Damage Estimation Using Muli-Objective Genetic Algorihtms

Faisal Shabbir, Piotr Omenzetter

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Faisal Shabbir[1], Piotr Omenzetter[2]

[1] *Department of Civil Engineering, The University of Engineering and Technology, Taxila, Pakistan*

[2] *The LRF Centre for Safety and reliability Engineering, The University of Aberdeen, Aberdeen, UK*

piotr.omenzetter@abdn.ac.uk

Abstract

It is common to estimate structural damage severity by updating a structural model against experimental responses at different damage states. When experimental results from the healthy and damaged states are available, the updated finite element models corresponding to the two states are compared. Updating of these two models occurs sequentially and independently. However, experimental errors, updating procedure errors, modelling errors and parametric errors may propagate and become aggregated in the damaged model in this approach. In this research, a multi-objective genetic algorithm has been proposed to update both the healthy and damaged models simultaneously in an effort to improve the performance of the damage estimation procedure. Numerical simulations of a simply supported beam damaged at multiple locations with noisy mode shapes were considered and improved model updating results were confirmed. It was found that the proposed method is more efficient in accurately estimating damage severity, less sensitive to discretization as well as experimental errors, and gives the analyst an increased confidence in the model updating and damage estimation results.

Keywords: *multi-objective optimization, genetic algorithms, model updating, damage estimation*

1   Introduction

This paper presents a novel damage estimation method which simultaneously updates the numerical model of a structure in different states (e.g. healthy and damaged) in a multi-objective optimization (MOO) process. The main aim of this paper is the application of a MOO technique to damage estimation problems to concurrently utilize the experimental data of two states of a structure. Contemporary damage detection and estimation methods which are based on model updating typically require an updated finite element (FE) model corresponding to each of the structural states. The updated damaged models are then compared with each other for assessment of damage severity. There might be many errors associated with this model updating technique, e.g. experimental errors, updating procedure errors or parametric errors. These errors will be aggregated in the subsequent model updating runs.

In this paper, a damage estimation algorithm is being proposed which simultaneously updates the healthy as well as damaged structure model in the MOO process for improved damage estimation. In order to evaluate the performance of the proposed approach, it is applied to a numerically simulated simply supported beam with 3% noise added to the 'experimental' mode shapes. A genetic algorithm (GA) has been used as the optimization tool in this paper as multi-objective GAs have been extensively implemented in the past [1]. Compared to the single-objective optimization (SOO), which gives one optimal solution, multi-objective formulation gives a set of alternative solutions. A desirable solution can then be selected based on a trade-off between the different objective functions (in this case related to the different states of damaged).

This paper is structured as follows. Section 2 describes the MOO. This is followed by model updating of a simulated beam with noisy data using SOO and MOO for comparison in Section 3. The results of damage estimation using SOO and MOO are also detailed in Section 3, and the conclusions drawn from the study are reported in Section 4.

2 MULTI-OBJECTIVE OPTIMIZATION

An optimal solution for a physical system modelled using one objective function can be found using SOO. However, when two or more objective functions are used concurrently, the task of finding one or more optimal solutions is referred to as MOO. Many real world optimization problems involve multiple objectives. The extremist principle which prioritizes one objective over the others may lead to erroneous results especially in cases where the rest of objectives are also important or interdependent. Selecting a solution which is optimal for only one objective may compromise the other objectives. Evolutionary algorithms are a popular approach to solve MOO problems using the concept of domination [2]. According to this concept, one solution dominates the other solution if the following two conditions are satisfied:

1. The first solution is not worse than the second solution in all objectives, and
2. The first solution is strictly better than the second solution in at least one objective.

Violation of any of the above conditions indicates that the first solution does not dominate the second solution. It is intuitive that if any solution dominates the other solution, then it is also better in the context of MOO. The concept of domination is used to find non-dominated solutions. When all pair-wise comparisons have been made for a given finite set of solutions, we expect to have a set comprising a number of solutions which do not dominate one another. An important property of this set is that each of its solutions dominates all other solutions outside of this set. In other words, all the solutions in this set are better compared to the rest of the solutions. This leads to the definition of Pareto optimality which states that among all solutions, a non-dominated set of solution are those which are not dominated by rest of the solutions. The concept of Pareto optimality leads to a set of solutions known as the Pareto optimal set. A rank is assigned to the set [3]. A plot of objective function values corresponding to Pareto optimal set gives the Pareto front. Two basic approaches have been mentioned in the literature to obtain the Pareto optimal set, namely, preference based and evolutionary algorithm based MOO. While a large number of optimization runs is required to construct a Pareto front using the preference based approach, only a single run is required using evolutionary algorithm based approach.

2.1 Multi-objective evolutionary algorithm

In multi-objective evolutionary algorithm (MOEA) based optimization, all the objectives are evaluated concurrently instead of converting them into a single-objective function. Evolutionary algorithms, such as GA, work concurrently on a population of genes and use genetic operators such as selection, crossover and mutation to obtain globally optimal solutions. This evolution mechanism helps to explore the trade-off between solutions with different blends and grades of objectives. Also, they do not require gradients of the objective function; their chance to reach global optimal solutions is increased. A detailed review of multi-objective techniques can be found in [4].

Many variants of MOO, based on Pareto front approach and using MOEAs, have been proposed [4]. These include niched Pareto genetic algorithm, non-dominated sorting genetic algorithm (NSGA), multi-objective messy genetic algorithm, and strength Pareto genetic algorithm. A variant of NSGA, referred to as NSGA-II, is one of the most popular and efficient MOEAs [3, 5-7]. Therefore NSGA-II has been adopted for this study to investigate its effectiveness for damage detection and estimation via dynamic model updating. The general steps involved in NSGA-II are as follows:

1. Termination criteria based on the accuracy required and total number of generations are selected.
2. A random population of chromosomes (solutions) is initialized.
3. Values of objective functions for each chromosome are obtained.
4. Different ranks are assigned to each of the solutions based on a non-dominated sorting algorithm to classify the population into Pareto fronts.
5. Offsprings of the parent population are created.
6. A tournament selection of best solutions obtained from the previous step is performed.
7. Cross over with assumed probability is performed on the parent solutions to form new offsprings.
8. The new offsprings are mutated with a mutation probability.
9. A non-dominated sorting is performed on the new offsprings and once again all the solutions are classified into Pareto fronts using a non-dominated sorting algorithm.
10. If the termination criteria are achieved, stop, or else go to step 6.

After some trial and error, the following parameters of NSGA-II have been used:

Population size = 1000

Maximum number of generations = 500

Minimum of objective function value = 1×10^{-10}

Cross over probability = 0.8

Mutation probability = starts at 0.2 and linearly decreases to 0 at maximum number of generations

Pareto fraction (fraction of solutions to be kept in the first front) = 0.35.

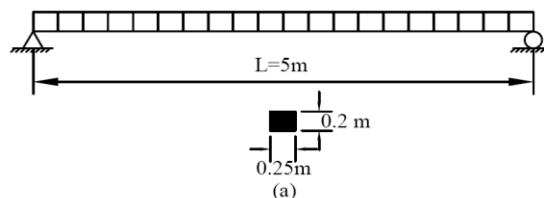## 3   APPLICATION TO DAMAGE ESTIMATION

In the previous decade, numerous studies have been performed to establish the health of structures under in-situ conditions [8, 9]. In the context of dynamic FE model updating, assessment of physical characteristics of the structure is done by comparing basic modal properties (such as natural frequencies and mode shapes) with their FE model counterparts.

A simulated simply-supported beam will be studied to demonstrate the effectiveness of the damage estimation method which simultaneously updates the undamaged as well as damaged structural model in a MOO process. The simulated beam has a total length of 5 m as shown in Figure 1. The beam has a total depth of 0.2 m and width of 0.25 m. The density of the beam was assumed as 2500 kg/m^3 and modulus of elasticity as 3.2 x 10^4 MPa. The area of the cross section is 0.05 m^2 and the moment of inertia (MOI) 1.66 x 10^{-4} m^4. The preliminary model of the beam was assumed as the one which has the above mentioned section properties with 20 discretised elements and is referred to as the initial FE model (Figure 1a). For updating, an a priori FE model was assumed with only 10 elements (Figure 1b). This introduces a discretization error in the process of damage estimation. In the simulated 'experimental' model for the damaged structure 1, MOI of elements No. 6 of the initial FE model has been reduced by 10% as shown in Figure 1c. For obtaining simulated 'experimental' modal parameters for the damaged structure 2, MOI of element No. 6 have been reduced by 15% and MOI of element No. 3 by 10 %. (Figure 1d). This methodology is advantageous to check the effectiveness of the proposed approach in updating both damaged models simultaneously.
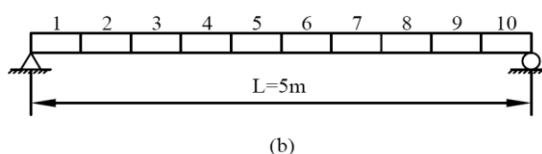
Modal analysis was carried out on both damaged beams to obtain first five natural frequencies and mode shapes. The values of first five natural frequencies for the initial FE model, the priori assumed model, damaged model 1 and damaged model 2 are shown in Table 1. It can be seen that maximum difference between the frequencies of the initial FE model and the undamaged beam is 1.16 % and of the damaged beam 2.62 %. Only 10 vertical degrees of freedoms are

assumed to be 'measured' to construct mode shapes (Figure 1b), as it is usual to have only limited measurements in actual experiments.
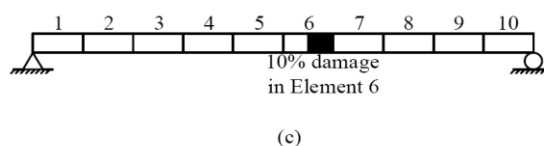
INITIAL MODEL

(a)

A PRIORI ASSUMED MODEL

(b)

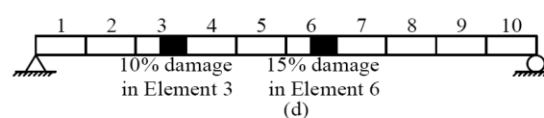**DAMAGED MODEL 1**

(c)

DAMAGED MODEL 2

(d)

Figure 1. Simulated simply supported beam: a) initial FE model without any reduction in MOI and with no discretization errors, b) a priori assumed model with discretization error, c) damaged model 1 with 10 % reduction in MOI of Element No. 6, and d) damaged model 2 with 15 % reduction in MOI of element No. 6 and 10 % in elements No. 3.

Table 1. Frequencies of the simulated beams.

Mode No.	Frequency (Hz)				Relative frequency difference (%)	
	Initial FE model	A priori assumed model	Damaged beam 1	Damaged beam 2	Damaged model 1 to initial	Damaged model 2 to initial
1	12.98	12.98	12.83	12.64	-1.16	-2.62
2	51.91	51.91	51.78	51.07	-0.25	-1.62
3	116.80	116.73	115.99	115.04	-0.69	-1.51
4	207.63	207.14	206.04	204.71	-0.77	-1.41
5	324.36	322.11	323.64	320.15	-0.22	-1.29

It can be assumed that modal frequencies are accurately determined in modal testing and experimental errors are more pronounced in mode shapes [10]. Consequently, random noise has been added to each of the k-th component of the j-th mode shape and the 'measured' component of the mode shape is given as:

$$\Phi_{jk,noise} = \Phi_{jk}\left(1 + \alpha_{noise}\varepsilon\right) \qquad (1)$$

where ε is a random number between -1 and +1 and α_{noise} is the level of noise. A noise level of 3% was added in the mode shapes [11], for checking the effectiveness of the proposed approach.

A combined objective function related to the frequencies and model assurance criterion [12] (MAC) is used in this study. The relative error between the experimental and analytical frequencies is:

$$\Pi_1 = \sum_{i=1}^{n} \left[\left(\omega_{a,i} - \omega_{e,i} \right) \Big/ \omega_{e,i} \right]^2 \qquad (2)$$

where ω represents modal frequency, subscripts a and e refer to the analytical and experimental values, respectively, and n is the total number of modes to be updated. The second objective function is related to the difference in mode shapes and can be defined in terms of MAC as:

$$\Pi_2 = \sum_{i=1}^{n} \left(1 - \sqrt{MAC_i} \right)^2 \Big/ MAC_i \qquad (3)$$

Two separate objective functions were defined respectively for the healthy Π_{health} and damaged structure Π_{dam2} as follows:

$$\Pi_{health} = \alpha \Pi_{health1} + \beta \Pi_{health2} \qquad (4)$$

$$\Pi_{dam} = \alpha \Pi_{dam1} + \beta \Pi_{dam2} \qquad (5)$$

where weighting factors α and β were both taken as 1.

Multi-objective techniques are able to handle multiple objective functions concurrently, therefore, in the context of this work, two more objective functions are proposed. The regularization expressions usually panelise parameter variations during iterations. Most frequently used conditions are: 1) parameter values be small, 2) parameter changes with respect to the reference model be small, and 3) parameter change between subsequent model updating iterations be small [13]. In this work, the second condition which mainly concerns the physical plausibility of the initial model is used. In order to give preference to a particular solution, the regularization terms for the two structural states are expressed as:

$$\Pi_{reg,health} = \left\| x_{i,health} - x_0 \right\|^2 \qquad (6)$$

$$\Pi_{reg,dam} = \left\| x_{i,dam} - x_0 \right\|^2 \qquad (7)$$

where x_0 is the value of parameters of the initial model, $x_{i,dam1}$ and $x_{i,dam2}$ are the values of parameters in current iteration i.

Model updating of the damaged beams has been performed in this section. The first five frequencies and mode shapes were selected to have a similar number of unknowns (MOI for each FE) as the number of knowns (frequencies and MACs). Parameter selection is an important step in model updating and prior damage detection and localization can result in an efficient selection of parameters. However, to check the effectiveness of the proposed approach in a challenging case, all the twenty parameters related to two structural states have been considered in model. Both SOO and MOO were performed to obtain the updated parameters to compare the performance of the two methods.

Following the conventional approach, an updated undamaged model was obtained using Equation (4) and SOO. A total of 10 parameters were updated with 10 knowns, i.e. five frequencies and five MACs of the damaged beam 1. After obtaining the updated undamaged model, the next step is to update the damaged model using Equation (5). Ten parameters were again updated with 10 knowns in this case, i.e. five frequencies and five MACs of the damaged beam 2.

For MOO, a total of 20 parameters need to be concurrently updated, in which case 10 parameters belong to the damaged beam 1 and ten parameters belongs to the damaged beam 2. All

the objective functions (i.e. Equation (4), (5), (6) and (7)) were concurrently used as objective functions during updating. The total number of knowns (frequencies and MACs) is 20 in this case and the number of unknowns (individual element MOIs) is also 20. The optimum solution is selected from the Pareto optimal set.

Figure 2a shows the mean relative errors in the stiffness ratio of the updated MOIs compared to their true values for 3% added noise for damaged beam 1, and Figure 2b shows the same for damaged beam 2. The maximum mean error for single-objective updating is found to be 0.10 and the maximum mean error for multi-objective case is 0.046 for damage beam 1 (Figure 2a). Likewise, the maximum mean error for single-objective updating is found to be 0.099 and the maximum mean error for multi-objective case is 0.059 for damage beam 1 (Figure 2b). It can be seen from the errors in the updated stiffness in Figure 2 that MOO has generally less error in the updated parameters than SOO, proving that the proposed approach is more efficient in updating both the damaged models. It also shows that MOO is less sensitive to experimental errors than SOO. It should be noticed that MOO has information available from both damage cases to the optimization algorithm in the search process.
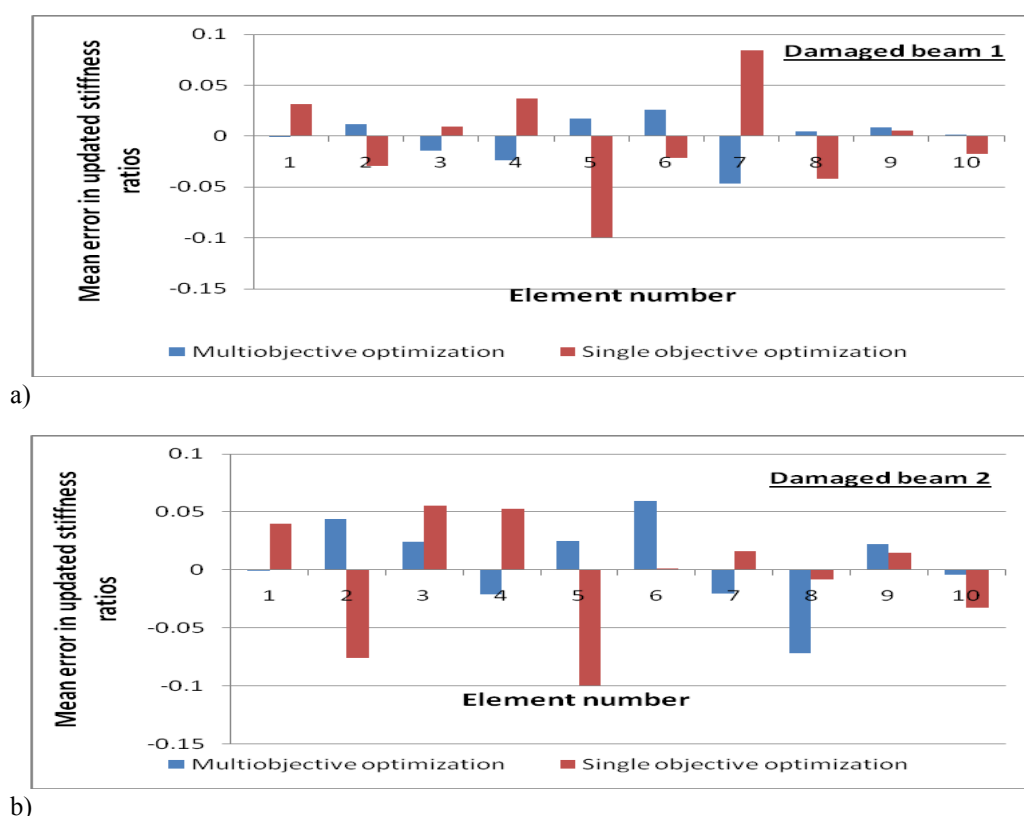

a)

b)

Figure 2. Relative mean error of estimated stiffness for 3% noise: a) damaged beam1, and b) for damaged beam 2.

For assessment of damage severity, the updated damaged model 1 is compared with the updated damaged model 2 for both SOO and MOO. Damage severity is quantified by subtracting the MOI ratios of the damaged model from that of the undamaged model. For example, the actual damage index has been calculated as 0.10 for element No. 6 in this study (as shown in Figure 1) for damage beam 1, which indicates a reduction of MOI of element No.6 from 1.0 to 0.9. Likewise, the actual damage index has been calculated as 0.15 for element No. 6 and 0.1 for element No. 3 for

damage beam 2. Damage severity is estimated for all the elements, and is shown in Figure 3a and b along with actual damage for damaged beam 1 and 2, respectively.

From the results, it can be seen that damage estimation from MOO is more accurate than from SOO. For example, the estimated damage index for element No. 6 for damaged beam 1 is found to be 0.074 for the single-objective case, whereas damage index for the same element is found to be 0.079 for the multi-objective case. This is a rather small adjustment, however, for other undamaged elements for damaged case 1, the wrong estimation of the maximum damage index for SOO is 0.099 for element No. 5 and for MOO is 0.046 for element No. 7, which are more significant improvements delivered my MOO over SOO. Furthermore, due to discretization error and experimental error assumed in the initial model, the SOO has led to erroneous detection of more damage in other elements of the beam than MOO. Especially the elements closer to the supports have less error in MOO than SOO. Generally speaking, SOO has shown more error in estimation of the damage severity than MOO.
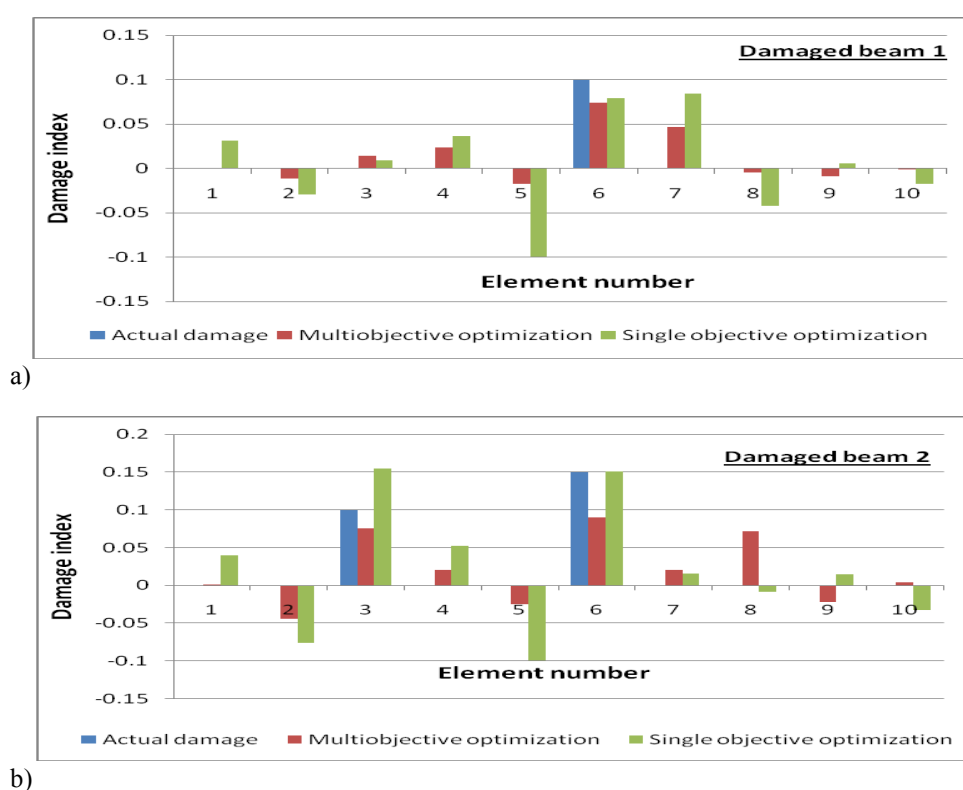

a)

b)

Figure 3. Damage severity index: a) damaged beam 1, and b) damaged beam 2.

CONCLUSIONS

This paper presents a damage estimation method which simultaneously updates the healthy as well as damaged structural model in a MOO process. The following is the summary and conclusions of this study:

1. Damage estimation via structural modal updating typically updates the models corresponding to different structural states separately. The associated experimental and numerical errors may propagate when these separately updated models are used for damage estimation. The use of multi-objective updating is an attempt to alleviate these issues.

2. A numerically simulated simply supported beam has been used as an example problem and noise level of 3% has been added to the identified mode shapes to assess the performance of the proposed procedure in accurate estimation of damage. It has been found that the proposed MOO method has been successful in updating both structural models concurrently.

3. The results have been compared with the results obtained from single-objective model updating and it has been found that the proposed method is more efficient in accurately estimating damage severity and is less sensitive to experimental errors.

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REFERENCES

1. Knowles, J., D. Corne, and K. Deb, Multiobjective problem solving from nature: from concepts to applications, 2008: Springer-Verlag, New York.
2. Kalyanmoy, D., Multi-objective optimization using evolutionary algorithms, 2001: John Wiley & Sons, New York.
3. Deb, K., et al., A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 2002. 6(2): p. 182-197.
4. Coello, C.A.C. and G.B. Lamont, Applications of multi-objective evolutionary algorithms, 2004: World Scientific, Singapore.
5. Li, H. and Q. Zhang, Multiobjective optimization problems with complicated Pareto sets, MOEA/D and NSGA-II. *IEEE Transactions on Evolutionary Computation*, 2009. 13(2): p. 284-302.
6. Chan, R.R. and S.D. Sudhoff, An evolutionary computing approach to robust design in the presence of uncertainties. *IEEE Transactions on Evolutionary Computation*, 2010. 14(6): p. 900-912.
7. Koppen, M. and K. Yoshida, Substitute distance assignments in NSGA-II for handling many-objective optimization problems. *Artificial Intelligence and Computer Science, Lecture Notes in Computer Science*, 2007. 4403 LNCS: p. 727-741.
8. Begambre, O. and J.E. Laier, A hybrid Particle Swarm Optimization - Simplex algorithm (PSOS) for structural damage identification. *Advances in Engineering Software*, 2009. 40(9): p. 883-891.
9. Hu, N., et al., Damage assessment of structures using modal test data. *International Journal of Solids and Structures*, 2001. 38(18): p. 3111-3126.
10. Udwadia, F.E., Structural identification and damage detection from noisy modal data. *Journal of Aerospace Engineering*, 2005. 18(3): p. 179-187.
11. Perera, R. and R. Torres, Structural damage detection via modal data with genetic algorithms. *Journal of Structural Engineering, ASCE*, 2006. 132(9): p. 1491-1501.
12. Möller, P.W. and O. Friberg, Updating large finite element models in structural dynamics. *AIAA Journal*, 1998. 36(10): p. 1861-1868.
13. Titurus, B. and M.I. Friswell, Regularization in model updating. *International Journal for Numerical Methods in Engineering*, 2008. 75(4): p. 440-478.