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## FINITE ELEMENT MODEL UPDATING USING POWER SPECTRAL DENSITY OF STRUCTURAL RESPONSE

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### ABSTRACT

Finite element models (FEM) are used for estimation of the physical behavior of the structures may not accurately represent real structural responses because of inaccurate assumption of mechanical properties. For the achievement of a reliable model, the numerical responses must be adjusted to the experimental or real life responses through a proper model updating scheme. The Power Spectral Density (PSD) of the structural response subjected to a random input is opted in this research for derivation of sensitivity based model updating method. Taylor series expansion of the PSD is used to develop the sensitivity equations. Upon the consideration of all the terms of the expansion, it is possible to take advantage from an already developed method based on frequency response function (FRF). The proposed sensitivity equation has also eliminated the need for model reduction or expansion to deal with incomplete measurement which is required for most of updating methods relying on the modal data. The proposed method is examined using a numerical example to predict changes in stiffness parameters.

**KEYWORDS :** *FE Model updating, Modal data, Power Spectral Density*

### INTRODUCTION

Nowadays great attention is dedicated to the development of computational methods for the purposes of finite element model updating and damage detection. Most of the already developed methods rely on the dynamic characteristics of the structure. Eigen value (Natural frequencies) and Eigen vectors (Shape modes) of the dynamic stiffness matrix and frequency response function (FRF) have been subjected to the researches done in this field [1-5]. The reason for the use of dynamic characteristics is their significant change due to the damage. Some of the methods are based on the minimization of the error between the measured response and the corresponding predicted response using FEM. Also, some methods are based on the estimation of the effect of structural change on the structural equation of motion (equation error methods)[6]. The error in output PSD of the structural response can also be adopted for development of a model updating method. An out-put error minimization approach was introduced by Kammer and Nimityongskul [6]. The frequency band averaged output Power Spectral Density with the central frequency of the band running over the full range of frequencies of interest was adopted as the objective function. In their study only the auto spectral components related to the measured DOFs of the PSD matrixes was considered and the algorithm does not lead to desirable results in case of zero percent band-averaging (point wise use of frequency points). Therefore, the selection range the range of frequency over which the averaging is done is an issue in that study.

In the present paper a derivation based sensitivity equation, based on the full Taylor expansion of the PSD matrix is developed. The developed method uses the advantages of an already developed

sensitivity formula based on FRF [5]. The method also uses both auto spectral and cross spectral elements of the PSD matrix. The method is examined by numerical example of a truss structure.

It is worth noting here, in development of a sensitivity method based on PSD a lot issues to be considered. The developed method still needs further investigation on the issues like, equation arrangement, selection of the proper measurement point with respect to the excitation point, the effect of techniques such as frequency band averaging on improvement of the method and the consideration of other types of random inputs rather than white noise.

A PSD based sensitivity equation has some advantages as compared to the FRF. (1) The random aspects of the structural response are considered in the study. (2) It is possible to consider the data from all the inputs at once. (3) The mutual effect of the excitation at different DOFs can be considered. (4) Upon the selection of the ideal frequency points for the structural update purpose; the method is more robust against measurement error. (5) The use of spectral density provides more options for equations set arrangement.

## 1 THEORETICAL DEVELOPMENT

The PSD of a structure subject to a stationary input is

$$S_{XX}(\omega) = H(\omega) \cdot S_{FF} \cdot H^*(\omega) \quad (1)$$

In this equation  $S_{XX}(\omega)$  is the PSD matrix of response in a sense that its diagonal terms are auto spectral density and the non-diagonal ones are cross spectral density terms.  $S_{FF}$  is the PSD of the input in all the active degrees of freedom and  $H(\omega)$  is the system frequency response function which is,

$$H(\omega) = (K - \omega^2 M + i\omega C)^{-1} \quad (2)$$

In which  $M$  is the system mass matrix,  $K$  is the system stiffness matrix and  $C$  is the damping matrix, which are symmetric for linear systems. The  $H^*(\omega)$  is the complex conjugate of  $H(\omega)$ . The PSD of a damaged structure can be estimated based on the full Taylor expansion of the PSD of the analytical model.

The Taylor expansion of the  $S_{XX}$  matrix could be expressed in the form of equation (3).

$$S_{XX}^d = S_{XX} + \sum_{r=1}^{n_p} \frac{\partial S_{XX}}{\partial p_r} \Delta p_r + \sum_{r=1}^{n_p} \sum_{m=1}^{n_p} \frac{\partial^2 S_{XX}}{\partial p_r \partial p_m} \Delta p_m \Delta p_r + \dots \quad (3)$$

In this equation,  $S_{XX}^d$  indicates the PSD of the damaged model,  $\Delta p$  is the change in the design variables ( $n_p$  is the number of design variables).

The derivative of the  $S_{XX}$  matrix with respect to  $r^{th}$  design variable is in the following format,

$$\frac{\partial S_{XX}}{\partial p_r} = -2 \cdot H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot S_{FF} \cdot H^* \quad (4)$$

The derivative of the of the  $S_{XX}$  matrix with respect to  $r$  and  $m$  design variables is in the following format,

$$\frac{\partial^2 S_{xx}}{\partial p_r \partial p_m} = 2 \left[ \begin{aligned} &H \cdot \frac{\partial Z}{\partial p_m} \cdot H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot S_{FF} \cdot H^* + H \cdot \frac{\partial^2 Z}{\partial p_r \partial p_m} \cdot H \cdot S_{FF} \cdot H^* + H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot \frac{\partial Z}{\partial p_m} \cdot H \cdot S_{FF} \cdot H^* + \dots \\ &+ H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot S_{FF} \cdot H^* \cdot \frac{\partial Z^*}{\partial p_m} \cdot H^* \end{aligned} \right] \quad (5)$$

Since in a linear structural system the impedance matrix is linear in design variables then  $\frac{\partial^2 Z}{\partial p_r \partial p_m} = 0$  and as a result the third term in the brackets in equation (5) is equal to zero. Therefore,

this equation can be written in the reduced form of,

$$\frac{\partial^2 S_{xx}}{\partial p_r \partial p_m} = 2 \left[ H \cdot \frac{\partial Z}{\partial p_m} \cdot H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot S_{FF} \cdot H^* + H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot S_{FF} \cdot H^* \cdot \frac{\partial Z^*}{\partial p_m} \cdot H^* \right] \quad (6)$$

Upon substitution of equation (4) and (6) in equation (3), the  $\delta S_{xx}$  can be expressed as,

$$\delta S_{xx} = \sum_{r=1}^{np} (-2 \cdot H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot S_{FF} \cdot H^*) \Delta p_r + \sum_{r=1}^{np} \sum_{m=1}^{np} 2 \left[ \begin{aligned} &H \cdot \frac{\partial Z}{\partial p_m} \cdot H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot S_{FF} \cdot H^* + \dots \\ &\dots + H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot \frac{\partial Z}{\partial p_m} \cdot H \cdot S_{FF} \cdot H^* + \dots \\ &\dots + H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot S_{FF} \cdot H^* \cdot \frac{\partial Z^*}{\partial p_m} \cdot H^* \end{aligned} \right] \Delta p_m \Delta p_r + \dots \quad (7)$$

The same procedure can be applied to the receptance matrix of the system, to form  $\delta H$  as

$$\delta H = \sum_{r=1}^{np} (-H \cdot \frac{\partial Z}{\partial p_r} \cdot H) \Delta p_r + \sum_{r=1}^{np} \sum_{m=1}^{np} \left[ \begin{aligned} &H \cdot \frac{\partial Z}{\partial p_m} \cdot H \cdot \frac{\partial Z}{\partial p_r} \cdot H + \dots \\ &\dots + H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot \frac{\partial Z}{\partial p_m} \cdot H \end{aligned} \right] \Delta p_m \Delta p_r + \dots \quad (8)$$

By factorizing the terms relating to Taylor series expansion of Frequency Response, in the  $\delta S_{xx}$ , it transforms as

$$\delta S_{xx} = 2 \cdot (\Delta H) \cdot S_{FF} \cdot H^* + \sum_{r=1}^{np} \sum_{m=1}^{np} 2 \left[ H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot S_{FF} \cdot H^* \cdot \frac{\partial Z^*}{\partial p_m} \cdot H^* \right] \Delta p_m \Delta p_r + \dots \quad (9)$$

By ignoring the terms like  $\sum_{r=1}^{np} \sum_{m=1}^{np} 2 \left[ H \cdot \frac{\partial Z}{\partial p_r} \cdot H \cdot S_{FF} \cdot H^* \cdot \frac{\partial Z^*}{\partial p_m} \cdot H^* \right] \Delta p_m \Delta p_r$ , which are generated while calculating the second and higher order derivatives of the  $S_{xx}$ , a formula for  $\delta S_{xx}$  is reached:

$$\delta S_{xx} = 2 \cdot (\delta H) \cdot S_{FF} \cdot H^* \quad (10)$$

The exact expression of the  $\delta H$  due to damage is [5]:

$$\delta H = -H_d \cdot \delta Z \cdot H \quad (11)$$

By substitution of the equation (11) in the (10), and noting that  $\delta Z = \delta K - \omega^2 \delta M + i\omega \delta C$  the change in the spectral density matrix can be expressed as

$$\delta S_{xx} = -2 \cdot H_d \cdot (\delta K - \omega^2 \delta M + i\omega \delta C) \cdot H \cdot S_{FF} \cdot H^* \quad (12)$$

It is worth noting that, if the effect of damping is negligible the H matrix is not complex and then  $H(\omega) = H^*(\omega)$ . Moreover,  $H_d(\omega)$  is the FRF of the damaged or real life structure.

In the presented sensitivity based model updating method, using all entries of the PSD matrix is not possible because of the problems associated with the measurement at some degrees of freedom. Therefore, the presented method should overcome the model incompleteness. Esfandiari et al. [5] has proposed an approximation formula for  $H_d$  as.

$$H_d^{approx}(\omega) = \sum_{j=1}^m \frac{\phi_j \phi_j^T}{\Omega_{jd}^2 - \omega^2 + 2i\xi_{jd} \Omega_{jd} \omega} + \sum_{j=m+1}^{nd} \frac{\phi_j \phi_j^T}{\Omega_j^2 - \omega^2 + 2i\xi_j \Omega_j \omega} \quad (13)$$

The second term of this equation improves the parameter identification process and makes it more precise by including the higher mode shapes of the intact structure. By using these definitions, sensitivity equations are extracted from equation (12) to update stiffness, damping and mass parameters.

$$S_s = -2 \cdot H_d \cdot (\delta K) \cdot H \cdot S_{FF} \cdot H^* \quad (14-1)$$

$$S_M = -2 \cdot H_d \cdot (\omega^2 \delta M) \cdot H \cdot S_{FF} \cdot H^* \quad (14-2)$$

$$S_c = -2 \cdot H_d \cdot (i\omega \delta C) \cdot H \cdot S_{FF} \cdot H^* \quad (14-3)$$

Final format of the sensitivity equations for calculating the change in the structural parameters is expressed as.

$$\delta S_{xx} = S_s \cdot \Delta p_s + S_M \cdot \Delta p_M + S_c \cdot \Delta p_c \quad (15)$$

The use of PSD gives more options for the arrangement of the equation set as compared to the FRF. In this study, the arrangement of the equations was considered in a way that  $(n_s \times n_s \times n_f)$  equations are generated to predict  $(n_p)$  unknown parameters, which is an over determined optimization problem. It should be noted that  $n_s$  is the number of sensors and  $(n_f)$  is the number of frequency points. The  $(n_e)$  unit white noise random input are considered to be excreted in a separate manner at the excited degrees of freedom. The resulting sensitivity equations of each excitation are added together; therefore the number of equations is not changed. This arrangement of equations leads to the consideration of cross spectral elements of the PSD matrix corresponding to the measurement points. This terms were not considered in the method which was developed [6]. The Bounded Value

Least Square (BVLS) algorithm is adopted for the solution, since the ratio of change in the structural parameters are limited ( $-1 \leq \Delta p_n \leq 1$ ).

## 2 NUMERICAL INVESTIGATION

The efficiency of the method is examined by modelling a thirty five member truss structure. The geometry of the model is introduced in the Figure 1. The truss model is consisted of eight spans of two meters length.

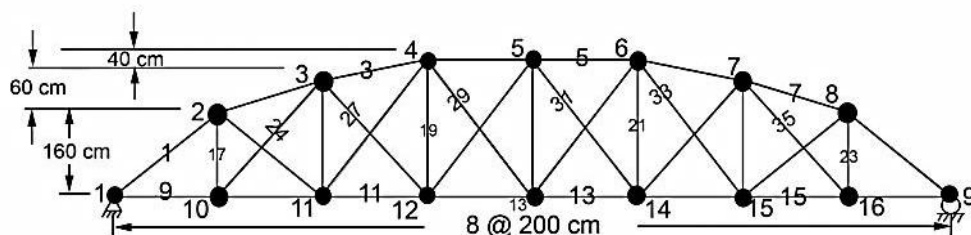


Figure1- Geometry of the studied truss model

The model is consisted of sixteen nodes, each of which has two Degree of freedom. The active DOFs of the truss are depicted in figure2.

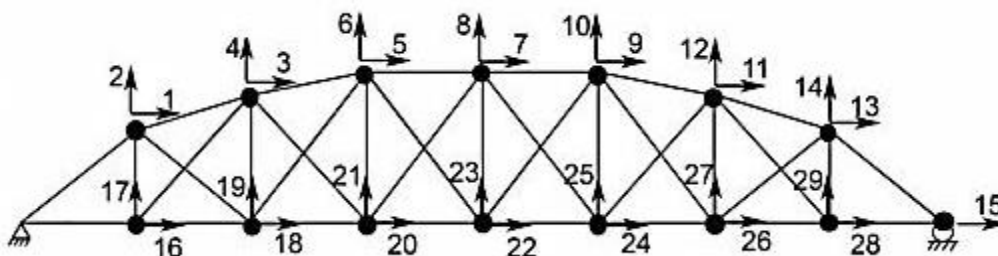


Figure 2. Active DOFs of the truss model

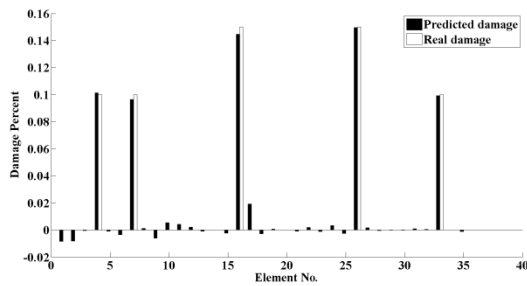
The cross sectional areas, modulus of Elasticity and mass per unit length of the truss members are presented in the table1.

Table1- Cross section of the elements of the truss

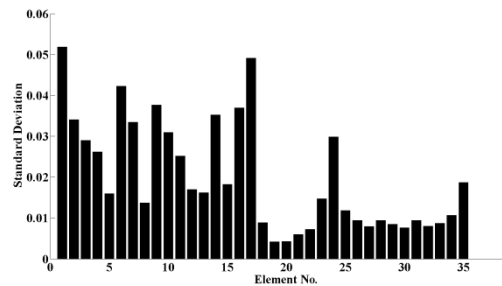
Member number	Area (cm <sup>2</sup> )	E (Gpa)	$\bar{m}$ (kg / m)
1-8	18	200	14.04
9-16	15	200	11.7
17-23	10	200	7.8

Some damage cases are considered in order to examine the ability of the method to detect the damages of various levels. In these cases the damage is modelled by introducing a level of change in the stiffness parameters of the structure  $EA$ . PSD data of the damaged models are simulated using FEM of the damaged model. Since the measurement of real structural response is contaminated by errors, in all damage scenarios, 15 percent normally distributed random error is added to the numerically simulated PSDs of the damaged structure. In order to investigate the stability of the proposed method against measurement error, Monte Carlo simulation was adopted. For each damage cases, 50 sets of noisy PSDs data were simulated and parameters estimation was carried out and the mean values of the estimated parameters were calculated. Moreover, in order to study the stability of the results against measurement noise, the COV was also taken into consideration. The Coefficient of variation (COV) of each parameter exhibits the sensitivity of the model updating method to measurement errors. The lower amount of COV of a parameter is a measure of robustness of method against measurement errors.

Figure 3 to 5 present the results of the simulated damage cases. The part (a) of these figures demonstrates the mean value of the predicted parameters and part (b) is the Coefficient of Variation of the estimated parameters. Figures 3, 4 and 5 represent the slight, mid and severe level of damage, respectively. The reported results prove that the proposed method is capable of predicting damages of all levels with high accuracy. In all three cases, the method is robust against the measurement error.

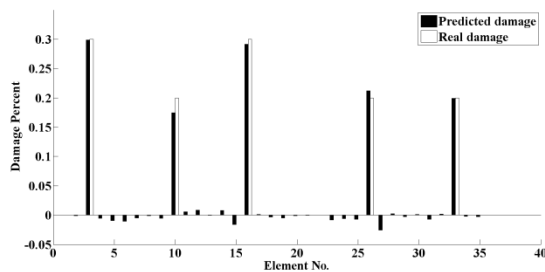


a) Actual and predicted truss stiffness change for slight damage case

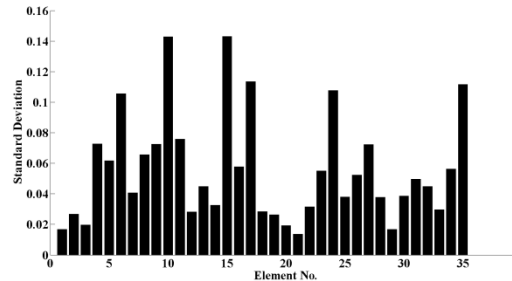


b) Slight damage case-COV of predicted parameters

Figure 3



a) Actual and predicted truss stiffness change for mid-level damage case



b) mid-level damage case-COV of predicted parameters

Figure 4

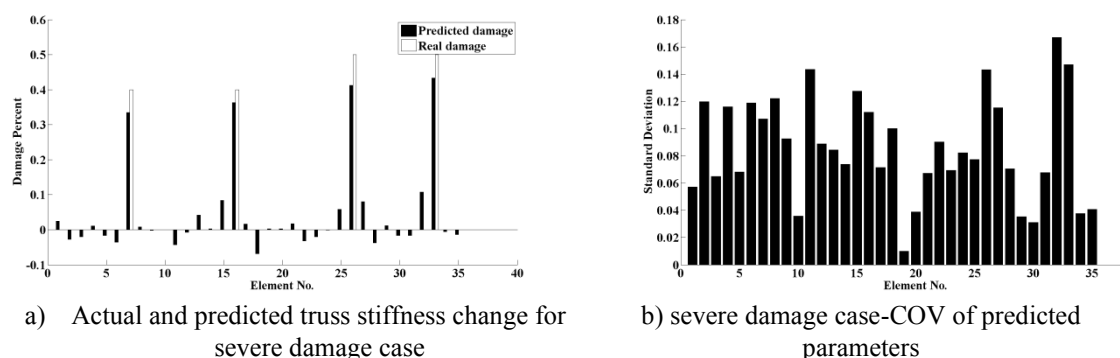


Figure 5

### 3 CONCLUSION

A new model updating method using PSD of the structural response is considered for deriving a sensitivity equation. The method is examined in stiffness parameter identification. It is capable of detecting various level of damage (slight to severe) by acceptable accuracy. The method is also robust against measurement noise if the frequency points are selected properly. The method converges without the need for techniques such as frequency band averaging which was necessary in for the algorithm presented in [6] to converge. Furthermore, the sensitivity of the cross spectral terms of the PSD matrix to the parameters change was taken into consideration. The developed method still needs further investigation on the issues like, equation arrangement, measurement point selection with respect to the excitation point, study of the effect of other techniques such as frequency band averaging on the improvement of the results and the consideration of other types of random inputs rather than white noise.

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