Finite Element Model-Based Structural Health Monitoring (SHM) Systems for Composite Material under Fluid-Structure Interaction (FSI) Effect

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7th European Workshop on Structural Health Monitoring
July 8-11, 2014. La Cité, Nantes, France

FINITE ELEMENT MODEL-BASED STRUCTURAL HEALTH MONITORING (SHM) SYSTEMS FOR COMPOSITE MATERIAL UNDER FLUID-STRUCTURE INTERACTION (FSI) EFFECT

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ABSTRACT

Advanced composite materials such as Carbon Fibre Reinforced Polymers (CFRP) tend to be used in aerospace industry to keep the weight at its minimum and yet retain a great strength. CFRP have a strong, stiff fibres in a matrix. The resulting material is very strong as it has the best strength to weight ratio of all construction materials. However, aircraft structures such as wings can break due to Fluid-Structure Interaction (FSI) oscillations or material fatigue. Material inspection by piezoelectric induced ultrasonic waves is a relatively new and an intelligent technique to monitor the health of CFRP for a damage detection in the Non-Destructive Test (NDT). To design a Structural Health Monitoring (SHM) system, it is important to understand phenomenologically and quantitatively the wave propagation in CFRP and the influence of the geomaterial and mechanical properties of the structures. The principal aim of this research is to explore and understand the design and operation from safety and economic points of view. To accelerate the design of SHM systems, the FSI effect on the wave propagation has to be considered. This research will focus on the mathematical modeling and numerical analysis of Navier-Stokes, elastodynamics and elastic waves equations in the arbitrary Lagrangian-Eulerian (ALE) framework in order to determine the wave propagation in moving domains and optimum locations for sensors. Since analytical solutions are only available in special cases, the equations need to be solved by numerical methods. For the implementation we chose the finite element library package `deal.ii` and DOPELIB due to its special characteristics.

KEYWORDS : *Finite elements based SHM, interaction between wave/field and structure/material, elastic wave propagation, fluid-structure interaction, adaptive finite element method.*

1. INTRODUCTION

Separating modeling and numerical errors in the simulated results from the material defects effects is a very challenging topic. It is of great importance for structural health studies and the design of appropriate monitoring systems. In this paper, we restrict ourselves to mathematical modeling and numerical analysis of coupled elastic wave propagation problems with Fluid-Structure Interaction (FSI) problems in a ALE framework. The main goal of this research is to apply higher order finite element methods (FEM) for the elastic wave propagation in a composite material under FSI effect. We present the state-of-the-art in computational methods and techniques of elastic wave propagation with FSI that go beyond the fundamentals of computational fluid and solid mechanics. In fact, the fundamental rule require transferring results from the computational fluid dynamics (CFD) analysis as input into the structural analysis and thus can be time-consuming, tedious and error-prone. This work consists of mathematical coupling, development of numerical approximation and solution techniques for elastic wave propagation with FSI problem. We will also consider the investigation of different time stepping scheme formulations for a elastic wave equation with a nonlinear fluid-structure interaction problem coupling the incompressible Navier-Stokes equations with a hyperelastic solid based on the well established Arbitrary Lagrangian Eulerian (ALE) framework [1–4]. Temporal discretization is based on finite differences and a formulation as one step-θ scheme, from which we can extract the implicit Euler, Crank-Nicolson, shifted Crank-Nicolson, and the fractional-step-θ schemes [1–6]. We will analysis variational space-time methods for the wave equation [7–9]. The ALE approach provides a simple, but powerful procedure to couple fluid flows with solid deformations by a monolithic solution algorithm. In such a setting, the fluid equations are transformed to a fixed reference configuration via the ALE mapping. The targets of this work are the development of concepts for the efficient numerical solution of FSI problem and the analysis of various fluid-mesh motion techniques, a comparison of different second-order time-stepping schemes. The time discretization is based on finite difference schemes whereas the spatial discretization is done with a Galerkin finite element scheme. The nonlinear problem is solved with Newton's method. To control computational costs, we apply a simplified version of a posteriori error estimation using the dual weighted residual (DWR) method. This method is used for the mesh adaption during the computation. For the implementation, we chose the FE software package DOpElib [10], which is a flexible modularized high-level algorithms toolbox based on the finite element library `deal.II` [11].

DOpElib, `deal.II` can be used to solve stationary and non-stationary PDE problems as well as optimal control problems constrained by PDEs and Dual-Weighted-Resiudal approach for goal-oriented error estimation. The principal aim of this research is to explore and understand the behaviour of engineering artefacts in extreme environments. To achieve the main ambition of this work, we split this research into two parts. The first part will consider the effect of fluid flow over a sample aircraft and the displacement of a control point under incompressible fluid flow. The second part will focus on the numerical approximation of wave propagation in composite material. The anticipated outcome of this research will be: (a) an overview of FSI effect on the sample aircraft wing, (b) optimum locations for sensors of SHM systems, (c) an outlook on the expected future of SHM systems and (d) modern techniques for FSI and wave propagation problem optimization [5].

In this project, we aim to solve second order hyperbolic equations coupled with a nonstationary fluid-structure interaction problem in arbitrary Lagrangian-Eulerian coordinates, where the mesh motion model is based on the solution of a biharmonic equation. To formulate the system of elasticity, let us assume that $\Omega \subset \mathbb{R}^d, d = 2,3$, is a bounded and convex domain at time $t = 0$ with the Lipschitzian boundary $\partial\Omega = \partial\Omega_D \cup \partial\Omega_N$, where $\partial\Omega_D$, $\partial\Omega_N$ denote Dirichlet and Neumann boundaries, respectively. The computational domain $\Omega := \Omega(t)$ is split into two time-dependent subdomains $\Omega_f(t)$ (for a homogeneous, newtonian and incompressible fluid) and $\Omega_s(t)$ (for a compressible hyperelastic structure). And the sensors of SHM systems would be optimally located in $\Omega_s(t)$. Both domains depend on time and their common boundary $\partial\Omega_i(t) = \partial\Omega_f(t) \cap \partial\Omega_s(t)$, where $partial\Omega_f(t) = \partial\Omega_{fD}(t) \cup \partial\Omega_{fN}(t) \cup \partial\Omega_i(t)$ and $\partial\Omega_s(t) = \partial\Omega_{sD}(t) \cup \partial\Omega_{sN}(t) \cup \partial\Omega_i(t)$.

Notations and Functional spaces

We begin this paper with some basic notation. For d a positive integer representing dimension, let $X \subset R^d$ denote an arbitrary bounded Lipschitz domain with boundary ∂X. As usual, let $L^2(X)$ denote the space of square integrable functions on X and define $\mathbf{L}^2(X) = (L^2(X))^d$. We will also utilize the standard Lebesgue space $L^p(X)$ where $1 \leqslant p \leqslant \infty$ that consists of measurable functions u, which are Lebesgue-integrable to the p-th power and their vectorial counterpart $\mathbf{L}^p(X)$. The set $L^p(X)$ forms a Banach space with the norm $|u|_{L^p(X)}$. The Sobolev space $W^{m,p}(X)$, $m \in N$, $1 \leqslant p \leqslant \infty$ is the space of functions in $L^p(X)$ that have distributional derivatives of order up to m, which belong to $L^p(X)$. For $p = 2, H^m(X) := W^{m,2}(X)$ is a Hilbert space equipped with the norm $\|.\|_{H^m(X)}$ $[5,10,11]$. Finally, the subspace $W^{m,p}(X)$ of functions indicate with zero trace on ∂X denoted by $W_0^{m,p}(X)$. Specifically, Hilbert space with zero trace on ∂X defined as $H_0^1(X) = \{u \in H^1(X) : u|_{\Gamma_D} = 0,\ where\ \Gamma_D = \partial X_D\}$, where ∂X_D is that part of the boundary ∂X at which Dirichlet boundary conditions are imposed. So for given set X, let us consider the Lebesque space $L_X := L^2(X)$, and $L_X^0 := L^2(X)/\mathbb{R}$. The functions in L_X with first-order distributional derivatives in L_X make up the Sobolev space $H^1(X)$. Furthermore, we can use the function spaces $V_X := H^1(X)^d$, $V_X^0 := H_0^1(X)^d$, and for time-dependent functions

$$\begin{aligned}
\mathscr{L}_X &:= L^2[0,T;L_X], \quad & \mathscr{V}_X &:= L^2[0,T;V_X] \cap H^1[0,T;V_X^*], \\
\mathscr{L}_X^0 &:= L^2[0,T;L_X^0], \quad & \mathscr{V}_X^0 &:= L^2[0,T;V_X^0] \cap H^1[0,T;V_X^*],
\end{aligned}$$

where, V_X^* is the dual of V_X.

2. Mathematical Models

To formulate the typical wave propagation problem with FSI effect, we consider the following definitions: the reference domains are denoted by $\widehat{\Omega}_f$ and $\widehat{\Omega}_s$, respectively, with their common interface $\partial\widehat{\Omega}_i$ at time, $t = 0$. The fluid external boundaries $\partial\Omega_{f_{in}}$ and $\partial\Omega_{f_{out}}$ are supposed to be fixed. The corresponding outward normal vectors to the fluid and solid boundaries are denoted by n_f and n_s, respectively. The variational ALE formulation of the fluid part is transformed from its Eulerian description into an arbitrary Lagrangian framework and stated on the (arbitrary) reference domain $\widehat{\Omega}_f$, while the structure part is formulated in Lagrangian coordinates on the domain $\widehat{\Omega}_s$, where $\widehat{\Omega} = \widehat{\Omega}_f \cup \partial\widehat{\Omega}_i \cup \widehat{\Omega}_s$. Moreover, here we solve the Laplace equation for the definition of the ALE mapping. Here, the continuity of velocity $\widehat{v}_f = \widehat{v}_s$ and $\widehat{u}_f = \widehat{u}_s$ across the common fluid-structure interface on $\partial\widehat{\Omega}_i(t) = \partial\widehat{\Omega}_f(t) \cap \partial\widehat{\Omega}_s(t)$. The ALE mapping is denoted by $\hat{T}$ and transforms the reference configuration $\hat{\Omega}_f$ of the fluid to the physical domain $\Omega_f(t)$. Furthermore, any function $\hat{q} \in \hat{\Omega}$ can be defined by:

$$q(x) = \hat{q}(\hat{x}) \quad \text{with} \quad x = \hat{T}(\hat{x},t) \tag{1}$$

In this FSI problem, the principal unknowns are the fluid domain displacement (mesh motion) $\hat{u}_f : \hat{\Omega}_f \times \mathbb{R}^+ \mapsto \mathbb{R}^3$, the fluid velocity $\hat{v}_f : \hat{\Omega}_f \times \mathbb{R}^+ \mapsto \mathbb{R}^3$, the fluid pressure $\hat{p} : \hat{\Omega}_f \times \mathbb{R}^+ \mapsto \mathbb{R}$ and the structure displacement $\hat{u}_s : \hat{\Omega}_s \times \mathbb{R}^+ \mapsto \mathbb{R}^3$. Thus $\hat{u}_f = Ext(\hat{u}_s|_{\partial\Omega_i(t)})$, the ALE mapping is defined through the extension of the structural displacement (for large mesh deformations without remeshing) into the fluid domain and determine by solving following biharmonic equation

$$\begin{aligned}
\Delta^2 \hat{u}_f &= 0 & &\text{in } \hat{\Omega}_f, \\
\hat{u}_f = \partial_n \hat{u}_f &= 0 & &\text{on } (\hat{\partial}\Omega_{f,in} \cup \hat{\partial}\Omega_{f,out}), \\
\hat{u}_f = \hat{u}_s \text{ and } \partial_n \hat{u}_f &= \partial_n \hat{u}_s & &\text{on } \hat{\partial}\Omega_i.
\end{aligned} \tag{2}$$

The ALE map is constructed by solving a mixed formulation of the biharmonic equation, where we introduce an auxiliary variable $\hat{w} = -\hat{\Delta}\hat{u}$ in $\hat{\Omega}$ and obtain

$$\begin{aligned}
\hat{w} &= -\hat{\Delta}\hat{u} & &\text{in } \hat{\Omega}, \\
-\hat{\Delta}\hat{w} &= 0 & &\text{in } \hat{\Omega}_f, \\
\hat{u}_f = \partial_n \hat{u}_f &= 0 & &\text{on } (\hat{\partial}\Omega_{f,in} \cup \hat{\partial}\Omega_{f,out}), \\
\hat{u}_f = \hat{u}_s \text{ and } \partial_n \hat{u}_f &= \partial_n \hat{u}_s & &\text{on } \hat{\partial}\Omega_i.
\end{aligned} \tag{3}$$

The ALE approach belongs to interface-tracking methods in which the mesh is moved such that it fits in all time steps with the FSI-interface. However, this leads to a degeneration of the ALE map. Methods to circumvent such as degeneration as long as possible are re-meshing techniques or to use (as suggested here) a biharmonic mesh motion technique. In this case the deformation gradient and its determinant become:

$$\hat{F} := I + \hat{\nabla}\hat{u}, \quad \hat{J} = \det(\hat{F}).$$

In the structure domain, $\widehat{T}$ takes the place of the Lagrangian-Eulerian coordinate transformation, while in the fluid domain, $\widehat{T}$ has no physical meaning but serves as ALE mapping. The corresponding equations are stated in the following:

Problem-1: The Navier-Stokes equations in ALE framework

Find $\widehat{v}_f \in \widehat{v}_f^D + \widehat{\mathscr{V}}^0_{\widehat{\Omega}_f}$ and $\widehat{p}_f \in \widehat{\mathscr{L}}_{\widehat{\Omega}_f}$, such that the initial data satisfiy $v_f(x,0) = v_f^0$ in Ω_f, and for almost all $t \in I$ it holds that:

$$\begin{aligned}
&\left(\widehat{J}_f\widehat{\rho}_f\partial_t\widehat{v}_f, \widehat{\phi}^v\right)_{\widehat{\Omega}_f} + \left(\widehat{J}_f\widehat{\rho}_f(\widehat{F}_f^{-1}(\widehat{v}_f - \partial_t\widehat{u}_f).\widehat{\nabla})\widehat{v}_f), \widehat{\phi}^v\right)_{\widehat{\Omega}_f}\\
&+\left(\widehat{J}_f\widehat{\sigma}_f\widehat{F}_f^{-T}, \widehat{\nabla}\widehat{\phi}^v\right)_{\widehat{\Omega}_f} - \langle\widehat{J}_f\widehat{\sigma}_f\widehat{F}_f^{-T}, \widehat{\phi}^v\rangle_{\widehat{\Gamma}_i} - \left(\widehat{J}_f\widehat{\rho}_f\widehat{f}_f, \widehat{\phi}^v\right)_{\widehat{\Omega}_f} - \langle\widehat{h}, \widehat{\phi}^v\rangle_{\widehat{\Gamma}_{f_N}} = 0 \quad \forall\widehat{\phi}^v \in \widehat{V}^0_{\widehat{\Omega}_f}\\
&\left(\widehat{div}(\widehat{J}_f\widehat{F}^{-1}\widehat{v}_f), \widehat{\phi}^p\right)_{\widehat{\Omega}_f} = 0 \quad \forall\widehat{\phi}^p \in \widehat{L}_{\widehat{\Omega}_f}
\end{aligned} \tag{4}$$

Problem-2: The Structural equations in Lagrangian framework

Find $\widehat{u}_s \in \widehat{u}_s^D + \widehat{\mathscr{V}}^0_{\widehat{\Omega}_s}$ and $\widehat{v}_s \in \widehat{v}_s^D + \widehat{\mathscr{V}}^0_{\widehat{\Omega}_s}$ such that the initial data satisfies $\widehat{u}_s(\widehat{x},0) = \widehat{u}_s^0(\widehat{x})$ in $\widehat{\Omega}_s$, and for almost all $t \in I$ it holds that:

$$\begin{aligned}
&(\partial_t\hat{u}_s - \hat{v}_s, \hat{\phi}^u)_{\widehat{\Omega}_s} = 0 \quad \forall\phi^u \in \widehat{V}^0_{\widehat{\Omega}_s}\\
&\left(\widehat{\rho}_s\partial_t\widehat{v}_s, \widehat{\phi}^v\right)_{\widehat{\Omega}_s} + (\widehat{J}_s\widehat{\sigma}_s\widehat{F}_s^{-T}, \widehat{\nabla}\widehat{\phi}^v)_{\widehat{\Omega}_s} - \langle\widehat{J}_s\widehat{\sigma}_s\widehat{F}_s^{-T}, \widehat{\phi}^v\rangle_{\widehat{\Gamma}_i} - (\widehat{\rho}_s\widehat{f}_s, \widehat{\phi}^v)_{\widehat{\Omega}_s} = 0 \quad \forall\phi^v \in \widehat{V}^0_{\widehat{\Omega}_s}
\end{aligned} \tag{5}$$

Let us define the density $\hat{\rho}$ and the Cauchy stress tensor $\hat{\sigma}$ for the whole domain by-

$$\hat{\rho}(\hat{x}) = \begin{cases} \hat{\rho}_f(\hat{x}), & \hat{x} \in \hat{\Omega}_f \\ \hat{\rho}_s(\hat{x}), & \hat{x} \in \hat{\Omega}_s \cup \partial\hat{\Omega}_i \end{cases} \qquad \hat{\sigma}(\hat{x}) = \begin{cases} \hat{\sigma}_f(\hat{x}), & \hat{x} \in \hat{\Omega}_f, \\ \hat{\sigma}_s(\hat{x}), & \hat{x} \in \hat{\Omega}_s \cup \partial\hat{\Omega}_i, \end{cases}$$

and by using monolithic approach, we get from **Problem-1** and **2**:

Problem-3: The FSI problem in ALE framework

Find $\{\hat{v}, \hat{u}, \hat{w}, \hat{p}\} \in \{\hat{v}^D + \hat{\mathscr{V}}^0_{\hat{\Omega}}\} \times \{\hat{u}^D + \hat{\mathscr{V}}^0_{\hat{\Omega}}\} \times \hat{\mathscr{V}} \times \hat{\mathscr{L}}_{\hat{\Omega}}$, such that $\hat{v}(0) = \hat{v}^0$ and $\hat{u}(0) = \hat{u}^0$, for almost all $t \in I$ it holds that:

$$\begin{aligned}
&(\hat{J}\hat{\rho}\partial_t\hat{v}, \hat{\phi}^v)_{\hat{\Omega}_f} + (\hat{\rho}\hat{J}(\hat{F}^{-1}(\hat{v} - \partial_t\hat{u}) \cdot \hat{\nabla})\hat{v}), \hat{\phi}^v)_{\hat{\Omega}_f}\\
&+(\hat{J}\hat{\sigma}\hat{F}^{-T}, \hat{\nabla}\hat{\phi}^v)_{\hat{\Omega}_f} - (\hat{J}\hat{\rho}\hat{f}_f, \hat{\phi}^v)_{\hat{\Omega}_f} - \langle\hat{h}, \hat{\phi}^v\rangle_{\hat{\Gamma}_N}\\
&+(\hat{\rho}\partial_t\hat{v}, \hat{\phi}^v)_{\hat{\Omega}_s} + (\hat{J}\hat{\sigma}\hat{F}^{-T}, \hat{\nabla}\hat{\phi}^v)_{\hat{\Omega}_s} - (\hat{\rho}\hat{f}_s, \hat{\phi}^v)_{\hat{\Omega}_s} = 0 \quad \forall\hat{\phi}^v \in \hat{V}^0_{\hat{\Omega}},\\
&(\hat{\alpha}_u\hat{w}, \hat{\phi}^w)_{\hat{\Omega}_f} + (\hat{\alpha}_u\hat{\nabla}\hat{u}, \hat{\nabla}\hat{\phi}^w)_{\hat{\Omega}_f} + (\hat{\alpha}_u\hat{\nabla}\hat{w}, \hat{\nabla}\hat{\phi}^w)_{\hat{\Omega}_s} = 0 \quad \forall\hat{\phi}^w \in \hat{V}_{\hat{\Omega}},\\
&\hat{\rho}(\partial_t\hat{u} - \hat{v}, \hat{\phi}^u)_{\hat{\Omega}_s} + (\hat{\alpha}_u\hat{\nabla}\hat{w}, \hat{\nabla}\hat{\phi}^u)_{\hat{\Omega}_f} = 0 \quad \forall\hat{\phi}^u \in \hat{V}^0_{\hat{\Omega}},\\
&(\widehat{div}(\hat{J}\hat{F}^{-1}\hat{v}), \hat{\phi}^p)_{\hat{\Omega}_f} + (\hat{p}_s, \hat{\phi}^p)_{\hat{\Omega}_s} = 0 \quad \forall\hat{\phi}^p \in \hat{L}_{\hat{\Omega}},
\end{aligned} \tag{6}$$

The stress tensors for the fluid and structure are implemented by

$$\hat{\sigma}_f = -\hat{p}I + \hat{\rho}_f \nu_f (\hat{\nabla}\hat{v}\hat{F}^{-1} + \hat{F}^{-T}\hat{\nabla}\hat{v}^T),$$
$$\hat{\sigma}_s = \hat{J}^{-1}\hat{F}(2\mu_s\hat{E} + \lambda_s tr\hat{E}I)$$

with the viscosity ν_f, the Lamé parameters μ_s, λ_s and $\hat{E} = \frac{1}{2}(\hat{F}^T\hat{F} - I)$.

Where, we notice that this problem is driven by a Dirichlet inflow condition. In this formulation, for momentum equations, integration by parts in both subdomains yields the boundary term on $\widehat{\Gamma}_i$ as:

$$\left(\widehat{n}_f.(\widehat{J}\widehat{\sigma}_s\widehat{F}^{-T}), \widehat{\phi}^v\right)_{\widehat{\Gamma}_i} + \left(\widehat{n}_s.(\widehat{J}\widehat{\sigma}_f\widehat{F}^{-T}), \widehat{\phi}^v\right)_{\widehat{\Gamma}_i} = 0.$$

By omitting this boundary integral jump over $\partial\hat{\Omega}_i$ the weak continuity of the normal stresses becomes an implicit condition of the fluid-structure interaction problem.

Our main target is to study the impact of FSI on modelling and simulation of wave propagation in composite materials, where we need an appropriate treatment of microstructure of composite material which typically cannot be resolved by finite element meshes. Simulation of the elastic wave propagation with the FSI effect on an aircraft wing is still in progress. In this project we will focus mostly on the concurrent numerical coupling techniques of **Problem-3** & **4**, where we will build a mathematical model that integrates the coupling methodology. In this case, we will use a monolithic approach in which all equations are solved simultaneously. Employing strongly coupled approach, the interface conditions, the continuity of velocity and the normal stresses, are automatically achieved at each time step. Here, a coupled monolithic variational formulation is an inevitable prerequisite for gradient based optimization methods, for rigorous goal oriented error estimation and mesh adaptation. However, this coupling leads to additional nonlinear behavior of the overall system.

Problem-4: The Elastic Waves equations in Lagrangian framework

Find $\widehat{u}_w \in \widehat{u}_w^D + \widehat{\mathscr{V}}^0_{\widehat{\Omega}_s}$ and $\widehat{v}_w \in \widehat{v}_w^D + \widehat{\mathscr{V}}^0_{\widehat{\Omega}_s}$ such that the initial data satisfies $\widehat{u}_w(\widehat{x},0) = \widehat{u}_w^0(\widehat{x})$ in $\widehat{\Omega}_s$, and for almost all $t \in I$ it holds that:

$$(\partial_t\hat{u}_w - \hat{v}_w, \hat{\phi}^u)_{\widehat{\Omega}_s} = 0 \quad \forall\phi^u \in \widehat{V}^0_{\widehat{\Omega}_s}$$
$$\left(\widehat{\rho}_s\partial_t\widehat{v}_w, \widehat{\phi}^v\right)_{\widehat{\Omega}_s} + (\widehat{J}_s\widehat{\sigma}_s\widehat{F}_s^{-T}, \widehat{\nabla}\widehat{\phi}^v)_{\widehat{\Omega}_s} - \langle\widehat{J}_s\widehat{\sigma}_s\widehat{F}_s^{-T}, \widehat{\phi}^v\rangle_{\widehat{\Gamma}_s} - (\widehat{\rho}_s\widehat{f}_w, \widehat{\phi}^v)_{\widehat{\Omega}_s} = 0 \quad \forall\phi^v \in \widehat{V}^0_{\widehat{\Omega}_s} \tag{7}$$

Here **Problem-4** is not strongly coupled with **Problem-3**. To slove the elastic wave equation in Lagrangian framework, for each time step we will need to consider the boundary conditions, $\langle\widehat{J}_s\widehat{\sigma}_s\widehat{F}_s^{-T}, \widehat{\phi}^v\rangle_{\widehat{\Gamma}_s}$ from **Problem-3**.

3. Numerical Approximation of FSI Problem

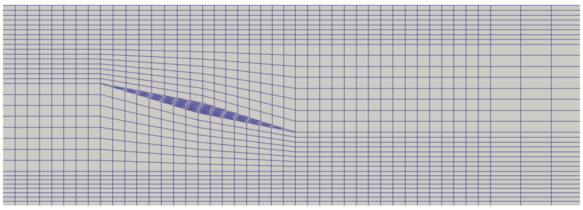
In this paper, we mainly deal with a double wedge airfoil ($2D$), where the left end of this airfoil is considered rigid and the control points $A(t)$ are fixed at the trailing edge with $A(t)|_{t=0} = (0.6, 0.147)$. We observed different behaviors of the double wedge airfoil with different FSI test cases that are treated with different inflow velocities (see Table-1) [1–4, 6]. The parameters are chosen such that a visible transient behavior of the double wedge airfoil can be seen. To ensure a 'fair' comparison of results, we calculate the comparison values using the ALE method.

Table 1 : Parameter setting for the FSI test cases

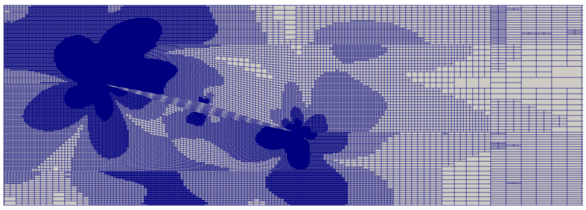
Parameter	Test-1	Test-2	Test-3
Structure model	STVK	STVK	STVK
$\rho_f[kgm^{-3}]$	1000	1000	1000
$\rho_s[kgm^{-3}]$	2710	2710	2710
$\nu_f[m^{-2}s{-}1]$	1×10^{-3}	1×10^{-3}	1×10^{-3}
ν_s	0.33	0.33	0.33
$\mu_s[kgm^{-1}s^{-2}]$	68.9×10^{6}	68.9×10^{6}	68.9×10^{6}
$U_m[ms^{-1}]$	0.2	1.0	2.0

One of the main issues of this numerical example is the automatic mesh adaptation within the ALE finite element approximation of the FSI problem. The computations presented below have been done on three different types of meshes [1–4, 6, 10, 11]:

- globally refined meshes obtained using several steps of uniform refinement of a coarse initial mesh,
- locally refined meshes obtained using a purely geometry-based criterion by marking all cells for refinement which have certain prescribed distances from the fluid-structure interface,
- locally refined meshes obtained using a systematic residual-based criteria by marking all cells for refinement which have error indicators above a certain threshold.



(a) DoFs 37056

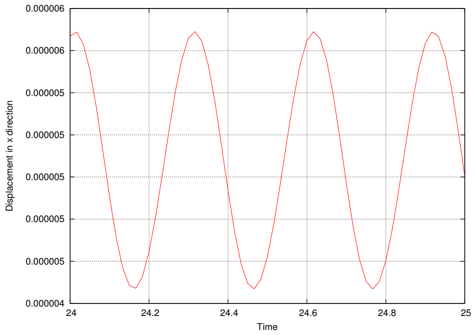


(b) DoFs 3265836

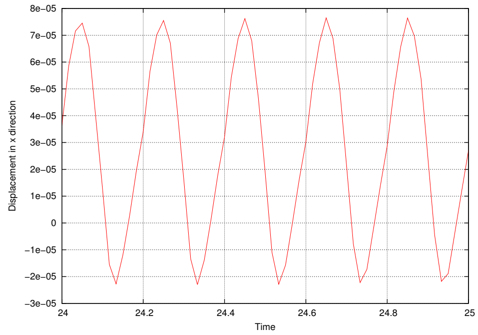
Figure 1: A regular mesh after eight cycles of patchwise adaptive refinement.

Finally the target is to apply the Dual Weighted Residual Method (DWR method) for the adaptive solution of FSI problems. The DWR method provides a general framework for the derivation of a goal-oriented posteriori error estimates together with criteria of mesh adaptation for the Galerkin discretization of general linear and nonlinear variational problems, including optimization problems. In order to incorporate also the time discretization into this framework, we have to use a fully space-time Galerkin method, i.e., a standard finite element method in space combined with the discontinuous Galerkin $dG(r)$ or continuous Galerkin $cG(r)$ method in time. The following discussion assumes such a space-time Galerkin discretization, though in our test computations, we have used the fractional-step-θ scheme which is a finite difference scheme. Accordingly, in this paper the DWR method is used only in its stationary form in computing either steady states or intermediate quasi-steady states within the time stepping process. We refer to [1–4, 6] for more details about the DWR method and goal-oriented posteriori error estimates.

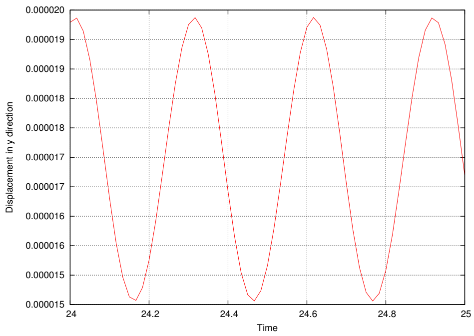
The computed values of the test-1, 2 & 3 are summarized in the Table-2. The results of the test-2 & 3 are displayed in the Figure-2. In test cases FSI-1, the deflection of trailing edge of the airfoil became steady, while in FSI test-2 and 3, we observed oscillating behavior. Therefore, we



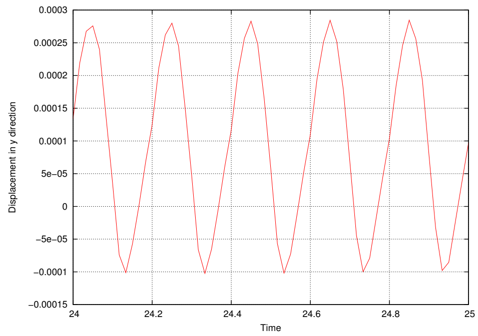
(a) Test Case-2, u_x



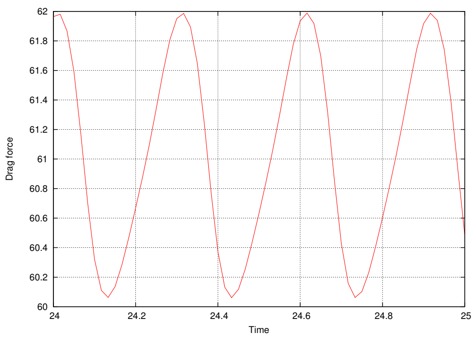
(b) Test Case-3, u_x



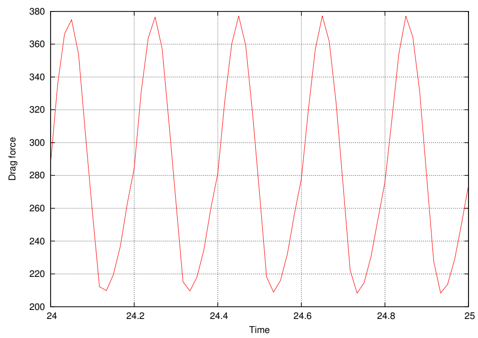
(c) Test Case-2, u_y



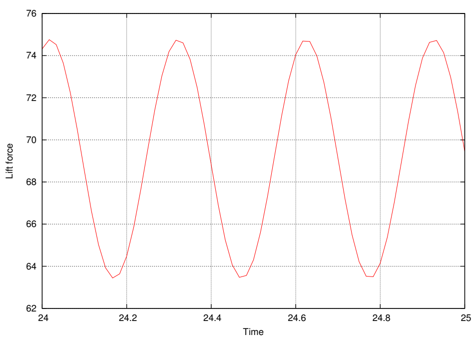
(d) Test Case-3, u_y



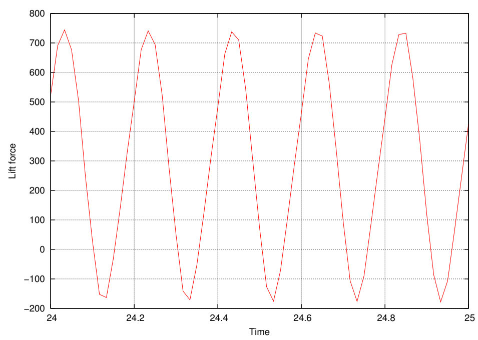
(e) Test Case-2, F_D



(f) Test Case-3, F_D



(g) Test Case-2, F_L



(h) Test Case-3, F_L

Figure 2: Variation of displacement (u_x, u_y), drag force (F_D) and lift force (F_L) over time (t)

Table 2 : Results for the test case 1, 2 and 3.

	Test-1	Test-2	Test-3
$DoFs$	3265836	3265836	3265836
Time step size $k[s]$	0.01	0.001	0.0001
$u_x(A)[\times 10^{-5}]$	0.16	0.51±0.061	2.68±4.97
$u_y(A)[\times 10^{-5}]$	0.56	1.75±0.241	9.22±19.22
F_D	15.69	61.03±0.96	292.73±84.5
F_L	24.99	69.11±5.61	285.01±456.1

conclude that the deflective behavior of structures becomes unsteady under high-speed fluid flow, which can be cause of massive fatalities.

4. REMARKS

For structural health studies and the design of appropriate monitoring systems it is of a great importance and a challenging issue to separate modelling and numerical errors in the simulated results from the material defects effects. Our work on the mathematical coupling of an elastic wave propagation in anisotropic and composite media with the FSI effect on an aircraft wing is still in progress. This project will focus mostly on the concurrent coupling techniques of **Problem-3** & **4**, where we will build a mathematical model that integrates the coupling methodology. Also part of this project aims to develop an efficient numerical method to design an integrate SHM systems, which combine modern techniques from PDE-constrained optimization, adaptive and multigrid simulation methods.

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