Dynamic Strain Prediction Using Modal Parameters

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ABSTRACT

Vibration monitoring is conventionally performed from measurements of acceleration, velocity or displacement. However, these are not the primer parameters of interest in most structural diagnosis. Dynamic stress and strain are the appropriate parameters, which may be used in this monitoring. In some cases, strain is measured using strain gages techniques. But, when it is required to measure strain at different points, these techniques become onerous, because the strain gages must be fixed in structure and cannot be reused. In this sense, some methods have been developed to predict the dynamic strain from vibration measurements on structures. These methods basically consist in the numerical differentiation of displacement, obtained by modal analysis. Here, the dynamic strain prediction, based on hybrid modal analysis and acceleration measurements, is carried out. We used the finite difference method in the displacement to strain transformation. The displacements were obtained from measurements of acceleration and operating deflection shapes technique. The predicted strains were compared with the measured strains in the time domain. The predicted results closely agree with the measured results.

KEYWORDS : *Dynamic strain, vibration,modal parameters .*

INTRODUCTION

In operating condition, machinery and equipment are subject to vibration and levels of strain and stress. In some situations, high levels of strain are achieved when operating conditions are changed or, when during the machine design the real standard operating condition is not considered. Operational drawbacks, as wear, misalignment and unbalance, can also cause vibrations. Fatigue failures can be the consequence of excessive vibration. Thus, to predicted theses failures, it is important to measure the dynamic strain distribution on structures. However, strain measurements with conventional strain gauges are not always possible and the strain gauges must be fixed in structure and cannot be reused.

In order to minimize or even avoid using the strain gauge techniques for dynamic strain identification, some methods have been developed to predict the dynamic strain from conventional vibration measurements. Bernasconi and Ewins [1] have shown how modal testing using both strain gages and displacement transducers can be used to determine (mass-normalized) modal strain fields. Okubo and Yamaguchi [2] predicted the distribution of dynamic strain under operating conditions, using the displacement to strain transformation matrix. In this case, the transformation matrix was obtained by the strain and displacement modes, which were identified using accelerometers and strain gauges, respectively. Sehlstedt [3] used the Hybrid Strain Analysis (HSA). HSA is interpreted as dynamic strain analysis based on Hybrid Modal Analysis (HMA), which was proposed by Dovstam [4]. HSA was tested experimentally, where the dynamic strain tensor was predicted at specific points in the frequency domain. Lee [5] considered the method to predict the strain responses from the measurements of displacement responses using the displacement to strain transformation matrix. Recently, we have used the HSA to identify operational strain modes on an aluminium plate using acceleration measurements [6].

Here, both conventional acceleration measurements and the hybrid modal analysis is carried out to predict strain in the time domain. We used the finite difference method in the displacement to strain transformation. The displacements were obtained from acceleration measurements. The predicted strains were compared with the measured strains.

1 THEORETICAL FUNDAMENTALS

1.1 Hybrid modal analysis

Hybrid Modal Analysis (HMA) was proposed by Dovstam [5]. This technique is based on the orthogonality of the system's eigenmodes and in the mean square convergence properties of generalized Fourier series. The eigenmodes are assumed to be solutions to an elastic eigenvalue problem corresponding to the true geometry of the analyzed body or structure. Numerical approximations of the eigenmodes can be obtained by Finite Element Method (FEM). It is assumed that the vibrational behavior is linear and for small deformations.
Please note the following guidelines for paper submission:

Since $\{\mathbf{Ue}_\mathrm{i}\}$ is a displacement vector in the ($\mathbf{i}$ = x, y or z) direction and consisting of N displacement responses [5]:

$$\{\mathbf{Ue_i}\} = \begin{Bmatrix} \mathrm{u_i}(1,\omega) \\ \mathrm{u_i}(2,\omega) \\ \vdots \\ \mathrm{u_i}(N,\omega) \end{Bmatrix}. \tag{1}$$

and since the modal displacement matrix $[\mathbf{\Phi}]_{N\times M}$ is:

$$\left[\mathbf{\Phi e_i}\right] = \mathbf{e_i}\begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1M} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{N1} & \phi_{N2} & \cdots & \phi_{NM} \end{bmatrix} \tag{2}$$

then the generalized Fourier series can be written in this way:

$$\{\mathbf{Ue_i}\} = \left[\mathbf{\Phi e_i}\right]\{\mathbf{C}\} + \{\mathbf{U}_\mathrm{res}\} \tag{3}$$

where $\{\mathbf{C}\}$ is the coefficient vector defined as:

$$\{\mathbf{C}\} = \{\mathbf{C}(\omega)\} = \{c_1(u) \quad c_2(u) \quad \ldots \quad c_M(u)\}^T \tag{4}$$

and $\{\mathbf{U}_\mathrm{res}\}$ is the residual vector. In most application the residual can be neglected.

Since N is the number of degree of freedom, or referred to the measured points, and M is the number of modes, the coefficient vector can be estimated as:

If $N = M$:

$$\{\mathbf{C}\} = \left[\mathbf{\Phi e_i}\right]^{-1}\{\mathbf{Ue_i}\} \tag{5}$$

If $N > M$, the coefficient vector thus is estimated as:

$$\{\mathbf{C}\} = \left[\mathbf{\Phi e_i}\right]^{+}\{\mathbf{Ue_i}\} \tag{6}$$

where $[\mathbf{\Phi e_i}]^{+}$ is the pseudo inverse matrix defined as:

$$[\mathbf{\Phi e}_{\mathrm{i}}]^{+} = \left([\mathbf{\Phi e}_{\mathrm{i}}]^{T}[\mathbf{\Phi e}_{\mathrm{i}}]\right)^{-1}[\mathbf{\Phi e}_{\mathrm{i}}] \tag{7}$$

In applications the M is chosen such that the angular frequency ω_M fulfills $\omega<\omega_{\max}<\omega_{\mathrm{M}}$, where $\omega_{\max}$ is the maximum frequency of the signal. The number of measurements N must then also be increased for the system of equations to be sufficiently overdetermined. This means that the number of measurements, N, should be large compared to the number of modes, M, so that the estimated Fourier coefficient are smooth enough in the frequency interval of interest [5].

From the hybrid modal analysis, it is possible to predict unmeasured response of the structure in other parts of it or, in other directions not considered in the measurement. The three-dimensional displacement information is essential to predict the strain using the strain tensor, which will be discussed in the next section.

1.2 Strain Tensor

The strain can be predicted taking into account the three-dimensional displacement vector $\mathbf{u}_{\mathrm{i}}$. Since u, v and w are the displacement components along the axes x, y and z, respectively, the strain tensor can be expressed in linearized matrix form, by [7]:

$$\varepsilon_{ij} = \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right) & \dfrac{\partial v}{\partial y} & \dfrac{1}{2}\left(\dfrac{\partial v}{\partial z}+\dfrac{\partial w}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial v}{\partial z}+\dfrac{\partial w}{\partial y}\right) & \dfrac{\partial w}{\partial z} \end{bmatrix} \tag{8}$$

or, for a specific case:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} \\ \gamma_{xy} &= \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \end{aligned} \tag{9}$$

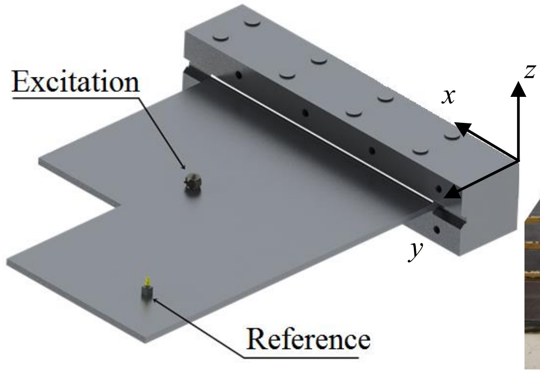
where ε_x and ε_y are the normal strain in the x and y direction, respectively, and γ_{xy} is the shear strain.

Since the displacement is known at discrete points on the structure, the tensor of Equation (8) is solved using numerical methods. In this case, finite difference method will be used.

2 EXPERIMENTAL TEST CASE

In this section the strain in the time domain is predicted based on acceleration measurements on a test plate. The predicted strains are compared with measured strains.

The dimensions of the test plate are $0.400\times0.500\times0.0095$ m (x,y,z) with a rectangular cutout having the dimensions $0.150\times0.200\times0.0095$ m. The plate was fixed at an aluminium block by weld. The block was fixed at a test bench using screws, as can be seen in Figure 1. The plate was excited with an electrodynamic shaker at a top location $0.200\times0.200\times0.0095$ m. The plate was subject to a harmonic force of magnitude A = 10 N and frequency f = 32 Hz. This frequency is close to the first natural frequency of the plate, which is approximately 35.8 Hz, and is also related to the rotation speed of most electric motors, used in machinery and equipment. Figure 1 illustrates the positions of the excitation and reference accelerometer. The excitation was placed close to gravity centre of the plate and the reference was placed in a large vibration amplitude region.



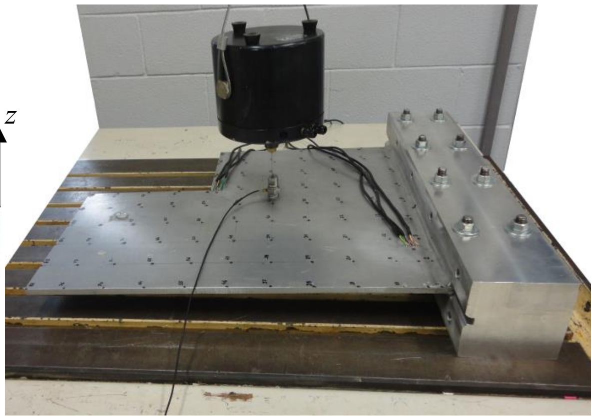


Figure 1: Test plate and position of the reference and excitation

The accelerations were measured in the *z* direction (normal to the top plate surface) for 24 locations, distributed along the plate surface. Piezoelectric accelerometers Delta Tron® type 4508 Brüel & Kjaer were used. The test plate and the measurements grid can be seen in Figure 2. During the measurements one accelerometer has remained fixed (reference accelerometer)

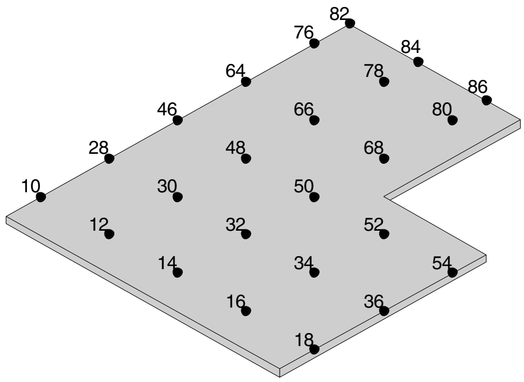


Figure 2: Test plate and position of the measurement points

As the measurements are not made simultaneously, we use the cross-spectral densities [8,9] between the measured signal and the reference signal for correction of relative phase between measurement points. The accelerations were converted to displacement by dividing by $-\omega^2 = -(2\pi f)^2$. The measured displacements in time domain were transformed to the frequency domain using the FFT algorithm in MATLAB ® 7.14.

The 16 lower-order tridimensional displacements modes shapes were obtained from the finite element model using ANSYS® 11.0. The finite element mesh consists of 1700 solid element type SOLID186 with 20 nodes in each element. The total number of nodes is 12353. Material properties used in the plate simulation were: density ρ = 2680 kg/m3, Young´s modulus E = 74 GPa and Poisson ratio ν = 0,33. The elastic eigenvalues were solved using the Block Lanczos algorithm. We used a simplified model of the plate without the block in the element finite simulation, as can be seen in Figure 3.

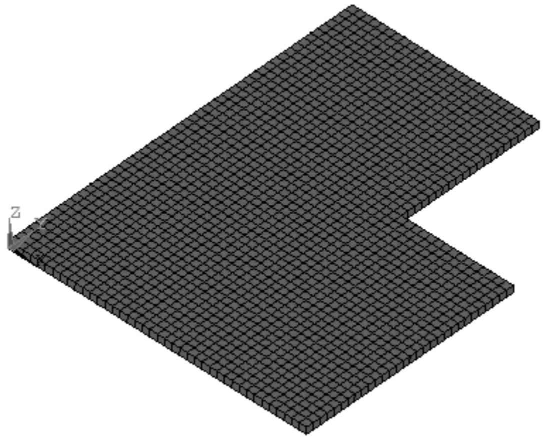


Figure 3: Simplified model and element mesh used

The 16 eigenmodes related to the 24 selected points and the z direction were conditioned in a modal matrix $[\mathbf{\Phi}]_{24\times16}$. This matrix and the frequency displacements were used to estimate the generalized Fourier coefficients by Equation (6).

A point close to the cut of the plate was chosen for validation. At this point the strains were predicted and measured. Rosette strain gage Kyowa KFG-1-120-D17-11 was used to measure strain. This point is located at a critical region of the plate because the stress concentration.

For estimation of the strain using the finite difference method, was selected eight points symmetrically arranged around the validation points, as can be seen in Figure 4. The distance between points is 10 mm. In Figure 4, point 90 corresponds to the validation points and the others are the points needed to calculate strain.

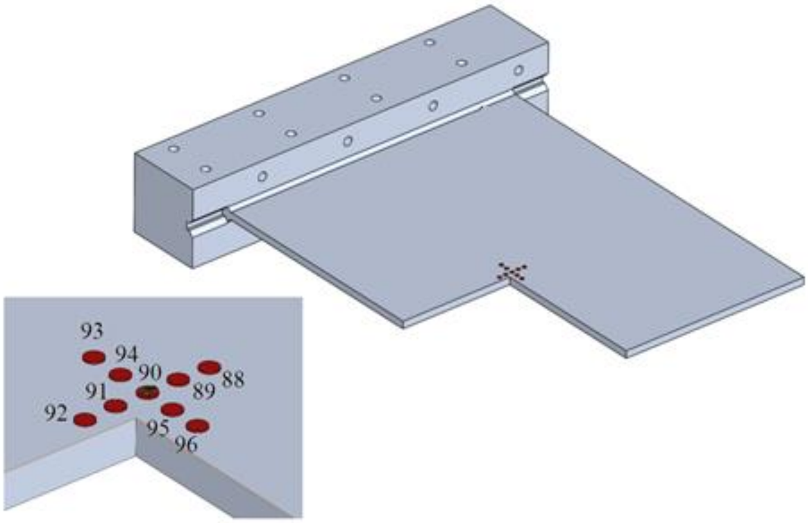


Figure 4: Validation points (90) and other necessary points to estimate strain

Using the Equation (3) and a new simulated modal matrix, it was possible to estimate the displacement in the x and y direction (planar direction) for the necessary points illustrated at Figure 4. The residual $\{U_{res}\}$ of Equation 3 was neglected. These estimated displacements in the frequency domain were transformed to time using the IFFT algorithm in MATLAB ® 7.14.

The strains at 90 validation point were predicted using the displacement in the planar direction and the following first order finite difference equations [10]:

$$\begin{aligned}
\varepsilon_x &\approx \frac{-u_{92} + 8u_{91} - 8u_{89} + u_{88}}{12h_x} \\
\varepsilon_y &\approx \frac{-v_{96} + 8v_{95} - 8v_{94} + v_{93}}{12h_y} \\
\gamma_{xy} &\approx \frac{-v_{92} + vu_{91} - vu_{89} + v_{88}}{12h_x} + \frac{-u_{96} + 8u_{95} - 8u_{94} + u_{93}}{12h_y}
\end{aligned} \tag{10}$$

where u_{i} and v_{i} are the displacement, in the time domain, estimated by hybrid modal analysis for the specific points and h_x and h_y are the distance between the points and equal to 0,010m.

2.1 Results

The predicted and measured strains of the validation point are shown in Figure 5.

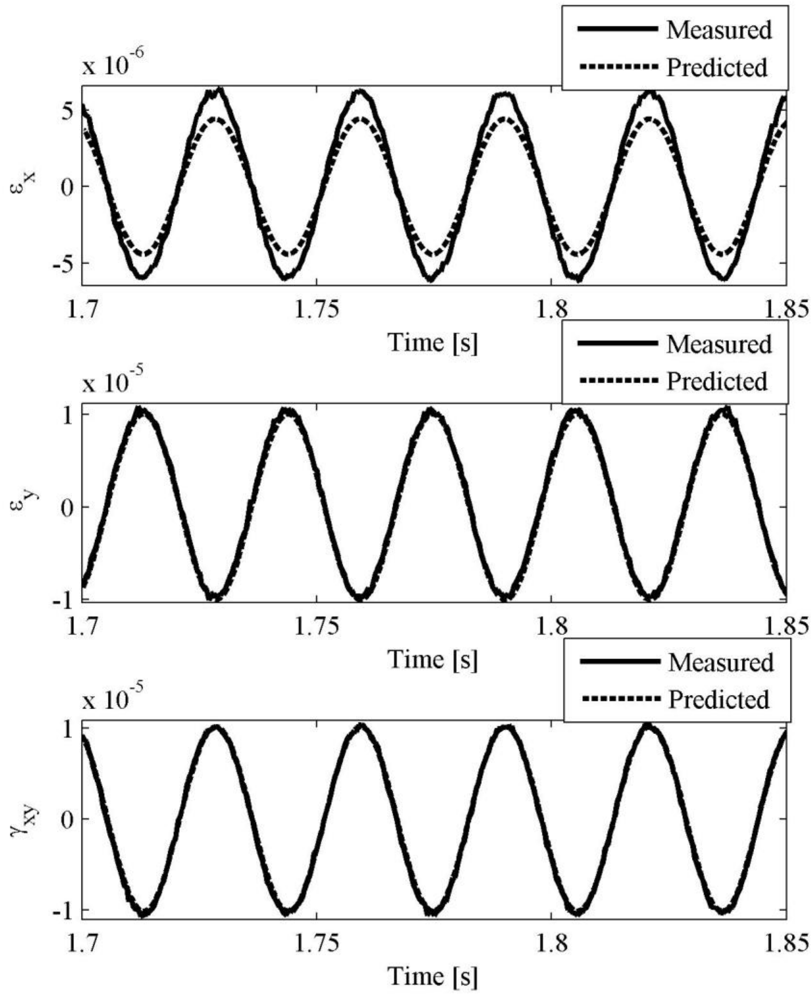


Figure 5: Predicted and measured strain at validation point

As can be seen in the Figure 5, the comparison between the predicted and measured strains, in the time domain, showed good results. The predicted strains are practically the same as the measured strains. Except the strain ε_x, where the predicted strain showed a lower amplitude than the measured strain. This difference may be associated with measurement errors and low strain values in the x direction.

The same results were obtained when was considered a point close to the block. The regions close to the cut and close to the block are important regions, which must be monitored, for instance, in real situation.

CONCLUSION

This study evaluated experimentally the strain prediction using the hybrid modal analysis in the time domain. Predicted strains and measured strains could be compared. The comparison showed good results. The strains were predicted at a critical region of the test plate.

In the experiment the excitation force was known. However, in the application of hybrid modal analysis, the excitation force need not be known. These results are promising and contribute to the equipment monitoring, especially in the operating conditions subject to unknown dynamic forces. Moreover, to know dynamic strain, the attachment of strain gages is not necessary. Instead, conventional acceleration measures can be used to predicted strain.

Is clear that a simulation is required to estimate the natural modes, but is not necessary the knowledge of the real material proprieties and boundary condition. In the evaluated case, a simplified model was used in the element finite simulation to estimate the natural modes.

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