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DAMAGE IDENTIFICATION OF NONLINEAR STRUCTURE WITH UNKNOWN EXCITATIONS USING QUADRATIC SUM SQUARE ERROR WITH AR MODEL

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ABSTRACT

The ability to accurately identify structural parameters, either on-line or almost on-line, based on vibration data measured from sensors, is essential for the structural health monitoring system. The problem is quite challenging, in particular when the external excitations are not completely measured and when the structural system is nonlinear. In practical applications, external excitations (inputs), such as seismic excitations, wind loads, traffic loads, etc., may not be measured or may not be measurable, and the structure may not always be linear. In this paper, a newly proposed parametric identification method, referred to as the quadratic sum-squares error with AR model (QSSE-AR), is used for estimating structural parameters of a nonlinear elastic structure and a nonlinear hysteretic structure. In this approach, external excitations and some structural responses may not be measured. The accuracy and effectiveness of the proposed approach will be demonstrated by numerical simulations without the measurement of external excitations. The simulation results indicate that the proposed damage detection technique is capable of identifying structural parameters, as well as predicting the unknown external excitations.

KEYWORDS : *structural health monitoring, parameteric identification, QSSE-AR, nonlinear structure.*

INTRODUCTION

Accurate estimation of structural parameters and assessment of structure conditions is important for a structure health monitoring (SHM) system. Analysis methodologies for parametric identification of structures, based on measured vibration data, have been studied in the literature [1, 2]. However, most of the methodologies available in the literature [1] require the measurement of external excitations. Due to practical limitations, either sensor may not be installed to measure all the external excitations (inputs) or the external excitations are not measurable, such as wind and traffic loads. In fact, it is highly desirable to install as few sensors as possible in the structural health monitoring system. Consequently, the purpose of this paper is to present a system identification methodology utilizing only limited measurements of responses (outputs), in order to reduce the number of sensors required in the health monitoring system. When the external excitations are not measured or not available, numerical iterative procedures have been proposed to identify the constant structural parameters without analytical solutions [3-5]. Recently, analytical recursive solutions with unknown external excitations have been proposed to identify the structural parameters based on: (i) the LSE approach [6], (ii) the extended Kalman filter technique [7], (iii) the sequential nonlinear least-square estimation (SNLSE) [8], and (iv) the quadratic sum-squares error (QSSE) [9].

However, when applying the above approaches, an extended unknown vector has to be formed consisting of both the unknown structural parameters and the unknown excitations. The time-

varying feature of this extended vector leads to a relative complicated derivation of its recursive solution and consequently makes it difficult to apply in real engineering practice. Thus, a new approach based on the basic concept of QSSE method and AR model has been proposed [10, 11], referred to as the QSSE-AR approach, aiming to improve the efficiency of estimating unknown structural parameters and unmeasured external excitations. In this approach, the unmeasured excitations will be expressed using AR model and instead of estimating the unknown inputs at every time step as in the QSSE-UI method, only constant ARMA parameters need to be computed, thus the computational efforts are significantly reduced. It has been proved that the proposed new approach [10, 11] is able to identify the unknown structural parameters as well as the unmeasured external excitation for linear structures. In this paper, the accuracy and effectiveness of the proposed approach will be demonstrated by numerical simulation of nonlinear structures.

1 QUADRATIC SUM-SQUARES ERROR WITH AR MODEL

The equation of motion of a m-DOF structure can be expressed as:

$$\mathbf{M} \ddot{\mathbf{x}}(t) + \mathbf{F}_c [\dot{\mathbf{x}}(t)] + \mathbf{F}_s [\mathbf{x}(t)] = \boldsymbol{\eta}^* \mathbf{f}^*(t) + \boldsymbol{\eta} \mathbf{f}(t) \quad (1)$$

in which $\mathbf{x}(t) = [x_1, x_2, \dots, x_m]^T = m$ -displacement vector; $\mathbf{M} = (m \times m)$ mass matrix; $\mathbf{F}_c [\dot{\mathbf{x}}(t)] = m$ -damping force vector; $\mathbf{F}_s [\mathbf{x}(t)] = m$ -stiffness force vector; $\mathbf{f}^*(t) = [f_1^*(t), f_2^*(t), \dots, f_r^*(t)]^T = r$ -unknown (or unmeasured) excitation vector; $\boldsymbol{\eta}^* = (m \times r)$ excitation influence matrix associated with $\mathbf{f}^*(t)$; $\mathbf{f}(t) = [f_1(t), f_2(t), \dots, f_s(t)]^T = s$ -known (or measured) excitation vector; and $\boldsymbol{\eta} = (m \times s)$ excitation influence matrix associated with $\mathbf{f}(t)$. In Eq.(1), $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_n]^T$ is an n-unknown parametric vector with θ_i ($i = 1, 2, \dots, n$) being the i th unknown parameter of the structure, including damping, stiffness, nonlinear and hysteretic parameters. For simplicity of derivation, we shall assume for the time being that the unknown parametric vector $\boldsymbol{\theta}$ is constant, i.e., $\boldsymbol{\theta} = \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \dots = \boldsymbol{\theta}_{k+1}$, where $\boldsymbol{\theta}_i = \boldsymbol{\theta}$ ($t = i\Delta t$) for $i = 1, 2, \dots, k+1$. In what follows, the bold face letter represents either a vector or a matrix.

Introducing a state vector $\mathbf{X}(t) = [\mathbf{x}^T \quad \dot{\mathbf{x}}^T]^T$ with a dimension of $2m$, we can transform the equation of motion in Eq.(1) into a state equation, i.e.

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{g}(\mathbf{X}, \boldsymbol{\theta}, \mathbf{f}^*, \mathbf{f}) + \mathbf{w}(t) \quad (2)$$

If discretizing the excitation time functions at a sampling time of $t = i \Delta t$, for unknown excitation, we have $\mathbf{f}_i^* = [f_{i,1}^*, f_{i,2}^*, \dots, f_{i,r}^*]^T$, in which $f_i^* = f^*(t = i\Delta t)$ and $f_{i,j}^* = f_j^*(t = i\Delta t)$ for $j = 1, 2, \dots, r$. Theoretically, $f_{i,j}^*$ is unknown for all $i = 1, 2, \dots$, and $j = 1, 2, \dots, r$. In order to reduce the number of unknowns and the required number of sensors, and unknown excitation can be expressed using the AR model as follows

$$f_{i,j}^* = \alpha_{j,1} \hat{f}_{i-1,j}^* + \alpha_{j,2} \hat{f}_{i-2,j}^* + \dots + \alpha_{j,p} \hat{f}_{i-p,j}^* = \bar{\Delta}_{i,j} \boldsymbol{\alpha}_j \quad (3)$$

for $i = 1, 2, 3, \dots, k+1$ and $j = 1, 2, 3, \dots, r$, where

$$\boldsymbol{\alpha}_j = [\alpha_{j,1}, \alpha_{j,2}, \dots, \alpha_{j,p}]^T \quad (4)$$

in which $\alpha_{j,q}$ for $q = 1, 2, \dots, p$ are ARMA constants of order p , and

$$\bar{\Delta}_{i,j} = [\hat{f}_{i-1,j}^*, \hat{f}_{i-2,j}^*, \dots, \hat{f}_{i-p,j}^*] \quad (5)$$

Let $\boldsymbol{\theta}^*$ denote the rp -unknown parametric vector containing all the ARMA constants for \mathbf{f}_i^* , i.e.,

$$\boldsymbol{\theta}^* = [\boldsymbol{\alpha}_1^T, \boldsymbol{\alpha}_2^T, \dots, \boldsymbol{\alpha}_r^T]^T \quad (6)$$

It is noticed that $\boldsymbol{\theta}^*$ is a constant unknown vector, i.e., $\boldsymbol{\theta}_1^* = \boldsymbol{\theta}_2^* = \dots = \boldsymbol{\theta}_{k+1}^* = \boldsymbol{\theta}^*$. Then, the unknown excitation \mathbf{f}_i^* can be expressed as

$$\mathbf{f}_i^* = \Delta_i \boldsymbol{\theta}^* = \Delta_i \boldsymbol{\theta}_i^* \quad (7)$$

in which Δ_i is a $(r \times rp)$ matrix,

$$\Delta_i = \begin{bmatrix} \bar{\Delta}_{i,1} & 0 & \dots & 0 \\ 0 & \bar{\Delta}_{i,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \bar{\Delta}_{i,r} \end{bmatrix} \quad (8)$$

Consequently, the equation of motion, Eq.(1), can be expressed as

$$\mathbf{F}_c[\dot{\mathbf{x}}(t), \boldsymbol{\theta}] + \mathbf{F}_s[\mathbf{x}(t), \boldsymbol{\theta}] + \mathbf{F}_{f^*}(\boldsymbol{\theta}^*, t) = \boldsymbol{\eta} \mathbf{f}(t) - \mathbf{M} \ddot{\mathbf{x}}(t) \quad (9)$$

where $\mathbf{F}_{f^*}(\boldsymbol{\theta}^*, i) = -\boldsymbol{\eta}^* \Delta_i \boldsymbol{\theta}_i^*$.

A nonlinear discrete equation (at $t=i \Delta t$) for an observation vector (measured responses) can be expressed as follows,

$$\mathbf{y}_i = \mathbf{h}[\mathbf{X}_i(\boldsymbol{\theta}_i), \boldsymbol{\theta}_i, \boldsymbol{\theta}_i^*, \mathbf{f}_i] + \mathbf{v}_i; \quad i = 1, 2, \dots, k+1 \quad (10)$$

in which \mathbf{y}_i is a l -dimensional observation (measured) vector, and \mathbf{f}_i is a measured s -dimensional excitation vector at $t=i \Delta t$ (sampling time step Δt), respectively, i.e., $\mathbf{y}_i = \mathbf{y}(t=i \Delta t)$, $\mathbf{X}_i = \mathbf{X}(t=i \Delta t)$ and $\mathbf{f}_i = \mathbf{f}(t=i \Delta t)$. In Eq.(9), $\mathbf{X}_i(\boldsymbol{\theta}_i)$ is an implicit function of the unknown parametric vectors $\boldsymbol{\theta}_i$ and $\boldsymbol{\theta}_i^*$, and \mathbf{v}_i is a measurement noise vector assumed to be a Gaussian white noise vector with zero mean and a covariance matrix $E[\mathbf{v}_i \mathbf{v}_j^T] = \mathbf{R}_i \delta_{ij}$ where δ_{ij} is the Kroneker delta.

The weighted sum-squares error (up to $t=(k+1)\Delta t$ between the observation data \mathbf{y}_i and the analytical expression h is given by

$$J_{k+1} = \sum_{i=1}^{k+1} \mathbf{e}_i^T \mathbf{R}_i^{-1} \mathbf{e}_i; \mathbf{e}_i = \mathbf{y}_i - \mathbf{h}[\mathbf{X}_i(\boldsymbol{\theta}_i), \boldsymbol{\theta}_i, \boldsymbol{\theta}_i^*, \mathbf{f}_i] \quad (11)$$

in which \mathbf{y}_i and $\mathbf{h}[\mathbf{X}_i(\boldsymbol{\theta}_i), \boldsymbol{\theta}_i, \boldsymbol{\theta}_i^*, \mathbf{f}_i]$ are given by Eq.(10), and the weighting matrix $\mathbf{R}_i = E[\mathbf{v}_i \mathbf{v}_i^T]$ is a $(l \times l)$ variance matrix of the measurement noises at $t=i \Delta t$.

Let $\hat{\boldsymbol{\theta}}_i$ be the estimate of $\boldsymbol{\theta}_i$, $\hat{\boldsymbol{\theta}}_i^*$ be the estimate of $\boldsymbol{\theta}_i^*$ at $t=i \Delta t$, and $\hat{\mathbf{X}}_{i|i-1}$ be the estimate of \mathbf{X}_i based on the estimated parametric vector $\hat{\boldsymbol{\theta}}_{i-1}$ and $\hat{\boldsymbol{\theta}}_{i-1}^*$, i.e., $\mathbf{X}(\hat{\boldsymbol{\theta}}_{i-1}) = \hat{\mathbf{X}}_{i-1}$. Since $\mathbf{h}[\mathbf{X}_i(\boldsymbol{\theta}_i), \boldsymbol{\theta}_i, \boldsymbol{\theta}_i^*, \mathbf{f}_i]$ is a nonlinear function of unknown parametric vectors $\boldsymbol{\theta}_i$ and $\boldsymbol{\theta}_i^*$, it will be linearized around the estimates of these two unknown vectors at the previous step, i.e., $\boldsymbol{\theta}_i = \hat{\boldsymbol{\theta}}_{i-1}$, $\boldsymbol{\theta}_i^* = \hat{\boldsymbol{\theta}}_{i-1}^*$ and $\mathbf{X}_i = \mathbf{X}(\boldsymbol{\theta}_i) = \mathbf{X}(\hat{\boldsymbol{\theta}}_{i-1}) = \hat{\mathbf{X}}_{i-1}$, as follows

$$\mathbf{h}[\mathbf{X}_i(\boldsymbol{\theta}_i), \boldsymbol{\theta}_i, \boldsymbol{\theta}_i^*, \mathbf{f}_i] = \mathbf{h}_i = \hat{\mathbf{h}}_i + \mathbf{H}_i (\boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_{i-1}) + \mathbf{D}_i (\boldsymbol{\theta}_i^* - \hat{\boldsymbol{\theta}}_{i-1}^*) \quad (12)$$

in which for simplicity of notation $\hat{\mathbf{h}}_i = \mathbf{h}(\hat{\mathbf{X}}_i, \hat{\boldsymbol{\theta}}_{i-1}, \hat{\boldsymbol{\theta}}_{i-1}^*, \mathbf{f}_i)$, and

$$\mathbf{H}_i = \mathbf{H}_{\boldsymbol{\theta},i} + \mathbf{H}_{\boldsymbol{\theta}^*,i} \mathbf{X}_{\boldsymbol{\theta},i} \quad (13)$$

$$\mathbf{H}_{\boldsymbol{\theta},i} = \mathbf{H}_{\boldsymbol{\theta}}(\hat{\mathbf{X}}_i, \hat{\boldsymbol{\theta}}_{i-1}, \hat{\boldsymbol{\theta}}_{i-1}^*, \mathbf{f}_i) = \left[\frac{\partial \mathbf{h}(\mathbf{X}_i, \boldsymbol{\theta}_i, \boldsymbol{\theta}_i^*, \mathbf{f}_i)}{\partial \boldsymbol{\theta}_i} \right]_{\mathbf{X}_i = \hat{\mathbf{X}}_i; \boldsymbol{\theta}_i = \hat{\boldsymbol{\theta}}_{i-1}; \boldsymbol{\theta}_i^* = \hat{\boldsymbol{\theta}}_{i-1}^*} \quad (14)$$

$$\mathbf{H}_{X,i} = \mathbf{H}_X(\hat{\mathbf{X}}_i, \hat{\boldsymbol{\theta}}_{i-1}, \hat{\boldsymbol{\theta}}_{i-1}^*, \mathbf{f}_i) = \left[\frac{\partial \mathbf{h}(\mathbf{X}_i, \boldsymbol{\theta}_i, \boldsymbol{\theta}_i^*, \mathbf{f}_i)}{\partial \mathbf{X}_i} \right]_{\mathbf{X}_i = \hat{\mathbf{X}}_i; \boldsymbol{\theta}_i = \hat{\boldsymbol{\theta}}_{i-1}; \boldsymbol{\theta}_i^* = \hat{\boldsymbol{\theta}}_{i-1}^*} \quad (15)$$

$$\mathbf{X}_{\boldsymbol{\theta},i} = \mathbf{X}_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}_{i-1}) = \left[\frac{\partial \mathbf{X}_i}{\partial \boldsymbol{\theta}_i} \right]_{\boldsymbol{\theta}_i = \hat{\boldsymbol{\theta}}_{i-1}; \boldsymbol{\theta}_i^* = \hat{\boldsymbol{\theta}}_{i-1}^*} \quad (16)$$

$$\mathbf{D}_i = \mathbf{D}_{\boldsymbol{\theta}^*,i} \quad (17)$$

$$\mathbf{D}_{\boldsymbol{\theta}^*,i} = \mathbf{D}_{\boldsymbol{\theta}^*}(\hat{\mathbf{X}}_i, \hat{\boldsymbol{\theta}}_{i-1}, \hat{\boldsymbol{\theta}}_{i-1}^*, \mathbf{f}_i) = \left[\frac{\partial \mathbf{h}(\mathbf{X}_i, \boldsymbol{\theta}_i, \boldsymbol{\theta}_i^*, \mathbf{f}_i)}{\partial \boldsymbol{\theta}_i^*} \right]_{\mathbf{X}_i = \hat{\mathbf{X}}_i; \boldsymbol{\theta}_i = \hat{\boldsymbol{\theta}}_{i-1}; \boldsymbol{\theta}_i^* = \hat{\boldsymbol{\theta}}_{i-1}^*} \quad (18)$$

Note that Eqs.(13) and (18) are obtained by the chain rule of partial derivative, with respect to the unknown vectors $\boldsymbol{\theta}_i$ and $\boldsymbol{\theta}_i^*$, respectively, and \mathbf{H}_i , $\mathbf{H}_{X,i}$, $\mathbf{X}_{\boldsymbol{\theta},i}$, $\mathbf{H}_{\boldsymbol{\theta},i}$, \mathbf{D}_i , and $\mathbf{D}_{\boldsymbol{\theta}^*,i}$ are $(1 \times n)$, $(1 \times 2m)$, $(2m \times n)$, $(1 \times n)$, $(1 \times rp)$, and $(1 \times rp)$ matrices, respectively. $\mathbf{X}_{\boldsymbol{\theta}}(\boldsymbol{\theta})$ is referred to as the sensitivity matrices of the state vector \mathbf{X} with respect to the parametric vectors $\boldsymbol{\theta}$, respectively, i.e.,

$$\mathbf{X}_{\boldsymbol{\theta}}(\boldsymbol{\theta}) = \frac{\partial \mathbf{X}}{\partial \boldsymbol{\theta}} = \begin{bmatrix} \mathbf{x}_{\boldsymbol{\theta}} \\ \dot{\mathbf{x}}_{\boldsymbol{\theta}} \end{bmatrix} \quad (19)$$

in which $\dot{\mathbf{x}}_{\boldsymbol{\theta}}$ and $\mathbf{x}_{\boldsymbol{\theta}}$ are $(m \times n)$ matrices,

$$\dot{\mathbf{x}}_{\boldsymbol{\theta}} = \frac{\partial \dot{\mathbf{x}}}{\partial \boldsymbol{\theta}}; \quad \mathbf{x}_{\boldsymbol{\theta}} = \frac{\partial \mathbf{x}}{\partial \boldsymbol{\theta}} \quad (20)$$

Substituting Eq.(12) into Eq.(11), we can obtain the following objective function, J_{k+1} , which is a quadratic function of unknown vectors $\boldsymbol{\theta}_i$ and $\boldsymbol{\theta}_i^*$, as follows

$$J_{k+1} = \sum_{i=1}^{k+1} [\bar{\mathbf{y}}_i - \mathbf{H}_i \boldsymbol{\theta}_i - \mathbf{D}_i \boldsymbol{\theta}_i^*]^T \mathbf{R}_i^{-1} [\bar{\mathbf{y}}_i - \mathbf{H}_i \boldsymbol{\theta}_i - \mathbf{D}_i \boldsymbol{\theta}_i^*] \quad (21)$$

where

$$\bar{\mathbf{y}}_i = \mathbf{y}_i - \hat{\mathbf{h}}_i + \mathbf{H}_i \hat{\boldsymbol{\theta}}_{i-1} + \mathbf{D}_i \hat{\boldsymbol{\theta}}_{i-1}^* \quad (22)$$

Let us define an extended unknown vector $\boldsymbol{\theta}_{e,i}$ and an extended matrix $\mathbf{H}_{e,i}$ at $t = i \Delta t$, i.e.,

$$\boldsymbol{\theta}_{e,i} = \begin{bmatrix} \boldsymbol{\theta}_i \\ \boldsymbol{\theta}_i^* \end{bmatrix}; \quad \mathbf{H}_{e,i} = [\mathbf{H}_i \quad \mathbf{D}_i] \quad (23)$$

in which $\boldsymbol{\theta}_{e,i}$ is a $(n + rp)$ -unknown vector.

Combining all $k+1$ time instants (from $i = 1$ to $i = k+1$), and assuming that $\boldsymbol{\theta}_{e,i}$ is a constant vector, i.e., $\boldsymbol{\theta}_{e,i} = \boldsymbol{\theta}_{e,k+1}$ for $k = 1, 2, \dots$, the objective function in Eq.(21) can be expressed as

$$\begin{aligned} J_{k+1} &= \sum_{i=1}^{k+1} [\bar{\mathbf{y}}_i - \mathbf{H}_{e,i} \boldsymbol{\theta}_{e,k+1}]^T \mathbf{R}_i^{-1} [\bar{\mathbf{y}}_i - \mathbf{H}_{e,i} \boldsymbol{\theta}_{e,k+1}] \\ &= [\mathbf{Y}_{k+1} - \boldsymbol{\Phi}_{e,k+1} \boldsymbol{\Theta}_{e,k+1}]^T \mathbf{W}_{k+1} [\mathbf{Y}_{k+1} - \boldsymbol{\Phi}_{e,k+1} \boldsymbol{\Theta}_{e,k+1}] \end{aligned} \quad (24)$$

where

$$\boldsymbol{\Theta}_{e,k+1} = \begin{bmatrix} \boldsymbol{\theta}_{k+1} \\ \boldsymbol{\theta}_{k+1}^* \end{bmatrix}; \quad \mathbf{Y}_{k+1} = \begin{bmatrix} \bar{\mathbf{y}}_1 \\ \bar{\mathbf{y}}_2 \\ \vdots \\ \bar{\mathbf{y}}_{k+1} \end{bmatrix}; \quad \boldsymbol{\Phi}_{e,k+1} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{D}_1 \\ \mathbf{H}_2 & \mathbf{D}_2 \\ \vdots & \vdots \\ \mathbf{H}_k & \mathbf{D}_k \\ \mathbf{H}_{k+1} & \mathbf{D}_{k+1} \end{bmatrix} \quad (25)$$

In Eqs.(24)-(25), $\Theta_{e,k+1}$ is a $[(n+rp)]$ -unknown column vector, Y_{k+1} is a $(k+1)l$ -known (measured) column vector at step $k+1$ with \bar{y}_i given by Eq.(22), H_i and D_i are given by Eqs.(13) and (17), respectively, for $i = 1, 2, \dots, k+1$, $\Phi_{e,k+1}$ is a $[(k+1)l \times (n+rp)]$ known (measured) data matrix at step $k+1$, and $W_{k+1} = \text{diag}(R_1^{-1}, R_2^{-1}, \dots, R_{k+1}^{-1})$.

Let $\hat{\Theta}_{e,k+1}$ be the estimate of $\Theta_{e,k+1}$ at $t = (k+1)\Delta t$, i.e., $\hat{\Theta}_{e,k+1} = [\hat{\theta}_{k+1}^T, \hat{\theta}_{k+1}^{*T}]^T$. Minimizing the quadratic objective function J_{k+1} in Eq.(24) with respect to $\Theta_{e,k+1}$, we obtain the estimate $\hat{\Theta}_{e,k+1}$ in the following [10, 11]

$$\hat{\Theta}_{e,k+1} = \hat{\Theta}_{e,k} + K_{e,k+1}(\bar{y}_{k+1} - H_{e,k+1} \hat{\Theta}_{e,k}) \tag{26}$$

in which

$$H_{e,k+1} = [H_{k+1} \quad D_{k+1}] \tag{27}$$

$$K_{e,k+1} = P_{e,k} H_{e,k+1}^T (R_{k+1} + H_{e,k+1} P_{e,k} H_{e,k+1}^T)^{-1} \tag{28}$$

$$P_{e,k+1} = (I - K_{e,k+1} H_{e,k+1}) P_{e,k} \tag{29}$$

In Eqs.(26)-(29) above, $K_{e,k+1}$ and $P_{e,k+1}$ are gain matrices, respectively, of size $[(n+rp) \times l]$ and $[(n+rp) \times (n+rp)]$.

For the numerical computation, \hat{h}_{k+1} is first obtained, in which $\hat{X}_{k+1|k}$ is the estimate of the state vector $X(t)$ at $t = (k+1)\Delta t$ based on $\theta = \hat{\theta}_k$ and $\theta^* = \hat{\theta}_k^*$, i.e.,

$$\hat{X}_{k+1|k} = \hat{X}_{k|k} + \int_{k\Delta t}^{(k+1)\Delta t} g(\hat{X}_{t|k}, \hat{\theta}_k, \hat{\theta}_k^*, f) dt \tag{33}$$

where $\hat{X}_{t|k}$ is the solution of Eq.(2) in $k\Delta t \leq t \leq (k+1)\Delta t$ with the initial condition $\hat{X}_{k|k}$ and $w = 0$.

Note that in Eqs.(26)-(29), R_{k+1} is the variance matrix of the measurement noises at $t = (k+1)\Delta t$, H_{k+1} and D_{k+1} are given by Eqs.(13) and (17), respectively, with i being replaced by $k+1$, and $H_{\theta,k+1}$, $H_{X,k+1}$ and $X_{\theta,k+1}$ are given by Eqs.(14)-(16), with i being replaced by $k+1$. Further, $P_{e,k}$ in Eq.(32) is the adaptation gain matrix at $t = k\Delta t$. Once $\hat{\Theta}_{e,k+1}$ is computed the unknown inputs f_{k+1}^* at $t = (k+1)\Delta t$ can be updated using Eq.(1). The analytical solution derived in Eqs.(26)-(29) is referred to as the quadratic sum-squares error with AR model (QSSE-AR), which is not available in the previous literature.

2 PARAMETRIC IDENTIFICATION

To demonstrate the accuracy and effectiveness of the proposed new QSSE-AR approach for parametric identifications of nonlinear structures, two 2-degree-of-freedom nonlinear building models subject to a white noise excitation applied on the top floor will be considered. In these examples, the absolute accelerations of each floor are measured while the external force is unknown. The sampling frequency is 1000Hz for all measured data.

2.1 2DOF Nonlinear Elastic Structure

Consider a 2-story Duffing-type nonlinear shear-beam building model with the equations of motion given by

$$\begin{aligned} m_1 \ddot{x}_1 &= -c_1 \dot{x}_1 + c_2 (\dot{x}_2 - \dot{x}_1) - k_{11} x_1 - k_{13} x_1^3 + k_{21} (x_2 - x_1) + k_{23} (x_2 - x_1)^3 \\ m_2 \ddot{x}_2 &= -c_2 (\dot{x}_2 - \dot{x}_1) - k_{21} (x_2 - x_1) - k_{23} (x_2 - x_1)^3 + f^*(t) \end{aligned} \tag{34}$$

in which x_i is the relative displacement between the i th floor and the ground, and $m_1 = m_2 = 1000$ kg, $c_1 = c_2 = 0.6$ kN.s/m, $k_{11} = 120$ kN/m, $k_{21} = 60$ kN/m, $k_{13} = 200$ kN/m³, and $k_{23} = -50$ kN/m³. As observed from the parametric values of the Duffing model above, the stiffness of the first story is strain hardening whereas that of the second story is strain softening. For the elastic structure with $k_{13} = k_{23} = 0$, the natural frequencies are $\omega_1 = 0.94$ Hz and $\omega_2 = 2.28$ Hz with the corresponding damping ratios $\zeta_1 = 2.2$ % and $\zeta_2 = 5.4$ %. Unknown parameters to be identified are: $c_1, c_2, k_{11}, k_{13}, k_{21}, k_{23}$, and the unmeasured external excitation $f^*(t)$.

In order to start the recursive solutions in Eqs.(28)-(31), the following initial values will be assumed: (i) the initial values of state variables and the unknown excitation are zero, (ii) the initial values for $c_1, c_2, k_{11}, k_{21}, k_{13}$, and k_{23} are: $c_{1,0} = c_{2,0} = 0.4$ kN.s/m, $k_{11,0} = 150$ kN/m, $k_{13,0} = 150$ kN/m³, $k_{21,0} = 40$ kN/m and $k_{23,0} = -40$ kN/m³, (iii) the initial value for ARMA constant is $\alpha_{1,0} = 0.1$, (iv) the initial error covariance matrix of the extended vector is chosen to be $P_{e,0} = \text{diag}[10^3, 10^5, 10^3, 10^4, 10^{-2}, 10^{-2}, 10^{-2}]$, and (v) the covariance matrix of the measurement noise vector $v(t)$ is chosen to be $R = I_2$, where I_j is a $(j \times j)$ unit matrix.

Based on the proposed QSSE-AR approach, the identified structural parameters are presented in Fig.1 as solid curves, whereas the dashed curves are the theoretical results. Fig.2 shows the comparison of identified and theoretical values of the unknown white noise excitation. Both Fig.1 and 2 indicate that the proposed technique is capable of tracking both linear and nonlinear system parameters as well as estimating the unmeasured external forces.

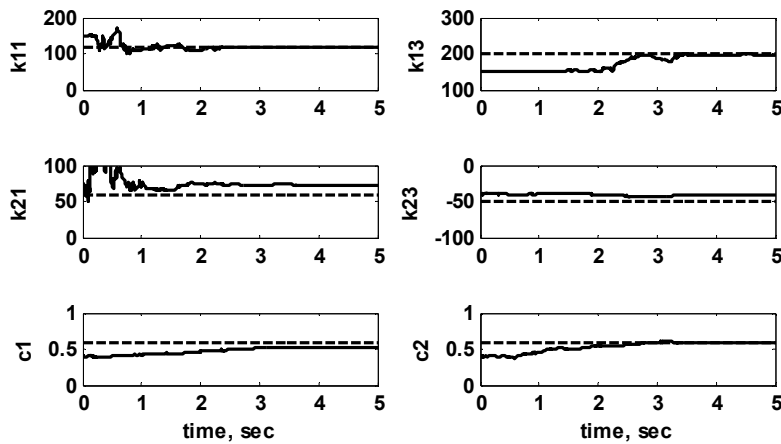


Figure 1: Identified structural parameters

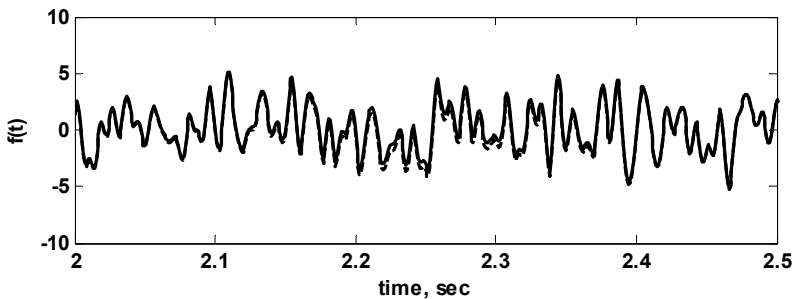


Figure 2: Identified unknown white noise excitation; unit of $f^*(t)$ in N.

2.2 2DOF Hysteretic Structure

Consider a 2DOF nonlinear hysteretic structural model with the equations of motion given by

$$\begin{aligned} m_1 \ddot{x}_1 &= -r_1 + r_2 \\ m_2 \ddot{x}_2 - f^*(t) &= -r_2 \end{aligned} \quad (35)$$

where

$$\begin{aligned} \dot{r}_1 &= c_1 \ddot{x}_1 + k_1 \dot{x}_1 - \beta_1 |\dot{x}_1| r_1^{\alpha-1} r_1 - \gamma_1 \dot{x}_1 |r_1|^\alpha \\ \dot{r}_2 &= c_2 (\ddot{x}_2 - \ddot{x}_1) + k_2 (\dot{x}_2 - \dot{x}_1) - \beta_2 |\dot{x}_2 - \dot{x}_1| r_2^{\alpha-1} r_2 - \gamma_2 (\dot{x}_2 - \dot{x}_1) |r_2|^\alpha \end{aligned} \quad (36)$$

in which $r = r(x, \dot{x})$ is the nonlinear hysteretic restoring force. The structural parameters are: $m_1 = m_2 = 125.53 \text{ kg}$, $c_1 = c_2 = 0.07 \text{ kN}\cdot\text{s/m}$, and $k_1 = k_2 = 24.2 \text{ kN/m}$, and the hysteretic parameters are: $\beta_1 = \beta_2 = 2$, $\gamma_1 = \gamma_2 = 1$, and $\alpha = 2$. For a small amplitude vibration (linear), the first natural frequency ω_1 and damping ratio ζ_1 are, respectively, $\omega_1 = 2.22 \text{ Hz}$ and $\zeta_1 = 2\%$. The unknown parameters to be identified are: c_i , k_i , β_i and γ_i ($i = 1, 2$), as well as the unmeasured external excitation $f^*(t)$.

The following initial values will be assumed: (i) the initial values of state variables and the unknown excitation are zero, (ii) the initial values for c_i , k_i , β_i and γ_i are $c_{i,0} = 0.1 \text{ kN}\cdot\text{s/m}$, $k_{i,0} = 10 \text{ kN/m}$, $\beta_{i,0} = 0$, and $\gamma_{i,0} = 0$ ($i = 1, 2$), respectively, (iii) the initial value for ARMA constant is $\alpha_{1,0} = 0.1$, (iv) the initial error covariance matrix of the extended vector is chosen to be $\mathbf{P}_{e,0} = \text{diag}[10^{-5}, 10^2, 10^{-1}, 10^{-1}, 10^{-5}, 10^2, 10^{-1}, 10^{-1}, 10^{-2}]$, and (v) the covariance matrix of the measurement noise vector $v(t)$ is chosen to be $\mathbf{R} = \mathbf{I}_2$, where \mathbf{I}_j is a $(j \times j)$ unit matrix.

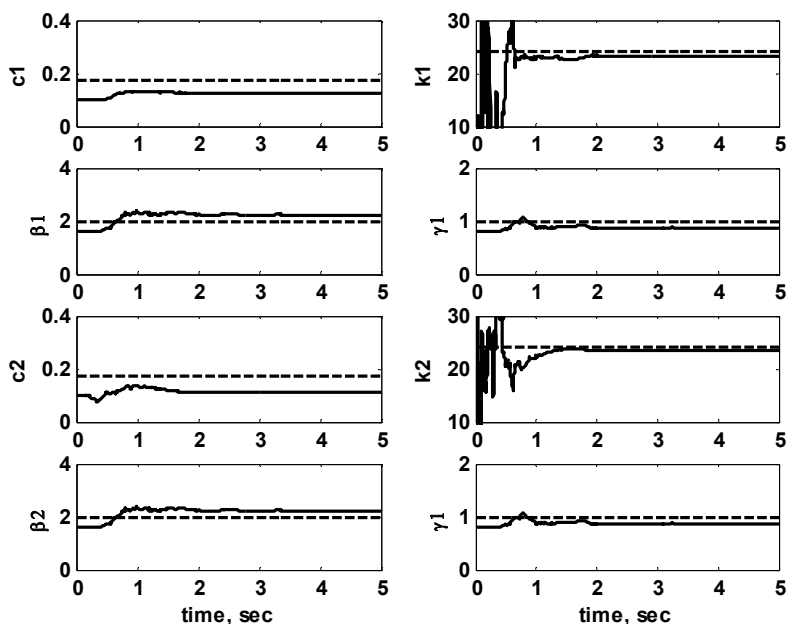


Figure 3: Identified structural parameters

Based on the proposed QSSE-AR method, the identified parameters are presented in Fig.3 as solid curves, whereas the theoretical results are shown by dashed curves for comparison. Again, the tracking capability of the proposed technique is plausible. Due to space limitation, the identified result for the unknown excitation is not shown here, which is similar to the previous example.

CONCLUSION

A new system identification technique, referred to as the quadratic sum-squares error with AR model (QSSE-AR), has been proposed for structural health monitoring. Analytical recursive solutions for such a new approach has been derived and presented. Numerical simulation of two different nonlinear structures has been carried out to verify that the QSSE-AR approach is capable of identifying the structural parameters and the unknown excitations. Further, with similar accuracy in the identification results, the QSSE-AR approach has a much simpler form of recursive solutions, and therefore, it is more preferable to be implemented in real engineering practice.

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