Calibration of the Finite Element Model of a Twelve-Span Prestressed Concrete Bridge Using Ambient Vibration Data

Xinghua Chen, Piotr Omenzetter, Sherif Beskhyroun

CALIBRATION OF THE FINITE ELEMENT MODEL OF A TWELVE-SPAN PRESTRESSED CONCRETE BRIDGE USING AMBIENT VIBRATION DATA

Xinghua Chen[1], Piotr Omenzetter[2], Sherif Beskhyroun[1]

[1] *Department of Civil and Environmental Engineering, University of Auckland, New Zealand*
[2] *The LRF Centre for Safety and Reliability Engineering, University of Aberdeen, UK*

piotr.omenzetter@abdn.ac.uk

ABSTRACT

The recently constructed Newmarket Viaduct in Auckland is a critical link in the New Zealand state highway network. Newmarket Viaduct is a 12-span, pre-cast, post-tensioned structure built using the balanced cantilever method. A continuous health monitoring system was designed and installed in the bridge. As a critical part in the SHM process, a baseline finite element (FE) model of Newmarket Viaduct was established. This paper describes the implementation of the FE model calibration using ambient vibration data. The initial model of the bridge was developed from the information provided in the design documentation, material testing data and site inspections. Two ambient vibration testing campaigns used some 60 wireless sensors in multiple setups to collect data to map with high density 3D mode shapes of the bridge. The output-only modal identification results obtained from the ambient vibration measurements of the bridge were used to update the FE bridge model. Different parameters of the model were calibrated using an automated procedure to improve the correlation between measured and calculated modal parameters. Careful attention was paid to the selection of the parameters to be modified during updating in order to ensure that the changes to the model were realistic and physically meaningful. The calibrated FE model reflecting the as-built structural condition and dynamic response mechanisms of Newmarket Viaduct will serve as a baseline model for assessment of structural health using continuous monitoring data.

KEYWORDS : *Bridge, model updating, baseline model, structural health monitoring, ambient vibration test*

INTRODUCTION

Calibration of finite element (FE) models of large civil engineering structures has recently received a great deal of attention. Accurate FE models that are representative of the actual structures are indispensable for studies such as validation of innovative structural designs, structural health monitoring, evaluation of seismic response, assessing post-earthquake condition, and structural control [1]. However, the initial FE models constructed on the basis of engineering blueprints and design drawings and specifications do not usually match the experimental results measured in the field [2-4]. The discrepancies are usually related to the variation in the material properties, uncertainties in geometry and boundary conditions and inaccuracy in the FE model discretisation. Although some of the discrepancies and uncertainties can be minimised by developing more detailed FE models, it is not possible to obtain a highly accurate match between the analytical and measured responses before calibration of the FE model is conducted.

Calibration of a FE model, also known as FE model updating, is a procedure to determine uncertain parameters in the initial model based on experimental results to achieve a model of the structure that better matches the experimental results [5]. Among the different types of field experiments, ambient vibration tests are a useful and popular approach to the identification of modal properties (natural frequencies, damping ratios and mode shapes), which has been used and reported by several authors [6-8]. Thus, the model calibration gaol is normally formulated mathematically as the minimisation of an objective function based on the residuals between the measured and analytically computed frequencies and modes shapes. In the optimisation process, the parameters of the FE model are adjusted to obtain a better match between measured and analytical modal responses. A number of model updating techniques have been proposed during the past decades [9-12]. There are basically two distinct FE model updating methodologies in structural dynamics: the direct methods [13] and the iterative methods [14]. The first approach directly updates the mass and stiffness matrices of the structure but it is very difficult to relate the changes inside the updated system matrices to physical properties of the FE model [5]. Furthermore, these methods can be very complicated for large structures with a detailed FE model and can result in ill-conditioned problems [9, 11]. Conversely, the iterative methods are more flexible and efficient in its application for large-scale structures with detailed FE models as the physical properties behind the FE model, such as material and geometric properties, can be updated. Sensitivity-based model updating approaches are an efficient iterative way of updating the structural parameters of the FE model [15].

This paper presents the implementation of a sensitivity-based approach to FE model calibration of a 12-span, pre-cast, post-tensioned concrete bridge using ambient test data. The bridge under investigation, Newmarket Viaduct, is a critical link in the New Zealand state highway network. The initial FE model of the bridge is developed to represent the bridge as realistically as possible from the information provided in the design documentation, material testing data and site inspections. Two ambient vibration testing campaigns used some 60 wireless sensors in multiple setups to collect data to map with high density 3D mode shapes of the bridge. The output-only modal identification results obtained from ambient vibration measurements are used to update the FE bridge model. Different parameters of the model are calibrated using a sensitivity-based automated procedure to improve correlation between the measured and calculated modal parameters. Careful attention is paid to the selection of the parameters to be modified during updating in order to ensure that the changes to the model were realistic and physically meaningful. The modal properties of the updated FE model match well with the field-measured natural frequencies and mode shapes.

1 SENSITIVITY-BASED MODEL CALIBRATION

The model calibration approach used in this paper is based on a sensitivity-based model updating procedure that seeks to minimise the error between the experimental and FE-computed modal characteristics of the structure. The sensitivity based model updating procedure generally comprises of three stages: (i) selection of responses as reference data, (ii) selection of physical parameters to update, and (iii) an iterative model tuning [16]. Experimental responses (**R**$_e$) are usually the natural frequencies and modes shapes measured on the real structure, whereas the updating parameters (**P**) are uncertain parameters in the FE model which can include geometric and material properties, and boundary and connectivity conditions related to stiffness and inertia. If accurate parameters (**P**$_a$) are not used as the input to the FE model, analytical responses (**R**$_a$) different from the measured responses will be obtained. The relationship between the actual and FE-computed modal frequencies can be expressed as a first order Taylor series with respect to the structural parameters and a sensitivity coefficient matrix [2] :

$$\mathbf{R}_e = \mathbf{R}_a + \mathbf{S}\left(\mathbf{P}_a - \mathbf{P}\right) \tag{1}$$

or

$$\Delta\mathbf{R} = \mathbf{S}\Delta\mathbf{P} \tag{2}$$

where $\Delta\mathbf{R}$ is the difference between the measured and FE-computed responses, $\Delta\mathbf{P}$ is the difference between the actual physical parameters and the parameter estimates used in the FE model, and **S** is the sensitivity matrix of the experimental responses with respect to the physical parameters:

$$\boldsymbol{S}_{ij} = \frac{\partial \boldsymbol{R}_{a,i}}{\partial \boldsymbol{P}_j} \tag{3}$$

Here $\mathbf{R}_{a,i}$ ($i=1, 2,\ldots,n$) and $\mathbf{P}_j$ ($i=1, 2,\ldots\ \ldots\ \ldots,m$) are the entries of the analytical structural response and the updating structural parameter vectors. In this research, an objective function related to the natural frequencies has been used. The sensitivity-based objective function to minimize the error between the measured and FE-computed modal frequencies ($\Delta\mathbf{f}$) can be written as:

$$J = \frac{1}{2}(\Delta\mathbf{f}\text{-}\mathbf{S}\Delta\mathbf{P})^T(\Delta\mathbf{f}\text{-}\mathbf{S}\Delta\mathbf{P}) \tag{4}$$

where superscript T denotes transposition. The first order optimization method and penalty function concept [5] is utilized to minimize the objective function. Equation (4) can be rewritten in term of a first order optimization function as

$$J = \frac{1}{2}(\Delta\mathbf{f}\text{-}\mathbf{S}\Delta\mathbf{P})^T(\Delta\mathbf{f}\text{-}\mathbf{S}\Delta\mathbf{P}) + \frac{1}{2}\alpha\Delta\mathbf{P}^T\Delta\mathbf{P} \tag{5}$$

where $\alpha\Delta\mathbf{P}^T\Delta\mathbf{P}$ represents the penalty on the constrained physics parameters and α is a weighting parameter (assumed 1 in the subsequent simulations). This method is known as the sensitivity-based penalty function method since the error due to the linear approximation by the Taylor series is penalized to find the smallest parameter changes $\Delta\mathbf{P}$ at each iteration. The optimization process used to obtain the optimum values of the physical parameters, **P**, is set as an iterative process as shown in Figure 1, where k refers to the iteration step. The sensitivity matrices are calculated at each iteration in order to optimize the sensitivity-based objective function. Additional constraints are also applied to prevent physically unreasonable solutions for the physical parameters. These are the lower bounds (b_l) and upper bounds (b_u) of the parameters used for updating the FE model. Finally, the convergence of the iterative solution is checked by evaluating the following two criteria: (i) the maximum error between the measured and calculated modal frequencies should be less than 3%, and (ii) the maximum relative changes of updated frequencies between two consecutive iterations should be less than 0.1%.

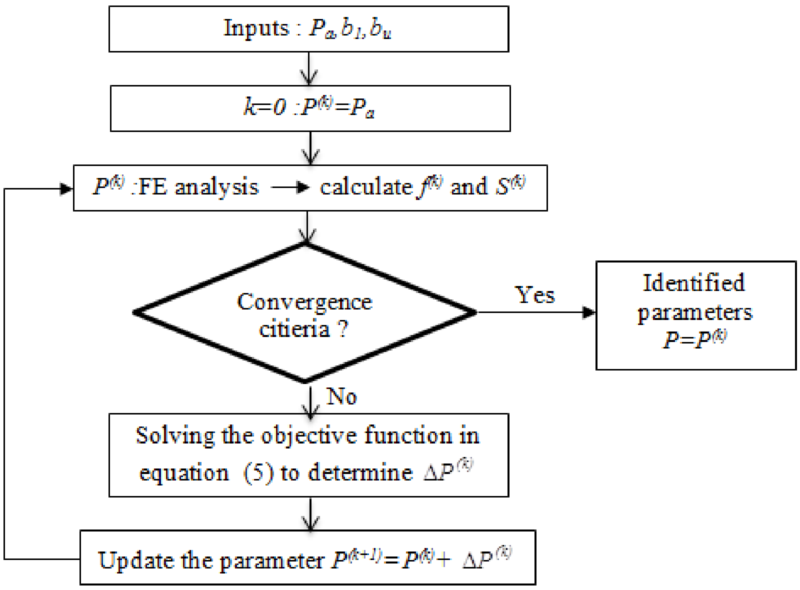

Figure 1: Sensitivity-based FE model updating procedure

2 FE MODEL OF THE BRIDGE

The bridge under investigation is the Newmarket Viaduct (Figure 2) located in Auckland, New Zealand. It is a curved 12-span post-tensioned concrete bridge, comprising two parallel, twin bridges (Northbound and Southbound). The bridge is supported by two abutments at both ends and 11 concrete piers. The total length of the bridge is 690 m, with twelve different spans ranging in length from 38.67 m to 62.65 m and average length of approximately 60 m. The superstructure of the bridge is a continuous twin-cell box girder of a total width of 30 m. The Northbound and Southbound Bridges are supported on independent pylons and only joined together via a cast in-situ concrete 'stitch' at the deck girder upper flange level. At the abutments and four interior supports, bridge deck is supported on bi-directional elastomeric seismic devices. For the other supports, the bridge bent bearings were fixed in all directions.

A detailed three-dimensional (3D) FE model of the as-built bridge was developed using SAP2000 finite element software to simulate realistic responses of the bridge. A view of the FE model of the bridge is shown in Figure 2. The concrete deck and all the piers were represented using solid elements. The elastic modulus of the concrete solid elements was initially computed based on the compressive strength of 60 MPa, which was first determined by the authors by testing 100×200 mm cylinder specimens that were cast during construction of deck slab and piers and then supplemented by the analogous tests conducted by the contractor. The bearings were modelled using link elements. The nominal value of the stiffness provided by the manufacturer was assigned to each link element.

3 CALIBRATION OF THE FE MODEL

3.1 Experimental results

The ambient vibration testing reported herein was conducted on November 29 and 30, 2012 under operational conditions and did not interfere with the normal flow of traffic over the bridge as the testing personnel worked exclusively inside the box girder. The accelerometers used for the test were two models of wireless USB accelerometers produced by the Gulf Coast Design Concepts (www.gcdataconcepts.com): X6-1A and X6-2. A total of 288 measurements points (24 for each span) on both sides inside the box girders were chosen for placing accelerometers in order to map accurately mode shapes. The accelerometers were 'lightly' glued to the internal surface of the bridge deck using silicone adhesives (Figure 3). Six test setups were used to cover the planned testing locations of both bridges. The sampling frequency was 160Hz and corresponding recording times were all approximately 1 hour for each setup.

The modal frequencies and mode shapes were identified using the Frequency Domain Decomposition (FDD) algorithm. The modal frequencies (*f*) were estimated at peak locations in the first singular value plots of spectral density functions (marked in Figure 3). This allowed the identification of 6 transverse, 6 torsional and 8 vertical modes. These first twenty extracted modal frequencies of the bridge are shown in Column 3 of Table 1. The standard deviations associated with the identified natural frequencies between the different setups are very small (between 0 and 0.04 Hz). Figure 4 shows selected mode shapes and the calculated modal assurance criteria (MAC) [17] values between the experimental and numeral model shapes are also presented in Table 1.

Figure 2: Newmarket Viaduct (left) and its 3D FE model (right).

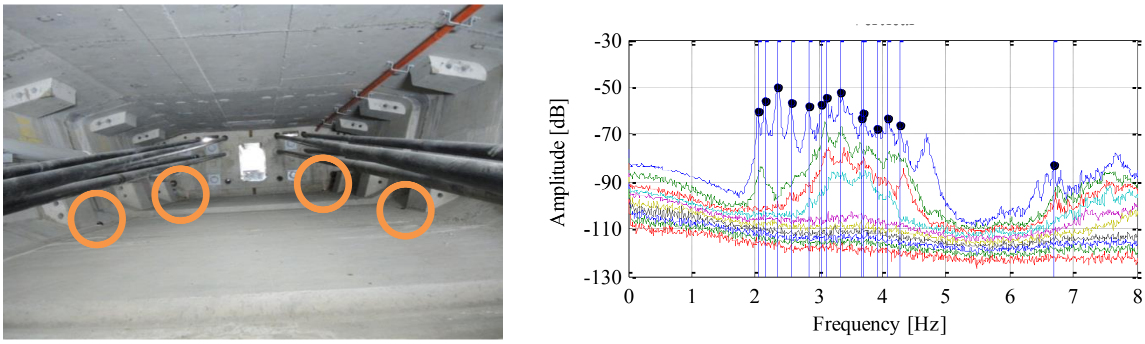

Figure 3: Location of accelerometers inside bridge girder (left) and singular values of spectral density functions of accelerations (right).

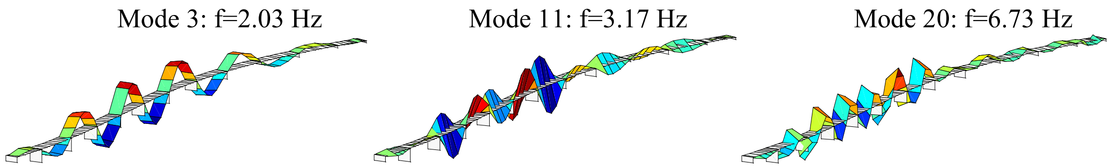

Figure 4: Selected mode shapes identified by FDD.

Table 1: Comparison of measured and FE computed results.

Mode	Type	FDD	FE Frequencies [Hz]		MAC values	
		f [Hz]	Initial Model	Updated Model	Initial Model	Updated Model
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Transverse	1.25	1.18	1.23	0.959	0.950
2	Transverse	1.56	1.47	1.54	0.828	0.892
3	Vertical	2.03	2.14	2.09	0.964	0.971
4	Transverse	2.15	2.01	2.13	0.752	0.790
5	Vertical	2.15	2.22	2.19	0.971	0.942
6	Vertical	2.34	2.43	2.40	0.875	0.887
7	Vertical	2.55	2.63	2.59	0.851	0.862
8	Transverse	2.81	2.68	2.76	0.619	0.713
9	Vertical	2.82	2.85	2.84	0.907	0.917
10	Vertical	3.09	3.15	3.16	0.550	0.921
11	Torsion	3.17	3.16	3.15	0.863	0.802
12	Torsion	3.34	3.38	3.34	0.895	0.872
13	Vertical	3.67	3.69	3.66	0.692	0.713
14	Torsion	3.71	3.78	3.73	0.694	0.682
15	Transverse	3.94	3.67	3.83	0.692	0.710
16	Torsion	3.94	3.97	3.92	0.563	0.836
17	Torsion	4.06	4.17	4.11	0.853	0.942
18	Torsion	4.27	4.40	4.38	0.879	0.858
19	Transverse	4.77	4.71	4.79	0.683	0.971
20	Vertical	6.73	6.66	6.64	0.775	0.824

Table 2: Parameters used for the FE model updating process and their values before and after updating.

Parameters	Allowable bounds	Initial values	Updated values
Elastic modulus of deck (GPa)	34.0-41.4	36.0	39.2
Elastic modulus of piers (GPa)	34.0-41.4	36.0	38.5
Thickness of the asphalt overlay (m)	0-0.050	0	0.042
Mass density of deck (kg/m^3)	2420-2680	2550	2433
Mass density of piers (kg/m^3)	2420-2680	2550	2420
Stiffness of bearings (MN/m)	50-300	80	162

3.2 Selection modal frequencies for calibration process

Although it is ideal to calibrate the FE models of bridges using as many modal frequencies and mode shapes as possible, including more vibration modes in the model updating problem can make the optimisation problem more complex. Minimising the error between the measured and FE-computed modal frequencies for higher modes may interfere with matching the lower modes of vibration. Therefore, it is important to identify which modes should be included and those that can be ignored during updating. Based on the approach proposed by Li etc. [18], the modal contribution coefficient for the first identified 20 modal frequencies was computed and used to determine the number of modes that contribute most significantly to the structural response of the bridge. The frequencies used to compute the modal contribution coefficients were obtained using the initial FE model of the bridge. The modes that cumulatively give 90% of the total deformation are considered as the most important. This corresponds to the first 14 modes of vibration. However, only the first nine natural frequencies (five vertical and four transverse) did not overlap with other close modal frequencies, and were therefore used in the FE model updating process. The remaining modal frequencies and all mode shapes were reserved for the purpose of validation of the updated FE model.

3.3 Selection of parameters

The selection of parameters in the calibration process is critical for the success of any such exercise. An excessive number of parameters compared to the number of available responses, or overparametrization, will lead to a non-unique solution, whereas insufficient number of parameters will prevent reaching a good agreement between the experiment and numerical model [19]. The changes in the selected parameters should potentially have a considerable effect on the vibration response of the bridge. These parameters are those that contribute significantly to the mass or stiffness properties of the structure. Therefore, the material properties of the major structural components (mass density and Young's modulus) and size/thickness of those structural components are potential parameters that can be selected for FE model calibration. Also, parameters are selected from among those whose exact values have high degrees of uncertainty. It is necessary, therefore, to select those parameters to which the numerical responses are sensitive and at the same time those whose values are uncertain in the initial model. After a careful consideration of the initial FE model and the available engineering drawings, the parameters selected for calibration in this study were the elastic modulus of concrete, thickness of the asphalt overlay, mass density of concrete and horizontal stiffness of the bearings. The selected parameters along with the allowable bounds of their values are listed in Table 2. Selecting the bounds on the allowable parameter variations during model updating is generally challenging and was done using engineering judgment.

A sensitivity analysis using the FEM model was conducted to confirm the selected updating parameters can influence the computed responses. Sensitivities were calculated using a finite difference method by changing the parameters by 0.1% with respect to their initial values. Paper length limit makes it impossible to show the sensitivity results, but the calculated sensitivities of the modal frequencies to the selected updating parameters demonstrated that all the selected parameters have an appreciable influence.

4 DISCUSSION OF THE FE MODEL UPDATING RESULTS

The initial estimates for the six selected model parameters were set to the average value between the lower and upper bounds listed in Table 1. The iterative algorithm in Figure 1 was implemented using a MATLAB script with MATLAB calling SAP2000 to determine the modal properties of the FE model. The modal frequencies computed from the SAP2000 model were then used to compute the new sensitivity matrices.

The final updated FE modal frequencies are presented in Columns 4 and 5 of Table 1. As seen in the Table 1, the differences between the measured and initially calculated modal frequencies vary from 0.54% for the 13th mode to 6.85% for the 15th mode. These differences improved after the model updating process for all the modal frequencies. The most improvement occurred for the fourth mode while the least improvement occurred for the 16th mode. These differences also improved on average by more than 3% for the first four modes. Although the mode shapes were not used in the objective function for the purpose of model updating, these MAC values are also calculated using the FE-computed and measured mode shapes and can be used to validate the model updating. The MAC values, shown in Columns 6 and 7 of Table 1, did not improve significantly for all the nine modes.

The updated values of the six parameters are shown in Table 2. The thickness of the asphalt overlay, the elastic modulus of concrete in the deck and piers nearly reached their upper limits. Mass density of concrete in the deck and piers reduced to their lower limits. In consultation with the bridge owner, the average thickness of asphalt overlay installed on is about 45 mm. As a result, this further confirmed the accuracy of the updated model parameters.

CONCLUSIONS

The paper presents the implementation of a sensitivity-based FE model updating process for a new twelve-span prestressed concrete bridge using ambient vibration data. The objective of the FE model updating process is to match the FE-computed and experimentally measured modal frequencies. The experimentally measured modal characteristics of the bridge were obtained using ambient vibration data measured using 60 wireless sensors. A FE model was developed in SAP2000 to simulate the dynamic response of the bridge as realistically as possible. A few physical parameters inside the model are selected for the updating process after performing a comprehensive sensitivity analysis of all major model parameters. Furthermore, the number of modal frequencies used in the optimisation process is minimised by identifying the most important modal frequencies of the bridge that participate the most in the global bridge response and are fully excited under ambient vibration conditions. Finally, a successful calibration of the Newmarket Viaduct FE model is demonstrated. The solution resulted in reasonable values for the updated parameters as well as a close match between the FE-computed and field-measured modal characteristics of the bridge. Finally, success of all the FE model calibration process depends significantly on fair engineering judgements about the level of detail used in developing the FE model, the unknown parameters used in the updating the FE model, reasonable objective functions and finally the solution selected.

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REFERENCES

[1] B. Jaishi, W. Ren. Structural Finite Element Model Updating Using Ambient Vibration Test Results, *Journal of Structural Engineering*, 131: 617-628, 2005.

[2] J. Brownjohn, P. Moyo, P. Omenzetter, Y. Lu. Assessment of Highway Bridge Upgrading by Dynamic Testing and Finite-Element Model Updating, *Journal of Bridge Engineering*, 8:162-172, 2003.

[3] S. Živanović, A. Pavic, P. Reynolds. Finite Element Modelling and Updating of a Lively Footbridge: The Complete Process, *Journal of Sound and Vibration*, 301:126-145, 2007.

[4] C. Bedon, A. Morassi. Dynamic Testing and Parameter Identification of a Base-isolated Bridge, *Engineering Structures*, 60:85-99, 2014.

[5] M. Friswell, J.E. Mottershead. *Finite Element Model Updating in Structural Dynamics*, Springer, 1995.

[6] J.M.W. Brownjohn, F. Magalhaes, E. Caetano, A. Cunha. Ambient Vibration Re-testing and Operational Modal Analysis of the Humber Bridge, *Engineering Structures*, 32:2003-2018, 2010.

[7] A. Altunişik, A. Bayraktar, B. Sevim. Output-Only System Identification of Posttensioned Segmental Concrete Highway Bridges, *Journal of Bridge Engineering*, 16:259-266, 2011.

[8] F. Magalhães, E. Caetano, Á. Cunha, O. Flamand, G. Grillaud. Ambient and Free Vibration Tests of the Millau Viaduct: Evaluation of Alternative Processing Strategies, *Engineering Structures*, 45:372-384, 2012.

[9] M.I. Friswell, J.E. Mottershead, H. Ahmadian. Finite–Element Model Updating Using Experimental Test Data: Parametrization and Regularization, *Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, 359:169-186, 2001.

[10] P.G. Bakir, E. Reynders, G.D. Roeck. An Improved Finite Element Model Updating Method by the Global Optimization Technique 'Coupled Local Minimizers', *Computers & Structures*, 86:1339-1352, 2008.

[11] H. Wang, A.-Q. Li, J. Li. Progressive Finite Element Model Calibration of a Long-span Suspension Bridge Based on Ambient Vibration and Static Measurements, *Engineering Structures*, 32:2546-2556, 2010.

[12] A.A. Mosavi, H. Sedarat, S.M. O'Connor, A. Emami-Naeini, J. Lynch. Calibrating a High-Fidelity Finite Element Model of a Highway Bridge Using a Multi-variable Sensitivity-based Optimisation Approach, *Structure and Infrastructure Engineering*, 10:627-642, 2013.

[13] Y.B. Yang, Y.J. Chen. A New Direct Method for Updating Structural Models Based on Measured Modal Data, *Engineering Structures*, 31:32-42, 2009.

[14] A. Teughels, G. De Roeck, J.A.K. Suykens. Global Optimization by Coupled Local Minimizers and Its Application to FE Model Updating, *Computers & Structures*, 81:2337-2351, 2003.

[15] P.G. Bakir, E. Reynders, G. De Roeck. Sensitivity-based Finite Element Model Updating Using Constrained Optimization with a Trust Region Algorithm, *Journal of Sound and Vibration*, 305:211-225, 2007.

[16] F. Butt, P. Omenzetter. Seismic Response Trends Evaluation and Finite Element Model Calibration of an Instrumented RC Building Considering Soil–structure Interaction and Non-structural Components, *Engineering Structures*, 65:111-123, 2014.

[17] R.J. Allemang, D.L. Brown. A Correlation Coefficient for Modal Vector Analysis, *Proceedings of the 1st International Modal Analysis Conference*, 110-116, 1982

[18] Z. Li, J. Swanson, A. Helmicki, V. Hunt. Modal Contribution Coefficients in Bridge Condition Evaluation, *Journal of Bridge Engineering*, 10: 169-178, 2005.

[19] B. Titurus, M.I. Friswell. Regularization in Model Updating, *International Journal for Numerical Methods in Engineering*, 75:440-478, 2008.