A generator of random convex polygons in a disc

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Introduction

- $\mathcal{D}$ a disc in $\mathbb{R}^2$ with radius 1 centered at o
- $(x_1, \dots, x_n)$ a sample of n points uniformly i.i.d. in $\mathcal{D}$
- P_n the convex hull of $(x_1, \dots, x_n)$
- $f_0(P_n)$ its number of vertices.

Then [2] :

$$\mathbb{E} f_0(P_n) = c\, n^{\frac{1}{3}} + o\left(n^{\frac{1}{3}}\right)$$

where $c > 0$ is constant.

- Generation of such a polygon:
  - Explicitly generate n points uniformly in $\mathcal{D}$.
  - Compute the convex hull.
- Faster?

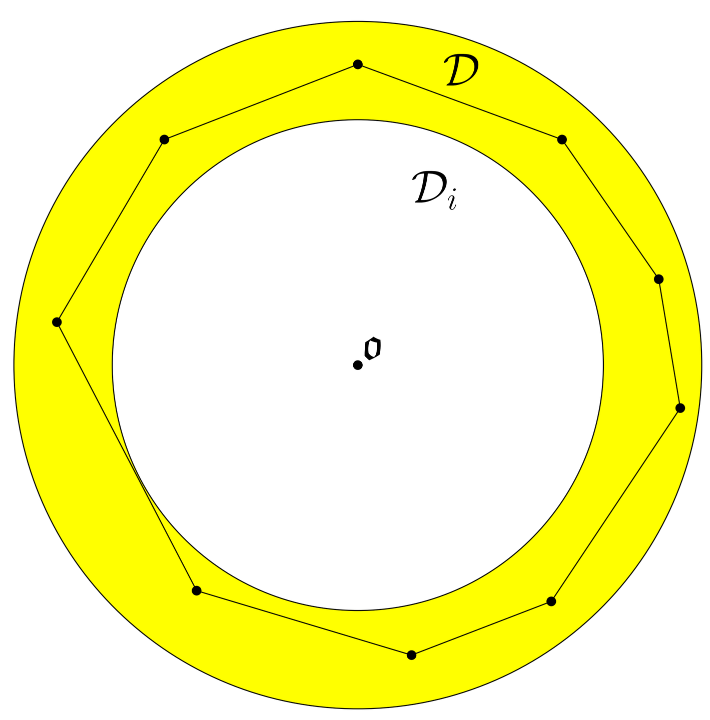
Result

- We propose an algorithm that **reduces the number of points generated**
- The expected number of points generated is only $O(n^{\frac{1}{3}} \log^2 n)$.

Outline of the Algorithm

Idea:

- Given the convex hull of small number of points
- The number of points deeply inside $\mathcal{D}$ can be computed without explicit generation.
- We **generate only the points that are close to the convex hull**.



Algorithm

- First, generate a small number of points in $\mathcal{D}$ and compute its convex hull
- While the number of points is $< n$ do
  - Compute the largest disc centered at o inscribed in the convex hull
  - Choose a number of points to simulate at this step
  - Simulate the number of points that falls in this inscribed disc at this step
  - Generate the rest of the points in the annulus defined by these two discs
  - Update the convex hull

Initialization

- Generate a small number of points in $\mathcal{D}$, such that o is in the convex hull.
- The probability that $o \notin P_n$ decreases exponentially.
- This initialization can be done in constant expected time.

Simulation of Points

- The polygon P_{m_i} is the convex hull of m_i points at step i.
- s_i is the number of new points simulated at step i.

$$s_i = \begin{cases} m_i & \text{if } m_i < n \log^{-2} n \\ n \log^{-2} n & \text{otherwise} \end{cases}$$

- $\mathcal{D}_i$ is be the largest inscribed disc in P_{m_i} centered in o, and r_i its radius.
- The number of points that falls in $\mathcal{D} \setminus \mathcal{D}_i$: Binomial variable $B(s_i, 1 - r_i^2)$.
- The points in $\mathcal{D}_i$ won't change the convex hull: avoid to generate them.
- We generate the rest of the points uniformly in the annulus $\mathcal{D} \setminus \mathcal{D}_i$.
- We update the convex hull using a Graham scan.

Size and Time Complexities

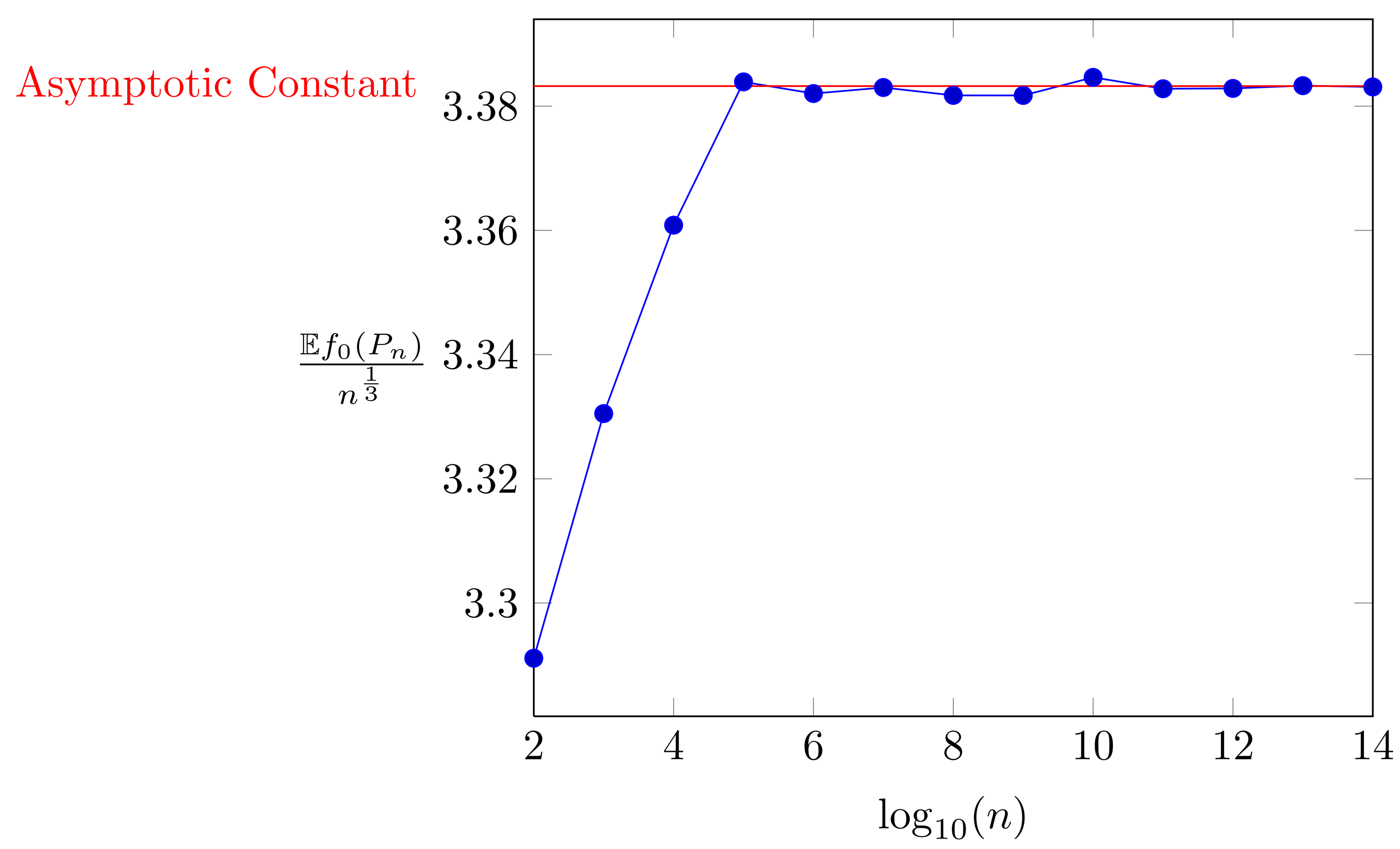
- At a given step i, the expected area of $\mathcal{D} \setminus \mathcal{D}_i$ is $O(m_i^{-\frac{2}{3}} \log^{\frac{2}{3}} m_i)$.
- The number of points generated is the size of the hull up to a factor $\log^2 n$.
- The expected memory needed is $O(n^{\frac{1}{3}})$ [1].

Trade-off:

- Increase the expected memory by a logarithmic factor.
- The expected time complexity is reduced from $O(n^{\frac{1}{3}} \log^2 n)$ to $O(n^{\frac{1}{3}} \log^{\frac{2}{3}} n)$.

Experiments

- Implementation in C++.
- Submitted for integration in the CGAL library.



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