Algorithm Fusion in Novelty Detection

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ABSTRACT

Algorithm fusion has received significant attention in the machine learning community in supervised learning mode but it appears little has been done at this point in a novelty detection framework. This paper examines the merit of a fusion strategy wherein metrics from multiple algorithms are treated as entries of a vector whose probability density is subsequently estimated and used for detection. In the present paper the framework is investigated using two algorithms: 1) a robust version of a whiteness test on Kalman filter innovations and 2) a robust version of a scheme that operates with residuals obtained from an orthogonality test. The density estimation part of the process is replaced by the Kernel PCA algorithm which provides a decision boundary without having explicit density estimates. The fused scheme is implemented in a change detection format and is show to provide notable improvements over the use of either algorithm independently.

KEYWORDS: Algorithm Fusion, Damage Detection, Kalman Filter

1. INTRODUCTION

Difficulties in damage detection derive from the fact that the healthy state is not a point but a space with dimensions that depend (among other things) on the fluctuations in the environmental conditions and from the fact that the structural state (in whatever form is characterized) is inevitably uncertain. In the end the ability to detect damage depends on the shape of the reference state "generic bubble" and on the sensitivity to damage of the various points in that space. This paper summarizes some results from a project that investigated, among other things, the merit of fusing algorithms as a way to improve performance in detection.

Examination of the literature shows that techniques for fusing algorithms have been mainly restricted to supervised learning problems and are heuristically supported [1]. Approaches that announce damage if ANY or ALL the metrics considered are above their associated thresholds are readily implemented in an unsupervised framework. In this paper we look at an alternative where the metrics from several algorithms are treated as components of a vector whose probability density is used as the system model. In this approach the correlation between algorithms is automatically considered and heuristics are eliminated. The two algorithms considered here are a subspace residual based technique [2] and a whiteness test based on Kalman filter innovations [3]. Instead of a density for the bi-variate vector from the two metrics the paper uses the kernel based PCA algorithm to establish a decision boundary and the announcement of damage or no damage is cast in a change detection format. The procedure is exemplified in a numerical simulation on a 3-D truss.

2. ROBUST STATISTICAL SUBSPACE BASED DAMAGE DETECTION

It is assumed that the underlying physical system has a state-space representation in discrete time given by

$$\begin{aligned} x_{k+1} &= A_d x_k + v_k \\ y_k &= C_d x_k + w_k \end{aligned} \tag{1}$$

where v_k and ω_k are assumed to be white noise processes. The output covariance is given by $R_j = \mathrm{E}(y_{k+j} y_k^T)$ and it is easily shown that $R_j = C_d A_d^{j-1} G$ where $G = \mathrm{E}(x_{k+1} y_k^T)$ [4]. The output covariance can be organized into a Hankel matrix, namely

$$\mathrm{H}_{p+1,q} = \begin{pmatrix} R_1 & R_2 & \dots & R_q \\ R_2 & R_3 & \dots & R_{1+q} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p+1} & \dots & \dots & R_{p+q} \end{pmatrix} \tag{2}$$

where p and q are user defined parameters. From the form of the covariance function in terms of the system matrices it follows that the Hankel matrix can be factorized as

$$\mathrm{H}_{p+1,q} = \mathrm{O}_{p+1}\mathrm{C}_q, \text{ where } \mathrm{O}_{p+1,q} = \begin{pmatrix} C_d \\ C_d A_d \\ \vdots \\ C_d A_d^p \end{pmatrix} \text{ and } \mathrm{C}_q = \begin{pmatrix} G & A_d G & \dots & A_d^{q-1} G \end{pmatrix}. \tag{3}$$

Designating the left null space of O_{p+1} in the reference state as S the characteristic property of the system being in the reference state writes as

$$S^T \mathrm{O}_{p+1} = 0 \text{ and thus } S^T \mathrm{H}_{p+1,q} = 0. \tag{4}$$

The damage detection exploits the orthogonality condition in eq.4. Namely, the residual vector

$$\zeta_N = \sqrt{N}\ \mathrm{vec}(S^T \hat{\mathrm{H}}_{p+1,q}) \tag{5}$$

computed from covariance estimates $\hat{R}_j = \frac{1}{N}\sum_{k=1}^{N} y_{k+j} y_k^T$ on test data containing N data points is, under the null hypothesis (i.e. that there is no damage) asymptotically Gaussian with zero mean and a covariance that can be estimated from data obtained while the structure is in the reference state. Damage is detected, therefore, by testing the hypothesis that the residual vector obtained for any data set is a realization from this distribution. In practice this is conveniently done by computing the Mahalanobis distance of the residual, namely,

$$\chi_N^2 = \zeta_N^T \Sigma_\zeta^{-1} \zeta_N \tag{6}$$

which, under the null is clearly chi square distributed with as many degrees of freedom as the dimension of the residual vector. In theory the threshold of the metric in eq.6 can be determined from the cumulative χ^2 distribution by selecting an acceptable Type I error but in practice, due to inevitable deviations between the assumptions and the real situation, the threshold is best established by inspecting results for a sequence of data sets from the reference state.

2.1 Changes in the Excitation Statistics

A limitation of the residual in eq.5 is the fact that it depends not only on changes in the system, but also in fluctuations in the disturbances. A modification that eliminates dependence on excitation amplitude [5] replaces the Hankel matrix at the time of enquiry by the left side singular vectors that define its span, namely, taking

$$\hat{H}_{p+1,q} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Delta_1 & \\ & \Delta_2 \end{bmatrix} V^T \tag{7}$$

the residual is then taken as

$$\zeta_N = \sqrt{N}\ \text{vec}(S^T U_1) \tag{8}$$

where the number of columns in U_1 is the selected order n.

3. KALMAN FILTER BASED DAMAGE DETECTION

The idea of using the Kalman Filter as a fault detector was introduced by Mehra and Peschon [6], soon after the filter was developed [7]. The strategy is based on the fact that when the filter is operating optimally (i.e., when the prevailing conditions satisfy the assumptions), differences between measurements and filter predictions, known as innovations, are realizations of a white noise process. Detection, therefore, is carried out by formulating the filter for some reference state, which is treated as the healthy condition, and subsequently carrying a hypothesis test on the innovation's whiteness. For damage detection purposes the physical meaning of the state vector is not relevant and this allows formulation of the filter from data measured in the reference state [8,9]. This section illustrates how the filter can be used as a damage detector. Discussion on how the filter can be extracted from reference state data can be found in [8]. The innovations form of the state space expressions are

$$x_{k+1}^{-} = Ax_k^{+} + AKe_k \tag{9}$$

$$y_k = Cx_k^{-} + e_k \tag{10}$$

where e is the innovation, K is the Kalman gain and the superscripts – and + indicate before and after information from the measurement is accounted for in the estimation. The algorithm to compute the innovations sequence can be summarized as follows:

- Let x_k^{-} be known or assumed
- The innovation at k is computed using eq.10
- The state at k is updated using

$$x_k^+ = x_k^- + Ke_k \tag{11}$$

- and it is advanced forward as

$$x_{k+1}^- = Ax_k^+ \tag{12}$$

The next step is to determine if it is reasonable to assume that the innovation sequence obtained is a realization from a white noise process.

3.1 Whiteness Test

Consider any one of the innovation signals, under the null hypothesis, the auto-correlation for the jth lag, $q_j = E(e_i e_{i+j})$ has an empirical estimate given by

$$q_j = \frac{1}{L}\sum_{i=1}^{L} e_i e_{i+j} \tag{13}$$

where L is the number of points selected for the summation. Under the null hypothesis, provided the innovations are normalized to unit variance, q_j has zero mean and a variance equal to 1/L. A χ^2 test for innovations whiteness can thus be carried out as follows:

- Normalize e(k) to unit variance
- Compute the correlation function q_j for lags n_1 to n_2

Compute the χ^2 statistic, γ, as

$$\gamma = L\sum_{j=n_1}^{n_2} (q_j)^2 \tag{14}$$

where n_1 is the first lag considered and n_2 the last. The null hypothesis, i.e., that the innovation sequence is a realization of a white noise process, is not rejected if γ is no larger than the (1-α)th quantile of the χ^2 with $\bar{n} = n_2 - n_1 + 1$ DOF.

Selection of n_1 and n_2

It has been shown [3] that the lag n_1 can be selected to minimize the sensitivity of the whiteness test to changes in the excitation covariance. A practical way to select n_1 is by using data from the healthy condition and checking for which level the metric in eq.14 begins to be reasonably stable, n_2 can be taken as $n_1 + \bar{n} = round(f_s / f_1)$ where f_s = sampling frequency and f_1 =fundamental frequency (from the identification). The whiteness test with the n_1 and $\bar{n}$ selection as described has been designated as the Lag Shifted Whiteness Test.

4. PSEUDO-DENSITY ESTIMATION - KERNEL PRINCIPAL COMPONENT ANALYSIS

Once the metrics for the two algorithms that are to be fused are available one can, if sufficient data is available, use a non-parametric Parzen estimator to estimate the density of the fused vector. An alternative to this approach is to obtain a decision boundary directly using the kernel based PCA algorithm [10]. We do not present the details of the KPCA algorithm due to space constraints but they can be found in [10]. The point to note, however, is that, given a set of training points and a desired Type I error rate the KPCA algorithm, without assuming a parametric form for the density of the reference state data provides a means for deciding if any test point should be considered an outlier.

5. CHANGE DETECTION

Advantage of the fact that the system state does not switch back and forth from healthy to damage can be realized by framing the damage detection problem as one of change detection. In this case the question posed is not whether a particular data point is an outlier but rather whether the aggregate of a sequence of points is such that a change should be announced. A convenient approach to process the data for change detection is by means of control charts devised in the process control field [11]. A control chart is a depiction in the *y*-axis of some metric whose probability depends on the state of the process vs. the sample number. The CUSUM chart is designed to perform a hypothesis test regarding the shift in any parameter of a distribution that is assumed to prevail when the null hypothesis is true. Due to space constraints we do not present the details of the change detection format here but the results of the numerical example in the next section are presented in a change detection format.

6. ILLUSTRATIVE EXAMPLE

We consider the 3-D tower shown in fig.1. The areas of the bars are all equal and the modulus of elasticity is taken such that the fundamental period is 1.27 sec when the mass matrix is the identity, damping is 2% in all the modes. The DOF are ordered as x,y,z starting in node5. The applied (ambient) loading is assumed to be restricted to the horizontal DOF and the sampling rate is 50Hz. Measurement noise with approximately 5% RMS is added to all the "measured" signals prior to processing.

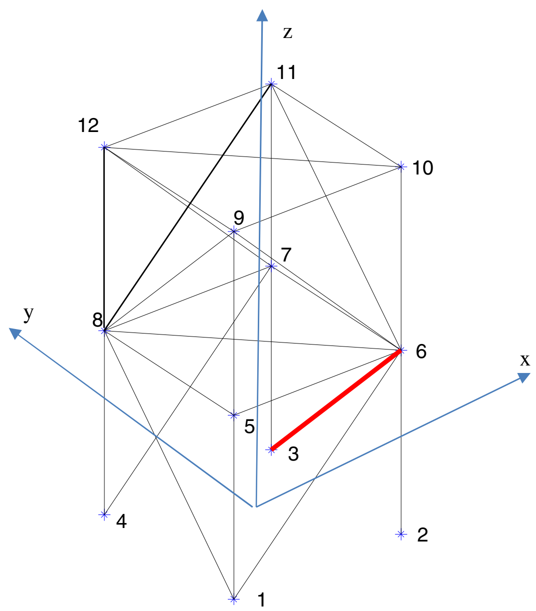

Fig.1 Tower structure with bar to be damaged indicated in red.

We select 6 accelerometers and damage as a 20% loss of stiffness in bar#3-6. Five different sensor arrangements (all with 6 sensors) are considered. Here we do not discuss the sensor placement problem but the arrangements are selected in order of decreasing anticipated performance, namely C1 is expected best, C2 following and so on. The reason for C3a and C3b is because these two arrangements are, according to the criterion that was developed for sensor placements, essentially the same. We begin by illustrating the performance of the isolated algorithms.

Kalman

Fig.2 shows the results for the Kalman detector. As can be seen, the detector is capable of identifying the damage for the sensor distributions C1 and C2 but fails otherwise.

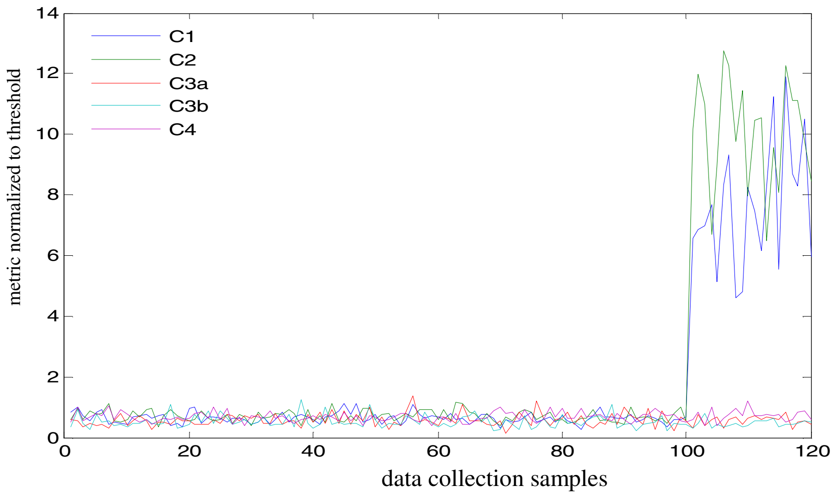

Fig.2 Performance of KALD on the structure of fig.1, damage = 20% on bar#3-6

Subspace

The results for the subspace approach are depicted in fig.3. As can be seen, the performance of the approach is rather poor.

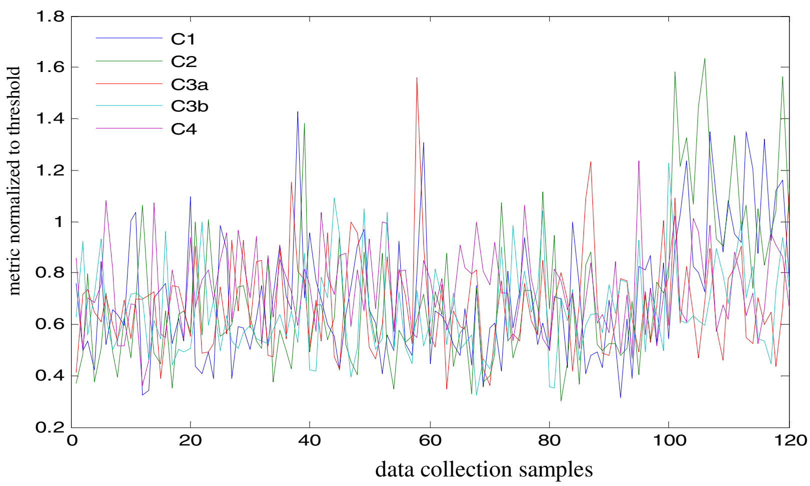

Fig.3 Performance of subspace approach on the structure of fig.1, damage = 20% on bar#3-6

Algorithm Fused Change Detection

The metrics depicted in fig.2 and 3 are fused using the KPCA algorithm and the results are shown, for all 5 sensor deployments, in a change detection format, in fig.4.

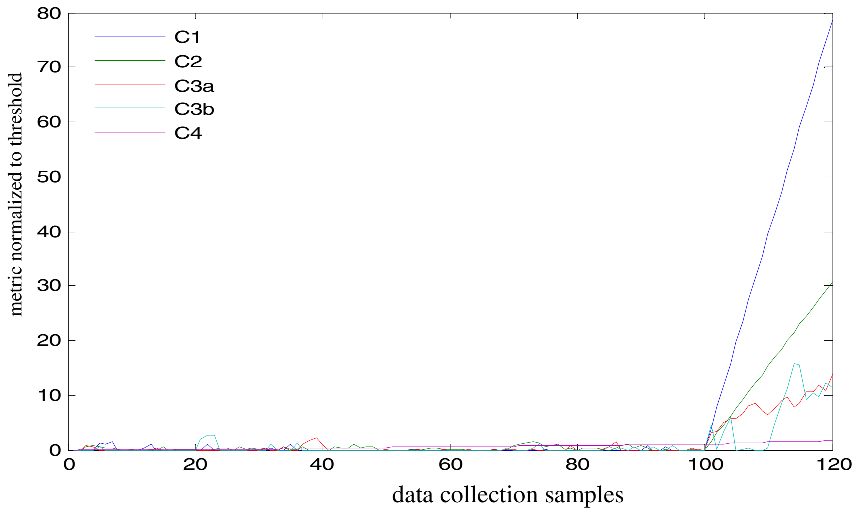

Fig.4 Performance of fused algorithms in change detection mode, damage = 20% on bar#3-6.

7. Concluding Comments

It is interesting to note that results for the fused strategy orders the performance of the sensor deployments in precisely the anticipated order. It is also worth noting that the fused strategy in the change detection mode identifies the damage not only with deployments C1 and C2 but also with the C3a and C3b arrangements, for which neither algorithm, independently, identifies damage. It appears, therefore, that algorithm fusion can improve damage detection performance and thus deserves further examination.

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