The Health Monitoring of a Prestressed Concrete Beam Using Inverse Modeling Technique and Measured Dynamic Response

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THE HEALTH MONITORING OF A PRESTRESSED CONCRETE BEAM USING INVERSE MODELING TECHNIQUE AND MEASURED DYNAMIC RESPONSE

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ABSTRACT

This paper presents an inverse modeling technique using dynamic strain responses to identify the prestressed concrete beam parameters: the pretension in the cable and the prestress in the concrete. The direct problem corresponds to 2D elastodynamics equations. To get the beam parameters, we minimize a classical data misfit function using a gradient-like algorithm. A low-cost computation of the functional gradient is performed using the adjoint problem.

KEYWORDS : *Inverse Problem, Adjoint Problem, Beam Parameters, Dynamic Response.*

INTRODUCTION

Prestressed beams are widely employed in civil engineering structures. Between 1945 and 1965, 1000 prestressed concrete bridges were constructed, 550 were of the VIPP type (independent span viaducts with prestressed beams) [1]. Thus, the maintenance of such structures is a central issue. To reduce maintenance costs, the damaged beams in the structure have to be identified early. To achieve this purpose, the beams should be monitored and non-destructive control methods should be used. Inverse techniques using dynamic responses have been developed in [2] to identify the prestress force in a beam, considering a one-dimensional (1D) beam modelisation. In the present article, we focus on inverse problems based on two-dimensional (2D) beam modelisation. As a matter of fact, by means of 2D modelisation, we can specify the 2D position of the sensors and we can precisely describe the geometry and the position of cables. Previous works [3] using 2D models and dynamic responses deal with the determinations of the steel bar cross section and the concrete Young Modulus in potential damage area. Using dynamic strain responses, we are interested in identifying the pretension in the cable and the prestressed in the concrete. For that, an inverse modeling technique based on the optimal control theory [4] is proposed.

METHODOLOGY TO IDENTIFY THE TENSION IN THE STEEL BAR AND THE PRESTRESS IN THE CONCRETE IN A 2D CONCRETE BEAM

In a 2D concrete beam we aim at identifying the pretension N_0 in the steel bar and the prestress σ_0 in the concrete from the knowledge of measured strain. Note that the dynamic loading applied to the concrete beam is considered known. To determine the beam mechanical parameters, an inverse modeling technique is employed. As usual in the optimal control theory, the pretension N_0 in the steel bar and the prestress σ in the concrete are determined by minimizing a data misfit function. The minimization process is performed in an iterative way using the steepest decent direction (gradient method). At each iteration, the mains steps are the followings:

- solve the direct problem (elastodynamics equations forward in time) considering the beam parameters N_0 and σ_0 of the previous iteration;
- solve the adjoint problem (elastodynamics equations backward in time);

- compute the functional gradient using the direct and the adjoint states;
- update the pretension N_0 applied in the steel bar and the prestress σ_0 in the concrete.

1. DIRECT PROBLEM

In the present study, the shear and the bending work in the cable are neglected. We also neglect the quantity of acceleration in rotation in the cable. We suppose that the cable and the concrete are perfectly adherent. The beam is instrumented with strain sensors.
We seek the displacement field $\underline{u} \in \mathscr{U}_0 = \{\underline{u}^* \in H_1(\Omega) \backslash \underline{u}^* = \underline{0} \text{ on } \partial\Omega_1, \underline{u}^*.\underline{y} = \underline{0} \text{ on } \partial\Omega_2\}$ such that :

$$\begin{aligned} &\int_\Omega \rho_c \underline{\ddot{u}}.\underline{u}^* d\Omega + \int_\Gamma \rho_b S_b \underline{\ddot{u}}.\underline{u}^* d\Gamma + \int_\Omega \underline{\underline{\varepsilon}}(\underline{u}) : \mathscr{K}_c \underline{\underline{\varepsilon}}(\underline{u}^*) d\Omega \\ &+ \int_\Gamma E_b S_b \frac{\partial u_x}{\partial x}\frac{\partial u_x^*}{\partial x} d\Gamma + \int_\Omega \underline{\underline{\sigma}}_0 : (\nabla \underline{u}.\nabla \underline{u}^*) d\Omega \\ &+ \int_\Gamma N_0 \frac{\partial u_x}{\partial x}\frac{\partial u_x^*}{\partial x} d\Gamma - \int_{\partial\Omega_f} \underline{F}_d.\underline{u}^* \partial\Omega = 0, \ \forall \underline{u}^* \in \mathscr{U}_0 \end{aligned} \tag{1}$$

The initial conditions are supposed to vanish.
where

$$\underline{\underline{\sigma}}_0 = \begin{bmatrix} \sigma_0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \text{and} \qquad \sigma_0 = -\frac{N_0}{h.p}.\left(1 - \frac{12.e.y}{h^2}\right) \tag{2}$$

- σ_0 : Prestressed concrete;
- N_0 : Tension on the cable;
- h : The height of the beam;
- p : The depth of the beam;
- e : The distance between the middle of the beam and the cable;

For solving the direct problem, the finite element method and Newmark time integration schema are used.

2. INVERSE PROBLEM

In order to determine the beam parameters, *i.e* the pretension N_0 in the cable and the prestress σ_0 in the concrete, an inverse modeling technique is employed. As usual in the optimal control theory, we seek N_0 and σ_0 such that it minimizes a data misfit function J. The minimization process is performed in an iterative way using the steepest descent direction (gradient method). The data misfit at sensors i is defined as:

$$\delta_i(t) = \int_\Omega \varepsilon_{xx}^{sim}(x,t)\psi_i(\underline{x} - \underline{x}_i) d\Omega - (\varepsilon_{xx}^{mes})_i(t) \tag{3}$$

We seek the normalized pretension $\bar{N_0}$ applied in the cable, minimizing the data misfit functional defined as:

$$J(\bar{N_0}) = \frac{1}{2}\sum_{i=1}^{n_s}\int_0^T [\delta_i(t)]^2 dt \tag{4}$$

where :

- n_s is the number of strain sensors;

- $(\varepsilon_{xx}^{mes})_i(t)$ and $(\varepsilon_{xx}^{sim})_i(t)$ are the xx measured and simulated strain respectively, by sensor i located at $\underline{x}_i$;
- $\psi_i(\underline{x}-\underline{x}_i)$ is spatial weight function associated to sensor i.
- $N_0 = \bar{N}_0.N_0{}^{ud}$, with: $\bar{N}_0 \in [0,1]$ and $N_0{}^{ud}$ is the undamaged pretension in the cable.

To get the functional gradient at a low computational cost, we use the adjoint state.
We seek the displacement $\underline{\tilde{u}} \in \mathscr{U}_0 = \{\underline{u}^* \in H_1(\Omega)\backslash \underline{u}^* = \underline{0}$ on $\partial\Omega_1, \underline{u}^*.\underline{y} = \underline{0}$ on $\partial\Omega_2\}$ satisfying:

$$\begin{aligned}
&\int_\Omega \rho_c \underline{\ddot{\tilde{u}}}.\underline{u}^* d\Omega + \int_\Gamma \rho_b S_b \underline{\ddot{\tilde{u}}}.\underline{u}^* d\Gamma + \int_\Omega \underline{\underline{\varepsilon}}(\underline{\tilde{u}}) : \mathscr{K}_c \underline{\underline{\varepsilon}}(\underline{u}^*) d\Omega \\
&+ \int_\Gamma E_b S_b \frac{\partial \tilde{u}_x}{\partial x}\frac{\partial u_x^*}{\partial x} d\Gamma + \int_\Omega \underline{\underline{\sigma}}_0 : (\nabla \underline{u}^*.\nabla \underline{\tilde{u}}) d\Omega \\
&+ \int_\Gamma N_0 \frac{\partial \tilde{u}_x}{\partial x}\frac{\partial u_x^*}{\partial x} d\Gamma - \sum_{i=1}^{n_s} \int_\Omega (\tilde{\sigma}_{0_{xx}})_i \frac{\partial u_x^*}{\partial x} d\Omega dt = 0,\ \forall \underline{u}^* \in \mathscr{U}_0
\end{aligned} \tag{5}$$

where $(\tilde{\sigma}_{0_{xx}})_i$ is the loading of the adjoint problem which corresponds to a xx internal stress in the vicinity of the i^{th} sensor location, it is defined as:

$$(\tilde{\sigma}_{0_{xx}})_i = \left[\int_\Omega \varepsilon_{xx}(\underline{x},t)\psi(\underline{x}-\underline{x}_i)d\Omega - (\varepsilon_{xx}^{mes})_i(t)\right]\psi_i(\underline{x}-\underline{x}_i) \tag{6}$$

Note that the problem (5) is solved using classical finite element method. At each iteration, we solve the direct problem (elastodynamics equations forward in time), we solve the adjoint problem (elastodynamics equations backward in time) and we compute the functional gradient using the direct and adjoint states. When considering the analytical format of the prestress depending on N_0, the functional gradient is given by:

$$\begin{aligned}
\frac{\partial J}{\partial \bar{N}_0} = &\int_0^T \int_\Omega \frac{1}{h.p}.\left(1 - \frac{12.e.y}{h^2}\right) N_0^{ud}.(\nabla \underline{u}.\nabla \underline{\tilde{u}})_{xx} d\Omega dt \\
&- \int_0^T \int_\Gamma N_0^{ud} \frac{\partial u_x}{\partial x}\frac{\partial \tilde{u}_x}{\partial x} d\Gamma dt
\end{aligned} \tag{7}$$

Once we get the functional gradient, several solves of the direct problem are performed to determine the optimal descent step and the new set of beam parameters are deduced.

NUMERICAL EXAMPLE

A 2D concrete beam with a single horizontal pretension cable is considered, Figure 1. A dynamic load $\underline{F}_d(t)$ is applied to the top of the concrete beam, it is defined as;

$$\underline{F}_d(t) = \begin{cases} -F_{max} sin\left(\dfrac{2\pi}{T_c}t\right)\underline{y},\ t \in [0, T_c/2[\\ 0,\ t \in [T_c/2, T] \end{cases} \tag{8}$$

In this study, the section of the steel bar S_b is assumed constant throughout the beam, the same for the Young Modulus of the concrete E_c, we take $S_b = 0.05$ and $E_c = 27GPa$. The Pretension N_0 in the cable and the prestrees σ_0 in the concrete are given by in the equation (2). We consider seven strain sensors in the concrete beam, there are located at :

$$\begin{aligned}
&S_1 : (3L/16, h/8); S_2 : (5L/16, h/8); S_3(7L/16, h/8); S_4(L/2, h/8); \\
&S_5(9L/16, h/8); S_6 : (11L/16, h/8); S_7(13L/16, h/8).
\end{aligned} \tag{9}$$

where L is the length of the beam ($L = 8m$)and h is the height ($h = 0.60m$).

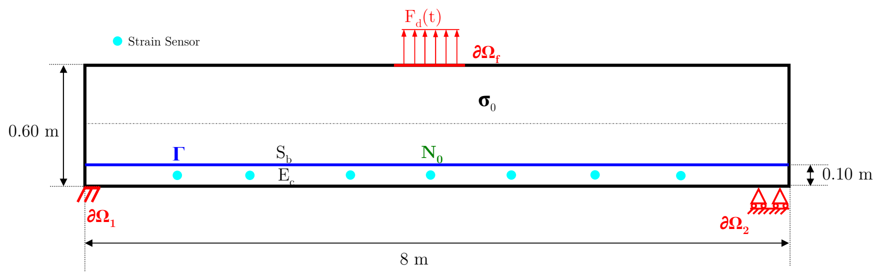

Figure 1 : Concrete beam with a horizontal steel bar - Beam instrumented with 7 strains sensors.

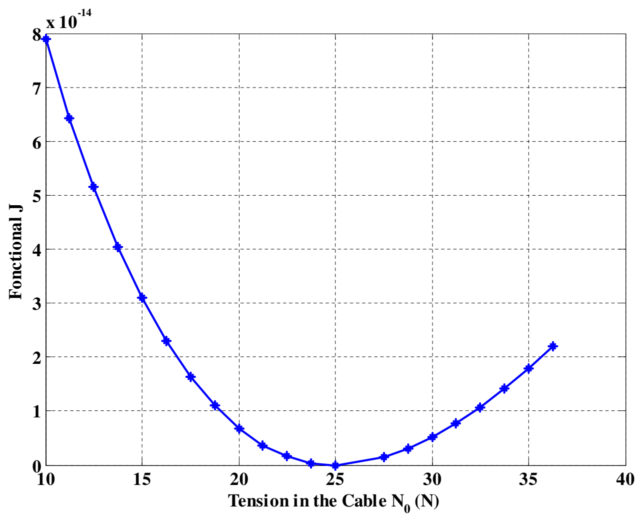

Figure 2 : Data misfit functional- 2D beam with 1 Cable - Reference pretension : $(N_0)_{ref} = 25.10^5 N$

Solution of the inverse modeling technique

Figure2 illustrates the convexity of the functional J described in equation 4. At the fourth iteration, Figure 3, we note that the error between the the reference pretension $(N_0)_{ref}$ and the reconstructed pretension N_0 is about 1% .

Conclusion

The identification technique based on 2D elastodynamics model and dynamic strain responses is proposed. On the 2D concrete beam with single cable, the pretension in the cable and the prestress in the concrete are identified after 4 iterations with an error less than 1%. In the futur work, real data outputs will be considered and more representative 2D beam model will be employed in the inverse technique.

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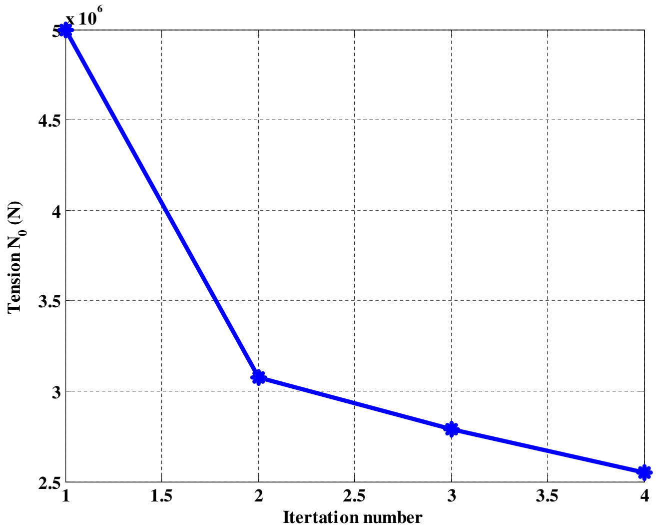

Figure 3 : The Tension N_0 at each iteration

France), CEDRAT, ESIEE Paris, NECS, NEWSYEO, SYROKKO and IFSTTAR.

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