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A STRUCTURAL DAMAGE DETECTION INDICATOR BASED ON PRINCIPAL COMPONENT ANALYSIS AND MULTIVARIATE HYPOTHESIS TESTING OVER SCORES*

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ABSTRACT

This paper is focused on the development of a damage detection indicator that combines a data driven baseline model (reference pattern obtained from the healthy structure) based on principal component analysis (PCA) and multivariate hypothesis testing. More precisely, a test for the plausibility of a value for a normal population mean vector is performed. The results indicate that the test is able to accurately clasify random samples as healthy or not.

KEYWORDS : *damage detection, PCA, multivariate statistical inference, SHM.*

1. INTRODUCTION

Among all the elements that integrate a structural health monitoring (SHM) system, methods or strategies for damage detection are nowadays playing a key role for improving the operational reliability of critical structures in several industrial sectors [1]. The essential paradigm is that a self-diagnosis and some level of detection and classification of damage is possible through the comparison of the in-service dynamic time responses of a structure with respect to baseline reference responses recorded in ideal healthy operating conditions [2]. These dynamic time responses recorded in each test, even in stable environmental and operational conditions, present the main characteristic that they are not repeatable. It means that always exist variation between measurements. Such variability may be caused by random measurement errors: measure instruments are often not perfectly calibrated and thus generating discordant interpretation and report of the results.

Since the dynamic response of a structure can be considered as a *random variable*, a set of dynamic responses gathered from several experiments can be defined as a *sample variable* and, all possible values of the dynamic response as the *population variable*. Therefore, the process to draw conclusions about the state of the structure from several experiments by using statistical methods is usually named as statistical inference for damage diagnosis. In SHM field, statistical inference can be considered as one of the emerging technologies that will have an impact on the damage prognosis process [3, 4].

In general, there are two kinds of statistical inference: (i) *estimation,* which uses sample variables to predict an unknown parameter of the population variable and, (ii) *hypothesis testing,* which uses sample variables to determine whether a parameter fulfils a specific condition and to test a hypotheses about a population variable. In this last context, classical hypothesis test is used to compare extracted statistical quantities from statistical time series models like mean, normalized autocovariance function, cross covariance function, power spectral density, cross spectral density, frequency response function, squared coherence, residual variance, likelihood function, residual sequences, among

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others [5]. A hypothesis testing technique called a sequential probability ratio test (SPRT) has been combined with time series analysis and neural networks for damage classification in [6]. The usefulness of the proposed approach is demonstrated using a numerical example of a computer hard disk and an experimental study of an eight degree-of-freedom spring-mass system. Afterwards, the performance of the SPRT is improved by integrating extreme values statistics, which specifically models behavior in the tails of the distribution of interest into the SPRT. A three-story building model was constructed in a laboratory environment to assess the approach [7]. Recently, a generalized likelihood ratio test (GLRT) is used to compare the fit of minimum mean square error MMSE model parameters in order to detect damages in a scaled wooden model bridge [8, 9].

In previous works, the authors have been investigating novel multi-actuator piezoelectric systems for detection and localization of damages. These approaches combine: (i) the dynamic response of the structure at different exciting and receiving points; (ii) the correlation of dynamical responses when some damage appear in the structure by using principal component analysis (PCA) and statistical measures that are used as damage indices; and (iii) the contribution of each sensor to the indices, what is used to localize the damage [10, 11].

Following the same framework and considering dynamic responses as random variables as in [12], this paper is focused on the development of a damage detection indicator that combines a data driven baseline model (reference pattern obtained from the healthy structure) based on principal component analysis (PCA) and multivariate hypothesis testing. As said before, the use of hypothesis testing is not new in this field. The novelty of the previous work [12] is based on (i) the nature of the data used in the test since we are using scores instead of the measured response of the structure [5] or the coefficients of an AutoRegressive model [13]; (ii) the number of data used since our test is based on two random samples instead of two characteristic quantities [14]. The proposed development starts obtaining the baseline PCA model and the subsequent projections using the healthy structure. When the structure needs to be inspected, new experiments are performed and they are projected onto the baseline PCA model. Each experiment is considered as a random process and the projections onto a predetermined number of principal component is a multivariate random variable. The objective is to analyse whether the distribution of the variable associated with the current structure is related to the healthy one.

2. Data driven baseline model based on PCA

In this work a particular experimental set-up based on the analysis of vibrational changes is used as an exemplifying configuration in order to justify, validate and test the methodology. The proposed methodology can also be applied to a more general structure.

2.1 Experimental set-up

Some experiments were performed in order to test the methods presented on this paper. In these experiments, four piezoelectric transducer discs (PZTs) were attached to the surface of a thin aluminum plate, with dimensions 250 mm x 250 mm x 1 mm. Those PZTs formed a square with 144 mm per side. The plate was suspended by two elastic ropes, being isolated from environmental influences. Figure 1 (left) shows the plate hanging on the elastic ropes.

As a response to an electrical excitation, a PZT produces a mechanical vibration, propagating, in this case, across the plate (forming Lamb waves, since a thin plate has been used). PZTs are also able to generate an electrical signal as a response to a mechanical vibration. In every excitation phase of an experiment, one PZT were used as actuator and the other three PZTs were used as sensors, recording the dynamical response of the plate.

500 experiments were performed over the healthy structure, and another 500 experiments were performed over the damaged structure with 5 damage types (100 experiments per damage type). Figure

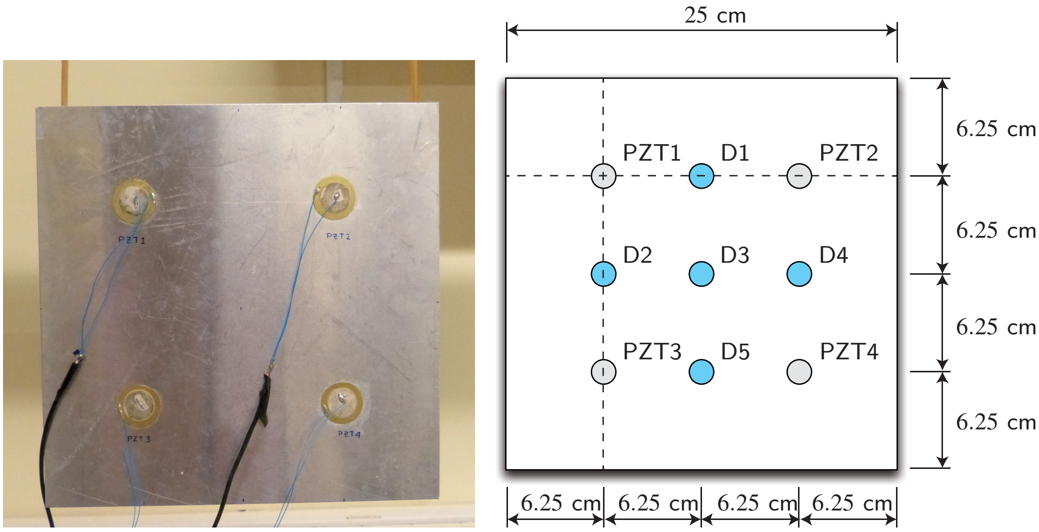

Figure 1 : Aluminium plate (left). Dimensions and piezoelectric transducers location (right).

1 (right) shows the position of damages 1 to 5 (D1 to D5). As excitation, a 50kHz sinusoidal signal modulated by a hamming window were used. Figure 2 shows the excitation signal and an example of the signal collected by PZT 1.

2.2 Principal component analysis (PCA): theoretical background

Let us initiate the analysis of a physical process by measuring different variables (sensors) at a finite number of time instants. In this work, the collected data are arranged in a $n \times (N \cdot L)$ matrix as follows:

$$\mathbf{X} = \begin{pmatrix} x_{11}^1 & x_{12}^1 & \cdots & x_{1L}^1 & x_{11}^2 & \cdots & x_{1L}^2 & \cdots & x_{11}^N & \cdots & x_{1L}^N \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1}^1 & x_{i2}^1 & \cdots & x_{iL}^1 & x_{i1}^2 & \cdots & x_{iL}^2 & \cdots & x_{i1}^N & \cdots & x_{iL}^N \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{n1}^1 & x_{n2}^1 & \cdots & x_{nL}^1 & x_{n1}^2 & \cdots & x_{nL}^2 & \cdots & x_{n1}^N & \cdots & x_{nL}^N \end{pmatrix} \tag{1}$$

Matrix $\mathbf{X} \in \mathscr{M}_{n\times(N\cdot L)}(\mathbb{R})$ –where $\mathscr{M}_{n\times(N\cdot L)}(\mathbb{R})$ is the vector space of $n \times (N \cdot L)$ matrices over $\mathbb{R}$– contains data from N sensors at L discretization instants with respect to n experimental trials. Consequently, each row vector $\mathbf{X}(i,:) \in \mathbb{R}^{N\cdot L}$, $i = 1,\ldots,n$ represents, for a specific experimental trial, the measurements from all the sensors at every specific time instant. Equivalently, each column vector $\mathbf{X}(:,j) \in \mathbb{R}^n$, $j = 1,\ldots,N\cdot L$ represents measurements from one sensor at one particular time instant in the whole set of experimental trials.

The main objective of principal component analysis (PCA) is to distinguish which dynamics are more relevant in the system, which are redundant and which can be considered as a noise [11]. This objective is essentially accomplished by defining a new coordinate space to re-express the original. This new coordinate space is used to filter noise and redundancies according to the variance-covariance matrix of the original data. In other words, the objective is to find a linear transformation orthogonal matrix $\mathbf{P} \in \mathscr{M}_{(N\cdot L)\times(N\cdot L)}(\mathbb{R})$ that will be used to transform the original data matrix $\mathbf{X}$ according to the following matrix multiplication

$$\mathbf{T} = \mathbf{XP} \in \mathscr{M}_{n\times(N\cdot L)}(\mathbb{R}). \tag{2}$$

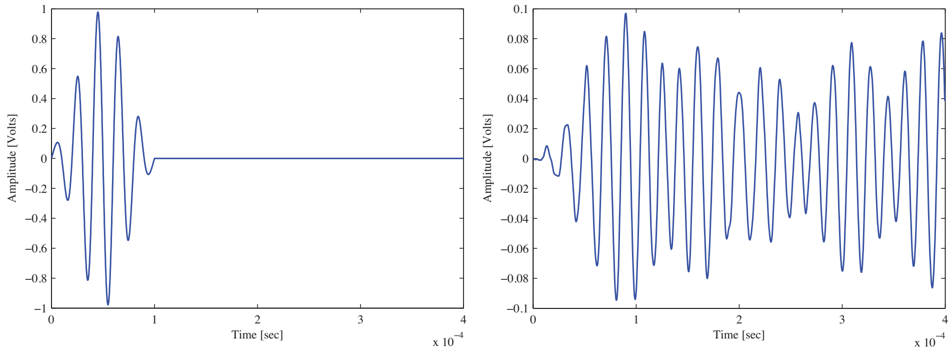

Figure 2 : Excitation signal (above) and, dynamic response recorded by PZT 1 (below).

Matrix $\mathbf{P}$ is usually called the principal components of the data set or loading matrix and matrix $\mathbf{T}$ is the transformed or projected matrix to the principal component space, also called score matrix. Using all the $N \cdot L$ principal components, that is, in the full dimensional case, the orthogonality of $\mathbf{P}$ implies $\mathbf{P}\mathbf{P}^{\mathrm{T}} = \mathbf{I}$. Therefore, the projection can be inverted to recover the original data as $\mathbf{X} = \mathbf{T}\mathbf{P}^T$.

Matrix $\mathbf{P}$ can be computed by means of the singular value decomposition (SVD) of the covariance matrix defined equation (3). Then, the principal components are defined by the eigenvectors and eigenvalues of the covariance matrix as follows:

$$\mathbf{C_X} = \frac{1}{N \cdot L - 1}\mathbf{X}^T\mathbf{X} \in \mathscr{M}_{(N \cdot L)\times(N \cdot L)}(\mathbb{R}), \tag{3}$$

$$\mathbf{C_X}\mathbf{P} = \mathbf{P}\Lambda, \tag{4}$$

where the columns of $\mathbf{P}$ are the eigenvectors of $\mathbf{C_X}$. The diagonal terms of matrix Λ are the eigenvalues λ_i, $i = 1, \ldots, N \cdot L$ of $\mathbf{C_X}$ whereas the off-diagonal terms are zero.The eigenvectors p_j, $j = 1, \ldots, N \cdot L$ representing the columns of the transformation matrix $\mathbf{P}$ are classified according to the eigenvalues in descending order and they are called the *principal components* of the data set. The eigenvector with the highest eigenvalue, called the *first principal component*, represents the most important pattern in the data with the largest quantity of information.

However, the objective of PCA is, as said before, to reduce the dimensionality of the data set $\mathbf{X}$ by selecting only a limited number $\ell < N \cdot L$ of principal components, that is, only the eigenvectors related to the ℓ highest eigenvalues. Thus, given the reduced matrix $\hat{\mathbf{P}} = (p_1|p_2|\cdots|p_r) \in \mathscr{M}_{(N \cdot L)\times \ell}(\mathbb{R})$, matrix $\hat{\mathbf{T}}$ is defined as $\hat{\mathbf{T}} = \mathbf{X}\hat{\mathbf{P}} \in \mathscr{M}_{n\times\ell}(\mathbb{R})$. Note that opposite to $\mathbf{T}$, $\hat{\mathbf{T}}$ is no longer invertible. Consequently, it is not possible to fully recover $\mathbf{X}$ although $\hat{\mathbf{T}}$ can be projected back onto the original $(N \cdot L)-$dimensional space to get a data matrix $\hat{\mathbf{X}}$ as $\hat{\mathbf{X}} = \hat{\mathbf{T}}\hat{\mathbf{P}}^T \in \mathscr{M}_{n\times(N \cdot L)}(\mathbb{R})$.

The difference between the original data matrix $\mathbf{X}$ and $\hat{\mathbf{X}}$ is defined as the *residual error matrix* $\mathbf{E}$ as follows: $\mathbf{E} = \mathbf{X} - \hat{\mathbf{X}}$, or, equivalenty, $\mathbf{X} = \hat{\mathbf{X}} + \mathbf{E} = \hat{\mathbf{T}}\hat{\mathbf{P}}^T + \mathbf{E}$. The residual error matrix $\mathbf{E}$ describes the variability not represented by the data matrix $\hat{\mathbf{X}}$.

Even though the real measures obtained from the sensors as a function of time represent physical magnitudes, when these measures are projected and the scores are obtained, these scores no longer represent any physical magnitude [12]. The key point in this approach is that the scores from different experiments can be compared with the reference pattern to try to detect a contrasting behavior.

2.3 PCA modelling

For the PCA modelling stage, we carry out a set of experiments as stated in Section 2.1. For each different phase (PZT1 will act as an actuator in phase 1, PZT2 will act as an actuator in phase 2 and so on) and considering the signals measured by the sensors, the matrix $\mathbf{X}_\mathrm{h}$ is defined and arranged as in equation (1) and scaled as stated in [11]. PCA modelling basically consists of computing the projection matrix $\mathbf{P}$ for each phase as in equation (2). Matrix $\mathbf{P}$, renamed $\mathbf{P}_\mathrm{model}$, provides an improved and dimensionally limited representation of the original data $\mathbf{X}_\mathrm{h}$. $\mathbf{P}_\mathrm{model}$ is considered as the model of the healthy structure to be used to detect structural damage.

3. DETECTION OF STRUCTURAL CHANGES BASED ON MULTIVARIATE STATISTICAL INFERENCE

A predetermined number of experiments is performed in the structure to be diagnosed and a new data matrix $\mathbf{X}_\mathrm{c}$ is constructed with the recorded data, as in equation (1). The number of experiments is not limited *a priori*. However, the number of sensors and recorded samples must correspond with the number of sensors and recorded samples in the PCA modelling stage; more precisely, the number of columns of $\mathbf{X}_\mathrm{c}$ and $\mathbf{X}_\mathrm{h}$ must agree. Matrix $\mathbf{X}_\mathrm{c}$ will be projected onto the PCA model as specified in Section 3.1. The projections onto the first components –the so-called *scores*– are used for the construction of the multivariate random samples to be compared and consequently to obtain the structural damage indicator.

3.1 Multivariate random variables and multivariate random samples

Let us start this section by specifying what we consider a random variable and how a multivariate random variable is built. Assume that for a specific actuator phase (for instance, PZTi as actuator, $i = 1,2,3,4$) and using the signals measured by the sensors in a *fully healthy state* the baseline PCA model (identified as $\mathbf{P}^i_\mathrm{model}$) is built as in sections 2.2 and 2.3. Assume also that an experiment as detailed in section 2.1 is further performed. The time responses recorded by the sensors are first discretized and then arranged in a row vector $r^i \in \mathbb{R}^{N\cdot L}$, where N is the number of sensors, L is the number of discretization instants and i refers to the current actuator phase. The number of sensors and discretization instants must be equal to those that were used when defining $\mathbf{P}^i_\mathrm{model}$. Besides, the size of each column is $N \cdot L$. Selecting the jth principal component ($j = 1,\dots,\ell$), $\mathbf{P}^i_\mathrm{model}(:,j) =: v^i_j \in \mathbb{R}^{N\cdot L}$, the projection of the recorded data onto this principal component is the dot product $t^i_j = r^i \cdot v^i_j \in \mathbb{R}$, as in equation (2).

Since the dynamic behaviour of a structure depends on some indeterminacy, its dynamic response can be considered as a stochastic process and the measurements in r^i are also stochastic. On the one hand, t^i_j acquires this stochastic nature and it will be regarded as a random variable to construct the stochastic approach in this paper. On the other hand, an s-dimensional random vector can be defined by considering the projections onto several principal components as follows

$$\mathbf{t}^i_{j_1,\dots,j_s} = \begin{bmatrix} t^i_{j_1} & t^i_{j_2} & \cdots & t^i_{j_s} \end{bmatrix}^T \in \mathbb{R}^s,\ s \in \mathbb{N},\ j_1,\dots,j_s \in \{1,\dots,\ell\}. \tag{5}$$

By reiterating this experiment several times on the undamaged structure and using equations (3.1)-(5) we have a multivariate random sample of the variable $\mathbf{t}^i_{j_1,\dots,j_s}$ that can be viewed as a baseline. When structural changes on the structure have to be detected, a new set of experiments should be performed to create the multivariate random sample that will be compared with the multivariate baseline sample. As an example, in Figure 3 two three-dimensional samples are represented; one is the three-dimensional baseline sample (left) and the other is referred to damages 1 to 3 (right). This illustrating example refers to actuator phase 1 and the first, second and third principal components. More precisely, Figure 3 depicts the values of the multivariate random variable $\mathbf{t}^1_{1,2,3}$.The diagnosis sample is made by 20 experiments and the baseline sample consists of 100 experiments.

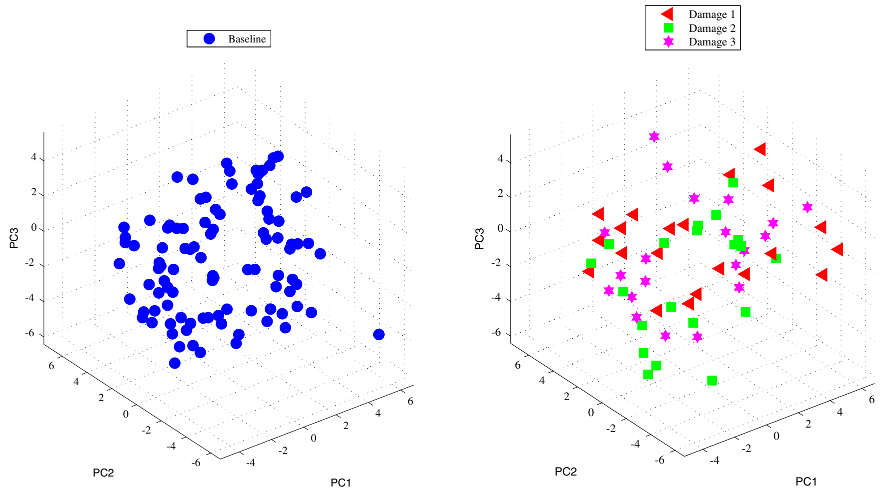

Figure 3 : Baseline sample (left) and sample from the structure to be diagnosed (right).

3.2 Detection phase

In this work, the framework of multivariate statistical inference is used with the objective of the classification of structures in healthy or damaged. With this goal, a test for multivariate normality is first performed. A test for the plausibility of a value for a normal population mean vector is then performed.

3.2.1 Testing a multivariate mean vector

The objecctive of this paper is to determine whether the distribution of the multivariate random samples that are obtained from the structure to be diagnosed (undamaged or not) is connected to the distribution of the baseline. To this end, a test for the plausibility of a value for a normal population mean vector will be performed. We will consider that: (a) the baseline projection is a multivariate random sample of a multivariate random variable following a multivariate normal distribution with known population mean vector $\boldsymbol{\mu}_{\mathrm{h}} \in \mathbb{R}^s$ and known variance-covariance matrix $\boldsymbol{\Sigma} \in \mathscr{M}_{s\times s}(\mathbb{R})$; and (b) the multivariate random sample of the structure to be diagnosed also follows a multivariate normal distribution with unknown multivariate mean vector $\boldsymbol{\mu}_{\mathrm{c}} \in \mathbb{R}^s$ and known variance-covariance matrix $\boldsymbol{\Sigma} \in \mathscr{M}_{s\times s}(\mathbb{R})$.

As said previously, the problem that we will consider is to determine whether a given s-dimensional vector $\boldsymbol{\mu}_{\mathrm{c}}$ is a plausible value for the mean of a multivariate normal distribution $N_s(\boldsymbol{\mu}_{\mathrm{h}}, \boldsymbol{\Sigma})$. This statement leads immediately to a test of the hypothesis $H_0 : \boldsymbol{\mu}_{\mathrm{c}} = \boldsymbol{\mu}_{\mathrm{h}}$ versus $H_1 : \boldsymbol{\mu}_{\mathrm{c}} \neq \boldsymbol{\mu}_{\mathrm{h}}$, that is, the null hypothesis is 'the multivariate random sample of the structure to be diagnosed is distributed as the baseline projection' and the alternative hypothesis is 'the multivariate random sample of the structure to be diagnosed is not distributed as the baseline projection'. In other words, if the result of the test is that the null hypothesis is not rejected, the current structure is categorized as healthy. Otherwise, if the null hypothesis is rejected in favor of the alternative, this would indicate the presence of some structural changes in the structure.

The test is based on the statistic T^2 –also called Hotelling's T^2– and it is summarized below. When a multivariate random sample of size $\nu \in \mathbb{N}$ is taken from a multivariate normal distribution $N_s(\boldsymbol{\mu}_{\mathrm{h}}, \boldsymbol{\Sigma})$, the random variable $T^2 = \nu \left(\bar{\mathbf{X}} - \boldsymbol{\mu}_{\mathrm{h}}\right)^T \mathbf{S}^{-1} \left(\bar{\mathbf{X}} - \boldsymbol{\mu}_{\mathrm{h}}\right)$ is distributed as $T^2 \hookrightarrow \frac{(\nu-1)s}{\nu-s} F_{s,\nu-s}$, where $F_{s,\nu-s}$ denotes a random variable with an F-distribution with s and $\nu - s$ degrees of freedom, $\bar{\mathbf{X}}$ is the sample vector mean as a multivariate random variable; and $\frac{1}{n}\mathbf{S}$ is the estimated covariance matrix of $\bar{\mathbf{X}}$.

At the α level of significance, we reject H_0 in favor of H_1 if the observed $t^2_{\text{obs}} = \nu(\bar{\mathbf{x}} - \boldsymbol{\mu}_{\text{h}})^T \mathbf{S}^{-1} (\bar{\mathbf{x}} - \boldsymbol{\mu}_{\text{h}})$ is greater than $\frac{(\nu-1)s}{\nu-s} F_{s,\nu-s}(\alpha)$, where $F_{s,\nu-s}(\alpha)$ is the upper (100α)th percentile of the $F_{s,\nu-s}$ distribution. In other words, the quantity t^2_{obs} is the damage indicator and the test is summarized as follows:

$$t^2_{\text{obs}} \le \frac{(\nu-1)s}{\nu-s} F_{s,\nu-s}(\alpha) \implies \text{Fail to reject } H_0 \tag{6}$$

$$t^2_{\text{obs}} > \frac{(\nu-1)s}{\nu-s} F_{s,\nu-s}(\alpha) \implies \text{Reject } H_0, \tag{7}$$

where $F_{s,\nu-s}(\alpha)$ is such that $P(F_{s,\nu-s} > F_{s,\nu-s}(\alpha)) = \alpha$. More precisely, we fail to reject the null hypothesis if $t^2_{\text{obs}} \le \frac{(\nu-1)s}{\nu-s} F_{s,\nu-s}(\alpha)$, thus indicating that no structural changes in the structure have been found. Otherwise, the null hypothesis is rejected in favor of the alternative hypothesis if $t^2_{\text{obs}} > \frac{(\nu-1)s}{\nu-s} F_{s,\nu-s}(\alpha)$, thus indicating the existence of some structural changes in the structure.

4. Experimental results

As said in Section 2.1, the experiments are performed in 4 independent phases: (i) piezoelectric transducer 1 (PZT1) is configured as actuator and the rest of PZTs as sensors; (ii) PZT2 as actuator; (iii) PZT3 as actuator; and (iv) PZT4 as actuator. In order to analyze the influence of each set of projections to the PCA model (score), the results of scores 1 to 5 (jointly) and scores 1 to 10 (jointly) have been considered. In this way, a total of 8 scenarios were examined. For each scenario, a total of 50 samples of 20 experiments each one (25 for the undamaged structure and 5 for the damaged structure with respect to each of the 5 different types of damages) plus the baseline are used to test for the plausibility of a value for a normal population mean vector, with a level of significance $\alpha = 0.60$. Each set of 50 testing samples are categorized as follows: (i) number of samples from the healthy structure (undamaged sample) which were classified by the hypothesis test as 'healthy' (fail to reject H_0); (ii) undamaged sample classified by the test as 'damaged' (reject H_0); (iii) samples from the damaged structure (damaged sample) classified as 'healthy'; and (iv) damaged sample classified as 'damaged'. The results for the 8 different scenarios presented in Table 2 are organized according to the scheme in Table 1. It can be stressed from each scenario in Table 2 that the sum of the columns is constant: 25 samples in the first column (undamaged structure) and 25 more samples in the second column (damaged structure). It is worth noting that Type I errors (false alarms) appear only when we consider scores 1 to 5 (jointly), while in the second case (scores 1 to 10), all the decisions are correct.

5. Concluding remarks

This paper has been focused on the development of a damage detection indicator that combines a data driven baseline model (reference pattern obtained from the healthy structure) based on principal component analysis (PCA) and multivariate hypothesis testing. A test for the plausibility of a value for a normal population mean vector has been performed. The results indicate that the test is able to accurately clasify random samples as healthy or not.

Table 1 : Scheme for the presentation of the results in Table 2.

	undamaged sample (H_0)	damaged sample (H_1)
Fail to reject H_0	Correct decision	Type II error (missing fault)
Reject H_0	Type I error (false alarm)	Correct decision

Table 2 : Categorization of the samples with respect to presence or absence of damage and the result of the test, for each of the four phases and considering scores 1 to 5 (jointly) and scores 1 to 10 (jointly).

	PZT1 act.		PZT2 act.		PZT3 act.		PZT4 act.	
	H_0	H_1	H_0	H_1	H_0	H_1	H_0	H_1
Scores 1 **to** 5								
Fail to reject H_0	21	0	23	0	21	0	20	0
Reject H_0	4	25	2	25	4	25	5	25
Scores 1 **to** 10								
Fail to reject H_0	25	0	25	0	25	0	25	0
Reject H_0	0	25	0	25	0	25	0	25

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