that exceed the threshold level and this is why they are shown in a different color. In this case a detailed inspection of the structure is warranted. If damage localization tools are also available, these should be used to determine the possible location and extend of damage, so that the bridge inspectors can have a better idea of where to look for damage.

It should be noted that if only a few measurements result in values that exceed the threshold levels, but subsequent measurements lead to results that are below the threshold levels (normal conditions), then these could be related to anomalous conditions detected by the SHM system. In such case, a detailed investigation of the SHM should be conducted and it should be determined if the channels of the system being used for the estimation of the damage indicators are the appropriate ones. This situation is illustrated below.

Figure 5. Example of damage control chart based on proposed subspace damage detection method.

C. Analysis of Pitt River Bridge data

The figure below (Fig. 6) shows the Control Chart of the results of application of the damage detection method described in this paper for a selected number of sensor installed at the Pitt River Bridge. The data analyzed is from the early 2012 until April 2014. As it can be seen in the chart below, there have been some instances where the threshold levels have been exceeded, and this has prompted an automatic notification to selected people through the BCSIMS project. However, it has been determined that these "exceedances" have been related to instrumentation issues (i.e., sensor malfunctions), rather than structural performance issues.

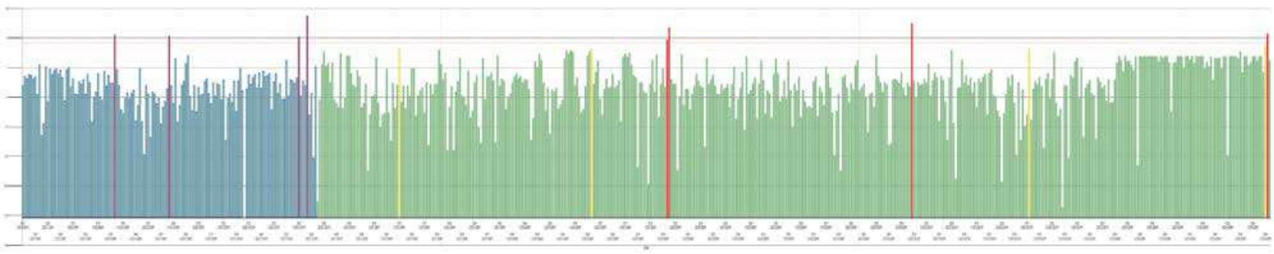

Figure 6. Control chart for Pitt River Bridge from 2012 to 2014.

The reference model for this particular structure has been based on daily measurements in order to include the natural dynamic variations of the structure due to e.g. temperature changes etc. The reference measurements are indicated as the gray background in the left part of the control chart. A selected portion of the chart above is presented in Fig. 7 for the months of April and May of 2014, in which more details of the results of the daily evaluations can be seen. It can also be seen on the right of the figure the different threshold levels that are being used to monitor the bridge performance. The significance level for the critical condition of the bridge has been selected to be 99%, while the corresponding one for what is defined as transition zone ("warning" level) is 95%. The corresponding χ^2 value for the threshold level is 6.87.

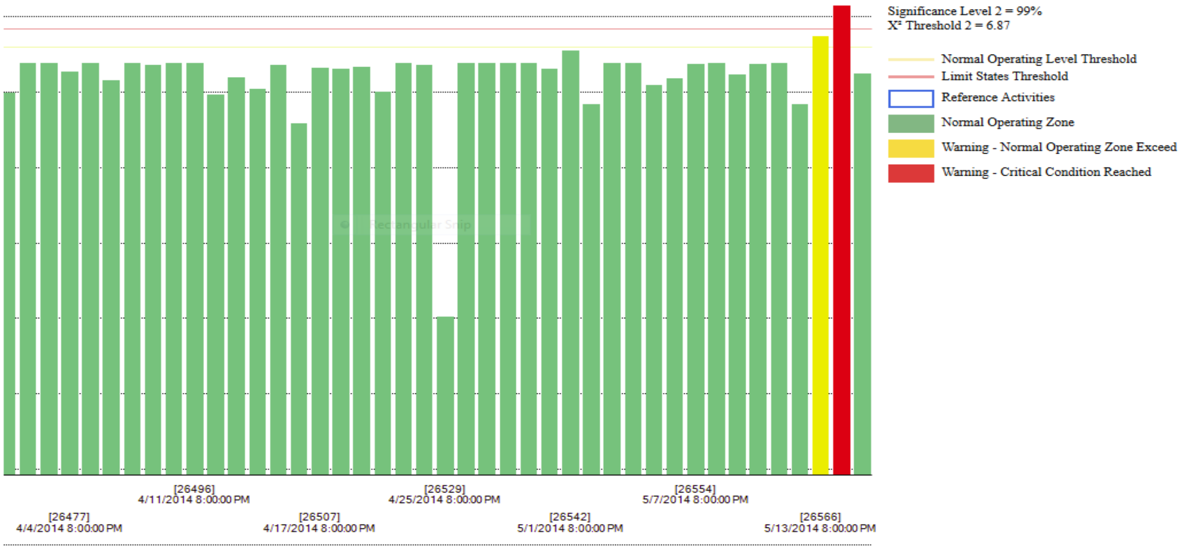

Figure 7. Detail of Control chart for Pitt River Bridge from April to May 2014.

As it can be seen in the figure above, only one measurement has reached the warning zone, and one measurement has exceeded the critical zone, but the follow up measurement shows a normal condition of the system. This is attributed to anomalous records obtained from one of the sensors attached to a stay cable.

It should be mentioned that through the development and implementation of the proposed damage detection methodology, all the data channels from the acquisition system at the bridge were used first. The results showed a significant number of excursions to above the threshold levels over the monitoring period of time. However, no damage reports for the bridge have been received from the bridge inspectors for this same monitoring period. A closer look at the implementation of the methodology revealed that these excursions were attributed to only a few sensors that consistently showed different measurements. Three of these channels were the ones corresponding to the downhole instrument installed 10 m below the ground to measure potential soil-structure interaction effects during an earthquake, but not designed to be integral part of the SHM for damage detection and localization. In addition to this, three data channels from sensors attached to stay cables showed signs of faulty connections. Therefore, all these data channels were removed from the suite of data sets, and the remaining dataset was reanalyzed using the procedure described above. The results presented in Figs. 6 and 7 correspond to this revised dataset. This clearly indicates that the selection of the data channels to be used for developing the control charts is a critical step in this process, and that this selection has to be based on the specific intended use of each measuring instrument.

V. SUMMARY AND FUTURE WORK

In this paper, a recently developed subspace-damage detection method has been used to monitor the behavior of an instrumented bridge in British Columbia, Canada. The method is now integral part of the BCSIMS bridge monitoring program and will be implemented shortly for the other structures being monitored as part of this program. The main benefits of the approach used to monitor instrumented structures using the proposed damage detection technique are

- No explicit modal parameter estimation needed.
- All modes in the frequency range of interest are tested for changes.
- A reference (baseline) model is established from multiple measurements making it statistically robust with respect to changes of the environmental conditions at the site.

- Measurements of a potential damaged state are tested against the reference model using a robust chi-squared ($\chi2$) test.
- Automatic estimation of thresholds to determine a zone of performance (safe-zone, critical-zone or unsafe-zone) can be readily conducted, and visually indicators of performance can be easily implemented.
- Due to the implicit inclusion of all the modes in the test, the damage evaluation is comprehensive and can help detect even small changes in the dynamic properties of the structure.

One significant advantage of the proposed approach is that two or more damage detection algorithms can be considered at the same time and the damage indicators from each of these methods can be unified into a to a single indicator using a Control Chart. This approach is currently being incorporated as part of the BCSIMS monitoring program. In addition to this, complementary studies by the authors on damage localization are presently underway and will eventually be incorporated as part of the SHM of the bridges and others structures that are part of the BCSIMS.

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