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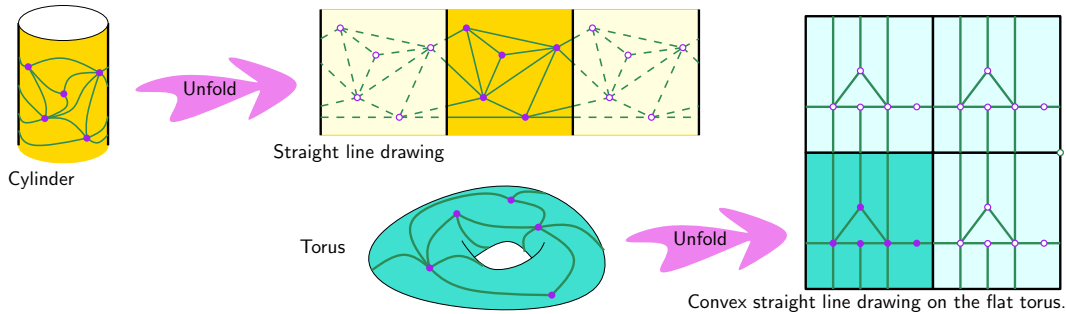
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# Crossing-free straight-line drawing of graphs on the flat torus\*

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## 1 Introduction

The problem of efficiently computing straight-line drawings of planar graphs has attracted a lot of attention in the past. In this paper we took interest on *straight line drawings* for graphs drawn on the cylinder or on the torus. More precisely, we look at straight line drawing of unfolded periodic representations of the cylinder and the torus (see Figure).

## 2 Main Result

**Theorem 1** For each essentially 3-connected<sup>1</sup> toroidal map  $G$ , one can compute in linear time a weakly convex crossing-free straight-line drawing of  $G$  on a periodic regular grid  $\mathbb{Z}/w\mathbb{Z} \times \mathbb{Z}/h\mathbb{Z}$ , where —with  $n$  the number of vertices and  $c$  the number of edges of the shortest non contractible loop—  $w \leq 2n$  and  $h \leq 1 + 2n(c + 1)$ . Since  $c \leq \sqrt{2n}$ , the grid area is  $O(n^{5/2})$ .

## 3 Sketch of Algorithm

We extend the incremental straight-line drawing algorithm of de Fraysseix, Pach and Pollack [2] (in the triangulated case) and of Kant [3] (in the 3-connected case).

The case of triangulations on a cylinder with the following properties: — no chords on the boundary, — no loops, — and no 2-cycles can be solved generalizing de Fraysseix *et al.* algorithm [2]. It needs to find a good *shelling* order to add the vertices and draw incrementally the triangulation preserving a good shape of its boundary. Allowing chords, loops, and 2-cycles is possible by a careful subdivision of the triangulation in pieces and gluing the result of the simple case. Generalizing to the torus can be done by cutting the graph along a short non contractible cycle. Adaptation from triangulation to maps is similar to the planar case [3].

## References

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- [2] H. de Fraysseix, J. Pach and R. Pollack. How to draw a planar graph on a grid. *Combinatorica*, 10(1):41–51, 1990. doi:10.1007/BF02122694.
- [3] G. Kant. Drawing planar graphs using the canonical ordering. *Algorithmica*, 16(1):4–32, 1996. doi:10.1007/BF02086606.

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<sup>1</sup> 3-connected in the periodic drawing,