Output-Only Statistical Time Series Methods for Structural Health Monitoring: A Comparative Study

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ABSTRACT

A comparative assessment of six well known Output-Only Statistical Time Series Methods (OO-STSMs) for Structural Health Monitoring (SHM) is presented via damage detection and identification in a GARTEUR type aircraft skeleton structure. A concise overview of the methods highlighting their principles is presented and their effectiveness for damage detection and identification is assessed via numerous experiments and various damage cases on the skeleton structure. What is more, issues such as the methods effectiveness based on local or remote vibration sensors as well as their computational complexity and ease of use are also investigated.

KEYWORDS : *output-only damage detection, output-only damage identification, structural health monitoring, statistical time series methods, vibration based methods.*

1. INTRODUCTION

Vibration based methods have been widely used for damage detection and identification of various types of damages in a multitude of real life and laboratory structures [1]. The methods which use only vibration response signals, also known as Output-Only (OO) methods, form an important category within the vibration based methods for Structural Health Monitoring (SHM). Their use is of high importance for structures such as bridges, aircraft, naval vehicles and others, where the excitation signal is not available.

The two main families of vibration based SHM methods are based on either: (a) detailed analytical models, such as Finite Element Models [2], or, (b) data-based models [3–5]. The Output-Only Statistical Time Series methods (OO-STSMs) fall within the second family of methods and may be classified into two subgroups: non-parametric and parametric [3, 4]. Non-parametric methods are based on corresponding time series representations (models), such as the Power Spectral Density (PSD) [6, 7], while parametric methods are based on scalar or vector parametric time series representations, such as the AutoRegressive Moving Average (ARMA) models. The latter have been used for damage detection and localization through the monitoring of: i) discrepancies in model residual properties, such as variance, whiteness and so on [8–10], or ii) discrepancies in model parameter [11, 12]. Actually such statistical time series methods have been widely used and their performance has been assessed in various comparative studies by using both excitation and response vibration signals [1,5–7] but this has not been yet done for the OO-STSMs.

Thus the aim of this study is a concise overview of six well known OO-STSMs and their critical comparison through the detection and identification of various types of damages on a GARTEUR type aircraft skeleton structure [13]. More specifically the methods which are employed in this study are: i) the PSD based method, ii) the Model Parameter based method, iii) the Residual Variance based method, iv) the Likelihood function based method, v) the Residual Uncorrelatedness (whiteness) based method and vi) the residual based method using the Sequential Probability Ratio Test. Full details for the considered methods are found in [3, 4, 7, 10]. Further issues such as the methods sensitivity to the various types of damages, their effectiveness based on local or remote vibration sensors, as well as their computational complexity and ease of use are investigated.

2. OO-STSMs for SHM - The general Framework

Damage detection, identification and magnitude estimation based on the OO-STSM is accomplished in two main phases: (a) the baseline phase (training of the methods) and, (b) the inspection phase (damage detection and identification). A concise presentation and the main steps of each phase follows; full details are found in [3,4,7,10].

Baseline phase: Data sets from known structural states that correspond to the healthy, say S_o, and damaged state of the structure are collected. If damage identification and magnitude estimation are pursued then data sets from various damage types and magnitudes, say $S_A, S_B, S_C, \ldots$, are also required. More than one data set for each structural state may lead to improved training of the methods and thus reduced false alarms and fault misclassifications (step 1).

Based on the collected data a proper characteristic quantity Q for each known structural state, say $\hat{Q}_o, \hat{Q}_A, \hat{Q}_B, \hat{Q}_C, \ldots$ is estimated (for each method) and a bank with these estimates is constructed (step 2). Based on these values critical statistical points (thresholds) are appropriately determined in a way that minimizes false alarms and fault misclassifications when damage detection and identification are pursued for the known structural states $S_o, S_A, \ldots$. For damage identification and magnitude estimation, different thresholds are used for each damage type and magnitude (step 3).

Inspection phase: A new data set from the structure in a current, unknown, structural state S_u is acquired and a characteristic quantity $\hat{Q}_u$, as that estimated in the baseline phase, is obtained. Damage detection, identification and magnitude estimation is thus achieved based on the comparison of the unknown $\hat{Q}_u$ with the known $\hat{Q}_l$ $(l = o, A, B, C, \ldots)$ quantities of the baseline phase (step 4). This comparison is based on appropriate test statistics and critical points (determined in step 3) as those shown in Table 1 for the methods that are investigated in the present study. Some remarks on these methods follows; see full details in [3,4,10].

The Power Spectral Density (PSD) of the vibration response signal is the characteristic quantity Q of the PSD based method and $\hat{Q}_u = \hat{Q}_l$ if the test statistic F that follows an F distribution lies within the critical points (see Table 1). Similarly, the characteristic quantity Q of the Model Parameter based method is the vector that includes the parameters of a parametric time series representation such as AR/ARMA models and $\hat{Q}_u = \hat{Q}_l$ if the test statistic X that follows a χ^2 is lower than the selected threshold (see Table 1). Functions of the AR/ARMA residual sequences, such as their Variance, their Likelihood function, their autocovariance and the Sequential Probability Ratio Test (SPRT) are also incorporated in appropriate hypothesis testing procedures for damage detection and identification in the framework of corresponding methods (see details in Table 1).

3. The structure, the damage cases and the experimental details

The experimental assessment of the OO-STSM is based on a GARTEUR type skeleton structure [13] designed by ONERA and manufactured at the University of Patras (Figure 1). This structure has been also used in [7] where more details about its characteristics may be found. The aircraft skeleton is suspended through a set of bungee cords and hooks from a long rigid beam sustained by two heavy-type stands. The suspension is designed in a way as to exhibit a pendulum rigid body mode below the frequency range of interest, as the boundary conditions are free-free. The excitation is broadband random stationary Gaussian applied vertically at the right wing-tip (Point X, Figure 1) through an electromechanical shaker (MB Dynamics Modal 50A, max load 225 N). Each data set consists of the vertical acceleration responses at Points Y1–Y3 (Figure 1) which are measured via lightweight accelerometers (PCB 352A10). The acceleration signals are driven through a conditioning charge amplifier (PCB 481A02) into the data acquisition system based on SigLab 20–42 modules. The damage

Table 1 : Test statistics and critical points of the considered OO-STSMs.

Method	Test statistic	Critical points
PSD based	$F = \frac{\hat{S}_L}{\hat{S}_u} \sim f(2K, 2K)$	$F \in [f_{\frac{\alpha}{2}}(2K, 2K), f_{1-\frac{\alpha}{2}}(2K, 2K)]$ for $S_l = S_u$
Model Parameter	$X = \delta\hat{\theta}^T \cdot (2P_l)^{-1} \cdot \delta\hat{\theta} \sim \chi^2(d)$	$X < \chi^2_{1-\alpha}(d)$ for $\delta\hat{\theta} = \hat{\theta}_l - \hat{\theta}_u = 0$
Res. Variance	$F = \frac{\hat{\sigma}^2_{lu}}{\hat{\sigma}^2_l} \sim f(N_u, N_l - d)$	$F < f_{1-\alpha}(N_u - 1, N_x - d - 1)$ for $\sigma^2_{lu} \leq \sigma^2_l$
Res. Likelihood	$X = N\frac{\hat{\sigma}^2_{lu}}{\hat{\sigma}^2_l} \sim \chi^2(N)$	$X < \chi^2_{1-\alpha}(N)$ for $\theta_l = \theta_u$
Res. Uncorrelatedness	$X = N(N+2) \cdot \sum_{\tau=1}^{r}(N-\tau)^{-1} \cdot \hat{\rho}^2[\tau] \sim \chi^2(r)$	$X < \chi^2_{1-\alpha}(r)$
SPRT	$L = n \cdot log\frac{\sigma_o}{\sigma_1} + \frac{\sigma_1^2 - \sigma_o^2}{2\sigma_o^2\sigma_1^2} \cdot \sum_{t=1}^{n} e^2_{lu}[t]$	$L \leq \frac{\beta}{(1-\alpha)}$ for $\sigma_u \leq \sigma_l$

Explanation of symbols:
$_{l,u}$: Designate the known l (baseline phase) and current (inspection phase), unknown u, structural states.
$\hat{S}$: Welch based estimate of the Power Spectral Density. $f_{\frac{\alpha}{2}}, f_{1-\frac{\alpha}{2}}$: F distribution's $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$ critical points.
α: Type I error or false alarm probability; the probability of rejecting the null hypothesis when it is true.
K: Number of non-overlapping segments used in Welch based spectral estimation.
θ: Model parameter vector. P_l: Parameter vector covariance matrix corresponding to structural state l.
σ^2_{lu}: Variance of the residual sequence obtained by driving the signals corresponding to structural state u through the model representing structural state l.
σ^2_l: Variance of the residual sequence obtained by driving the signals corresponding to structural state l through the model representing structural state l.
N_l, N_u: The number of samples used in estimating the residual variance in l, u structural states respectively.
d: Parameter vector dimensionality. N: Signal length in samples. r: Maximum lag. $\hat{\rho}[\tau]$: Normalized autocovariance.
σ_o, σ_1: User defined values for the residual standard deviation under healthy and damaged states respectively.
e_{lu}: Residual sequence obtained by driving the the signals of structural state u through the model representing structural state l. n: Number of samples with $n \leq N$.
β: Type II error or missed damage probability; the probability of accepting the null hypothesis when it is true.

cases used in the study correspond to the *complete loosening* of one or more bolts at different joints of the structure. Five distinct damage types are considered and are summarized in Table 2. 164 data sets are obtained for each structural state from which 20 are used in the baseline phase for the training of the methods. Furthermore the Welch-based PSD estimates for the healthy and damaged states of the structure based on one of the used sensors are shown in Figure 2. The effects of the considered damages are not important in the 0.5- 60 Hz range while they are significant in the 60-200Hz range.

4. DAMAGE DETECTION AND IDENTIFICATION RESULTS

4.1 The Power Spectral Density (PSD) based Method

The PSD based method achieves accurate damage detection based on the measurements at all points (Y1-Y3) with no false alarms and missed damages as the test statistic does not exceed the critical points for the healthy test cases, while it is beyond them for all damage cases (see Table 3). The method identifies also the different damage types on the structure, with no misclassifications in almost all considered test cases; only a single case of damage A (0.69%) is not successfully identified (Table 4). Damage detection and identification results are achieved at the $\alpha = 10^{-5}$ risk level, while for the Welch based estimation of the PSDs, 1024 sample long windows with 0% overlapping and frequency resolution of $\delta f = 0.5Hz$ are used. Indicative damage detection and identification results based on the PSD method for three damage cases are shown in Figure 3(a).

4.2 The Model Parameter based Method

The Model Parameter based method employs six AR models per measurement point, identified (Matlab function: *arx.m*) in the baseline phase for the healthy state and the five damage types. An AR model is also identified in the inspection phase with the structure in an unknown structural state based

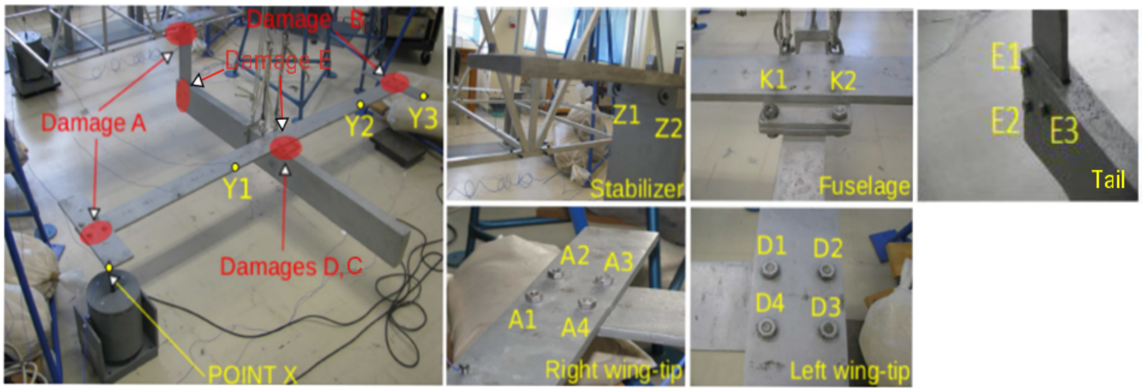

Figure 1 : The aircraft scale skeleton structure and the experimental set-up: The force excitation (Point X), the vibration measurement locations (Points Y1–Y3), and the bolts connecting the various elements of the structure.

Table 2 : The considered damage types and details on the acquired data sets.

Structural State	Description	Total Number of Data Sets
Healthy	—	164 (20 baseline)
Damage type A	loosening of bolt A1, A4, Z1 and Z2	164 (20 baseline)
Damage type B	loosening of bolts D1, D2 and D3	164 (20 baseline)
Damage type C	loosening of bolt K1	164 (20 baseline)
Damage type D	loosening of bolt K1 and K2	164 (20 baseline)
Damage type E	loosening of bolt K1, E1, E3	164 (20 baseline)
Sampling frequency: $f_s = 512$ Hz, Signal bandwidth: $[0.5 - 200]$ Hz		
Signal length N in samples (s): Non-parametric analysis: $N = 10\,000$ (19.53 s)		
Parametric analysis: $N = 10\,000$ (19.53 s)		

on the signals of each measurement point. The AR models are identified and validated through typical procedures, based on the Bayesian Information Criterion (BIC) and the RSS/SSS (Residual Sum of Squares / Signal Sum of Squares) criteria [14, pp. 498-514]. See model identification details in Table 5. A summary of the damage detection and identification results is presented in Tables 3 and 4, respectively. Damage detection is flawless in all considered cases with no false alarms or missed damages at the $\alpha = 10^{-2}$ risk level. Indicative damage detection results based on measurement point Y1 are pictorially presented in Figure 3(b). Damage identification is achieved at the $\alpha = 10^{-6}$ risk level where the method identifies correctly all damage types based on measurements at point Y1, while very few misclassifications are observed based on points Y2 (0.69% for damage type E) and Y3 (1.39% for damage type A, 0.69% for damage type E).

4.3 Residual Variance based method and Likelihood function based method

These methods employ the residual sequences which are obtained by driving the signals from points Y1, Y2 and Y3 through the corresponding AR models identified in the baseline phase (see Table 5). More specifically the residual variance and the likelihood function are the characteristic quantities which are used by the methods for damage detection and identification. Thus damage detection and identification is based on the comparison between the values obtained in the inspection phase with their counterparts of the baseline phase and this is achieved through appropriate test statistics that are shown in Table 1. It is noted that in the present study adjusted thresholds are selected in a way that minimizes false alarms and missed damages by applying the damage detection and identification

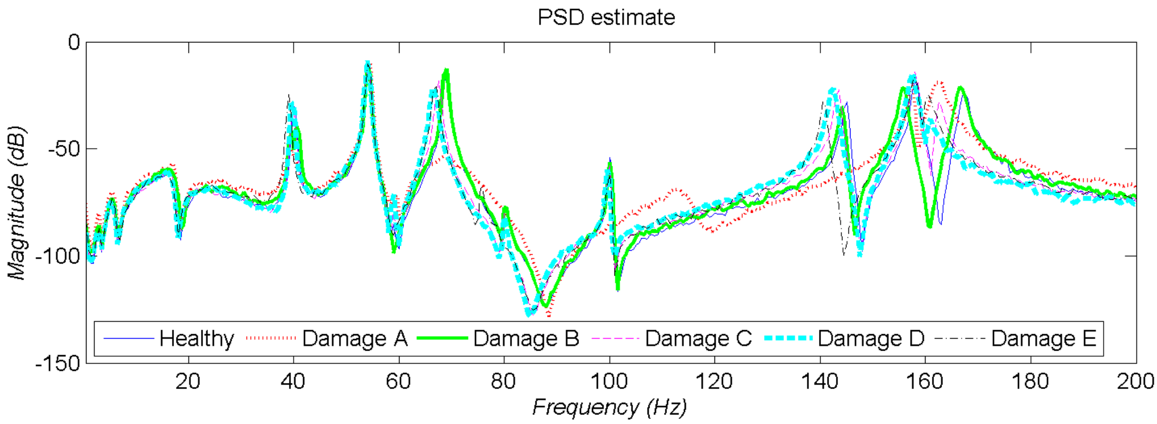

Figure 2 : Welch-based PSD for the healthy and damaged structure (window type: Hamming; window length 1024 samples; overlap: 90%; a single experiment for each test case).

Table 3 : Damage detection results (inspection phase).

Damage Detection

Method	Correct Detection (%)	False Alarms (%)	Missed damage (%) damage A	damage B	damage C	damage D	damage E
PSD based	100/100/100	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
Mod. Par.	100/100/100	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
SPRT	100/100/100	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
Res. Lik.	100/100/100	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
Res. Variance	100/100/100	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
Res. Uncorr.	100/100/100	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0

$(\%)/(\%)/(\%)$: results based on response Y1, Y2 and Y3 respectively. 144 data sets each of $N = 10000$ samples per test case.
PSD method: $\alpha = 10^{-5}$; **Mod. Par.:** $\alpha = 10^{-2}$; **SPRT method:** $\alpha = 10^{-3}$, $\beta = 10^{-4}$, $q = 1.08$;
Other methods: adjusted threshold.

procedures for the known structural states of the baseline phase. A summary of the damage detection and identification results of these methods is shown in Tables 3 and 4, respectively. Damage detection results for both methods based on measurement point Y1 are also pictorially presented in the corresponding Figures 4(a) and (b). These results demonstrate the flawless damage detection in all considered cases while in damage identification based on measurements of point Y1 damage type C is strongly identified as D $(86,11\%)$, although it is also correctly identified as type C at 98.61%. The same applies for damage E which is also identified as damage D (63.19%) based on data obtained from point Y2 (also see Table 4). This happens because these three types of damages have common characteristics (see Table 2) which lead to similar residual variances – variance is the main component of both test statistics of these methods (see Table 1) – when these are obtained from AR(124) for damages C and D and from AR(113) for damages D and E (also see Table 5).

4.4 The Residual Uncorrelatedness based method

This method is based on the autocovariance of the residual sequences obtained as in the previous two methods when the current signals pass through the models of Table 5. Adjusted thresholds are also used for damage detection and identification based on the corresponding test statistic (see Table 1). The damage detection results are summarized in Table 3 and indicative results are also shown in

Table 4 : Damage identification results (inspection phase).

Damage Identification

Method	Damage identification success rate (%)				
	damage A	damage B	damage C	damage D	damage E
PSD based	99.31/99.31/99.31	100/100/100	100/100/100	100/100/100	100/100/100
Mod. Par.	100/100/98.61	100/100/100	100/100/100	100/100/100	100/99.31/99.31
SPRT	100/100/100	100/100/100	100/100/100	100/100/100	100/100/100
Res. Lik.	100/100/100	100/100/100	98.61/100/100	95.83/100/100	100/100/93.06
Res. Variance	100/100/100	100/100/100	98.61/100/100	95.83/100/100	100/100/93.06
Res. Uncorr.	100/97.22/99.31	100/100/100	100/100/100	100/100/98.61	100/98.61/100

$(\%)/(\%)/(\%)$: results based on response Y1, Y2 and Y3 respectively. 144 data sets each of $N = 10000$ samples per test case.

PSD method: $\alpha = 10^{-5}$; **Mod. Par.:** $\alpha = 10^{-6}$; **SPRT:** $\alpha = 10^{-8}$, $\beta = 10^{-8}$, $q = 1.1$;

Other methods: adjusted threshold.

Table 5 : The AR models which are used for the parametric OO-STSMs and corresponding identification details.

Response	Selected Model	No of estimated parameters	SPP*	BIC	RSS/SSS (%)
Y1	AR(124)	124 parameters	81	−2.79	5.29
Y2	AR(113)	113 parameters	89	−5.79	0.26
Y3	AR(108)	108 parameters	93	−5.76	0.26

*Samples Per Parameter

Figure 4(c), while damage identification results are in Table 4. Damage detection is again excellent while damage types A, D, and E are not identified only in few cases based on remote measurements from points Y2 and Y3 (see Table 4).

4.5 The residual based method using the SPRT

Similarly this method employs an AR model for each measurement point (Table 5) and for each structural state from which residual sequences are obtained by driving the current response signals through them. Damage detection and identification is then achieved via the Sequential Probability Ratio Test (see Table 1), where σ_o is the mean estimate of the residual standard deviation obtained based on the 20 data sets of the baseline phase that correspond to the healthy state, while σ_1 is determined through the ratio $q = \frac{\sigma_1}{\sigma_o}$ with q being a user defined value that adjusts the method's sensitivity to the damage extent. The damage detection and identification results of this method are also presented in Tables 3 and 4, respectively, while the method's operation is presented through an indicative case with the healthy structure in Figure 3(c). The method achieves effective damage detection with zero false alarms and missed damages for all considered cases by using $\alpha = 10^{-3}$, $\beta = 10^{-4}$, $q = 1.08$. Moreover, the method is capable of identifying all considered damage types although the identification of damage C (identified also as damage D at 89.02% through data of point Y1) and E (identified also as damage D 82.93% through data of point Y2) exhibit similar misclassification issues with those of the Residual Variance and Likelihood based methods. Damage identification is achieved by setting $\alpha = 10^{-8}$, $\beta = 10^{-8}$ and $q = 1.1$. See [10] for the complete procedure of α, β, q determination.

5. CONCLUDING REMARKS

A comparative study for six well known OO-STSMs for SHM were presented through damage detection and identification of various damage types and numerous experiments in a GARTEUR type skeleton structure. Damage detection results were excellent for both non-parametric and parametric

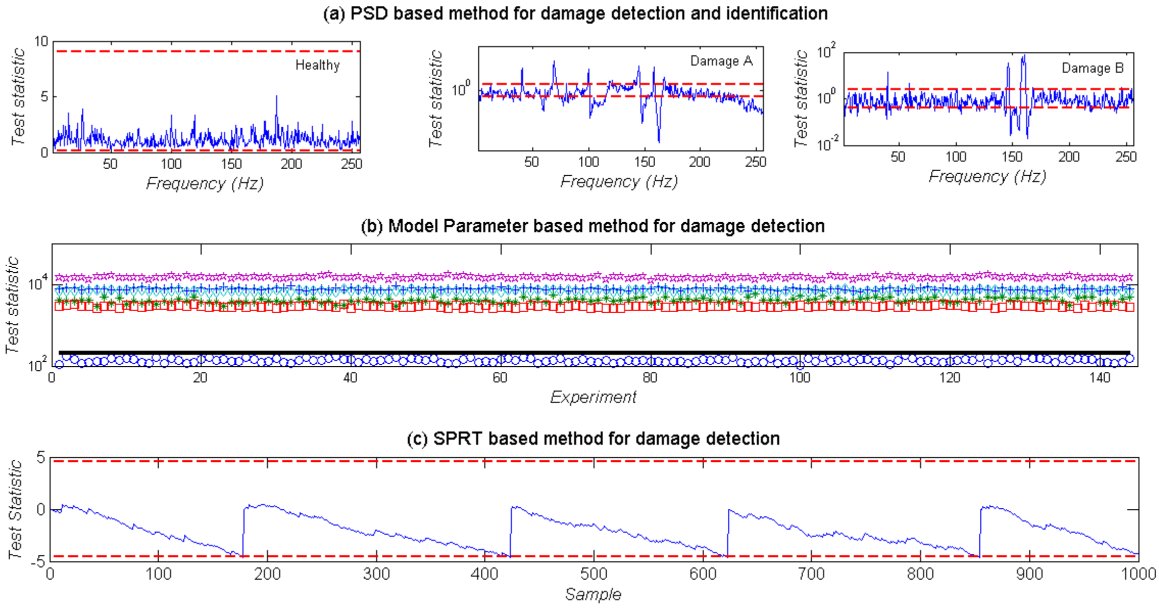

Figure 3 : (a): Indicative damage detection and identification results based on the PSD based method ($\alpha = 10^{-5}$, window 1024, 0% overlapping; damage is detected if test statistic exceeds the critical points: - - -). (b): Damage detection results based on the model parameter method and output Y1 (144 data sets for each structural state; ○: Healthy state +: Damage A, ∗: Damage B, □: Damage C, ◇: Damage D, ⋆: Damage E; —: threshold at $\alpha = 10^{-2}$ level; damage is detected if test statistic is over the threshold). (c): Indicative damage detection results based on the SPRT method (- - -: thresholds with $\alpha = 10^{-3}$, $\beta = 10^{-4}$, $q = 1.08$; single data set from point Y1 corresponding to healthy structure; a damage is detected when the test statistic exceeds the upper threshold while the structure is healthy when the test statistic lies below the lower threshold; the area between thresholds constitutes an uncertainty zone).

methods through local and remote sensors. PSD based method which is of the lowest complexity from the presented methods and limited user expertise is necessary for its application, achieved almost excellent damage identification results with a single misclassification. On the other hand the SPRT based method was the most elaborate as its training procedure necessitates the determination of α, β and q. However it is the only method from those presented that may be also applied for online SHM without any modification. From the parametric methods the Model Parameter and Residual Uncorrelatedness based methods found to achieve the best damage identification results. The Residual Variance, the Lihelihood function and the SPRT based methods identified some certain damage cases as belonging at two different damage types simultaneously. This is due to the fact that these methods are based on functions of the residual variance which may present a similar behavior under related small damages. Overall, the OO-STSMs presented a very good performance for SHM based only on a single local or remote vibration response signal.

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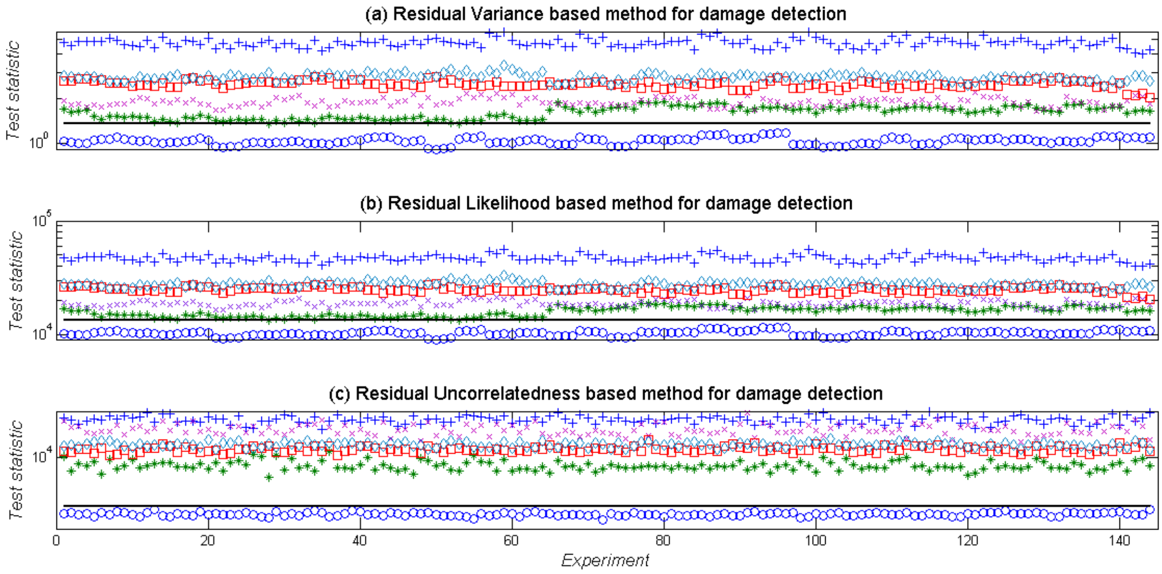

Figure 4 : Indicative damage detection results based on measurements at point Y1 and the: (a) Residual Variance metdod; (b) Residual Likelihood method; (c) Residual Uncorrelatedness method (144 data sets for the healthy state; 144 data sets for each damage state; ○: Healthy state , +: Damage A, ∗: Damage B, □: Damage C, ◇: Damage D, ⋆: Damage E, —: adjusted threshold; damage is detected if test statistic is over the threshold).

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