Crossing-free straight-line drawing of graphs on the flat torus

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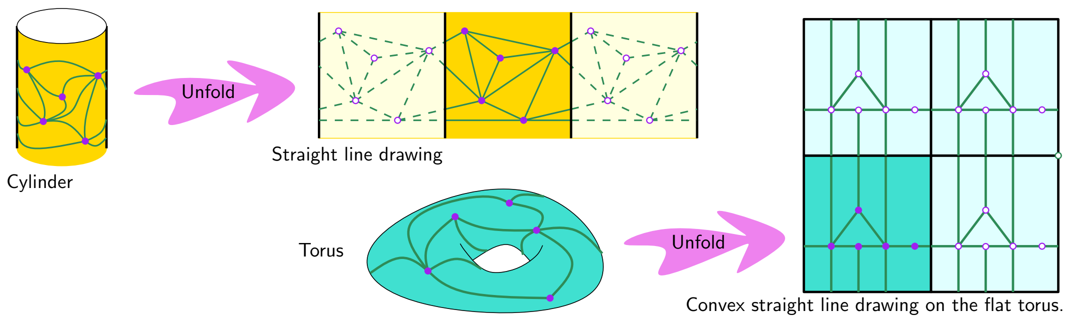

1 Introduction

The problem of efficiently computing straight-line drawings of planar graphs has attracted a lot of attention in the past. In this paper we took interest on *straight line drawings* for graphs drawn on the cylinder or on the torus. More precisely, we look at straight line drawing of unfolded periodic representations of the cylinder and the torus (see Figure).

2 Main Result

Theorem 1 *For each essentially 3-connected*[1] *toroidal map G, one can compute in linear time a weakly convex crossing-free straight-line drawing of G on a periodic regular grid $\mathbb{Z}/w\mathbb{Z} \times \mathbb{Z}/h\mathbb{Z}$, where —with n the number of vertices and c the number of edges of the shortest non contractible loop— $w \leq 2n$ and $h \leq 1 + 2n(c+1)$. Since $c \leq \sqrt{2n}$, the grid area is $O(n^{5/2})$.*

3 Sketch of Algorithm

We extend the incremental straight-line drawing algorithm of de Fraysseix, Pach and Pollack [2] (in the triangulated case) and of Kant [3] (in the 3-connected case).

The case of triangulations on a cylinder with the following properties: — no chords on the boundary, — no loops, — and no 2-cycles can be solved generalizing de Fraysseix *et al.* algorithm [2] . It needs to find a good *shelling* order to add the vertices and draw incrementally the triangulation preserving a good shape of its boundary. Allowing chords, loops, and 2-cycles is possible by a careful subdivision of the triangulation in pieces and gluing the result of the simple case. Generalizing to the torus can be done by cutting the graph along a short non contractible cycle. Adaptation from triangulation to maps is similar to the planar case [3].

References

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[2] H. de Fraysseix, J. Pach and R. Pollack. How to draw a planar graph on a grid. *Combinatorica*, 10(1):41–51, 1990. doi:10.1007/BF02122694.

[3] G. Kant. Drawing planar graphs using the canonical ordering. *Algorithmica*, 16(1):4–32, 1996. doi:10.1007/BF02086606.

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†LIX - École Polytechnique, Palaiseau, France, amturing,fusy@lix.polytechnique.fr

‡INRIA, France, olivier.devillers@inria.fr

[1] 3-connected in the periodic drawing,