Voronoi diagrams and Bolza surface

Mikhail Bogdanov, Monique Teillaud

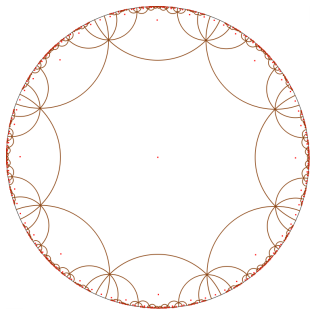
Voronoi diagrams and Bolza surface

Mikhail Bogdanov* Monique Teillaud†

A periodic Delaunay triangulations in a Euclidean space can be seen as delaunay triangulations in a closed Euclidean (aka. flat) manifold. The case of a manifold E^d/G, where G is a crystallographic group, was addressed in 2D and 3D [7,9], and more recent work [3,4] led to CGAL packages for the flat torus [5,8].

To the best of our knowledge, there were no known similar results in hyperbolic spaces.

A periodic triangulation in the hyperbolic plane is defined by an infinite point set that is the image of a finite point set by some (non commutative) discrete group generated by hyperbolic translations.
We focus here on the group defining the Bolza surface, homeomorphic to a torus having two handles. This setting is used very diverse fields [1,6,10].
The talk will show a few properties of Voronoi diagrams on the Bolza surface. Intuition is challenged there, in particular because hyperbolic translations do not commute in general.



Details and more general results can be found in [2].

References

[1] Agnès Bachelot-Motet. Wave computation on the hyperbolic double doughnut. *Journal of Computational Mathematics*, 28:1–17, 2010. doi.

[2] Mikhail Bogdanov and Monique Teillaud. Delaunay triangulations and cycles on closed hyperbolic surfaces. RR 8434, INRIA, 2013. url.

[3] Manuel Caroli and Monique Teillaud. Computing 3d periodic triangulations. In *European Symposium on Algorithms*, volume 5757 of *LNCS*, 37–48, 2009. url.

[4] Manuel Caroli and Monique Teillaud. Delaunay triangulations of point sets in closed Euclidean d-manifolds. In *Proceedings 27th Annual Symposium on Computational Geometry*, 274–282, 2011. doi.

[5] Manuel Caroli and Monique Teillaud. 3D periodic triangulations. In *CGAL User and Reference Manual*. CGAL Editorial Board, 4.4 edition, 2014. url.

[6] P. Chossat, G. Faye, and O. Faugeras. Bifurcation of hyperbolic planforms. *Journal of Nonlinear Science*, 21:465–498, 2011. doi.

[7] Nikolai P. Dolbilin and Daniel H. Huson. Periodic Delone tilings. *Periodica Mathematica Hungarica*, 34:1-2:57–64, 1997.

[8] Nico Kruithof. 2D periodic triangulations. In *CGAL User and Reference Manual*. CGAL Editorial Board, 4.4 edition, 2014. url.

[9] M. L. Mazón and Tomás Recio. Voronoi diagrams on orbifolds. *Comput. Geom.*, 8:219–230, 1997. url.

[10] F. Sausset, G. Tarjus, and P. Viot. Tuning the fragility of a glassforming liquid by curving space. *Physical Review Letters*, 101:155701(1)–155701(4), 2008. url.

*This author has left INRIA end 2013
†INRIA Sophia Antipolis - Méditerranée, France