Examination of Two Sensor Placements Schemes in Damage Detection

Dionisio Bernal, Michael Döhler, Danny Parker

EXAMINATION OF TWO SENSOR PLACEMENTS SCHEMES IN DAMAGE DETECTION

Dionisio Bernal[1], Michael Döhler[2], Danny Parker[3]

[1] *Northeastern University, Center for Digital Signal Processing, 360 Huntington Ave., Boston, 02115, MA, USA*
[2] *Inria, Campus de Beaulieu, 35042 Rennes, France*
[3] *AVNIK, 7262 Governors West Drive, Huntsville, 35806, AL, USA*

ABSTRACT

Deciding on the position of sensors by optimizing the utility of the monitoring system over a structure's lifetime is typically forbidden by computational cost. Sensor placement strategies are, instead, usually formulated for a pre-selected number of sensors and are based on cost functions that can be evaluated for any arrangement without the need for simulations. This paper examines the performance of two such schemes, the first one is derived directly from a technique that detects damage from the shift of a chi-square distribution from central to non-central and takes the optimal arrangement as the one that maximizes the sensitivity of the non-centrality to all parameter changes of equal norm. The second scheme selects the sensor arrangement as that which maximizes a weighted version of the norm of the sensitivity of the covariance of the output to all feasible changes in system parameters. The performance of the two schemes is tested in simulations.

KEYWORDS : Optimal sensor placements, damage detection, chi-square distribution

1 INTRODUCTION

A sensor arrangement is optimal, in a strict sense, if it maximizes the utility of the monitoring system over the structure's lifetime. Nevertheless, explicit treatment of this utility is difficult so in practice one generally settles for a less ambitious goal and places sensors in arrangements that maximize some scalar function of the probability of detection for the feasible damage patterns [1]. We note from the outset that the solution for the optimal arrangement when the damage detection is done with one algorithm is not necessarily the same as when it's done with another, indicating that the term optimal must be interpreted in a restricted, conditional sense. In any event, since the Fisher information in the measurements is algorithm independent one anticipates that dependency of the solution on the damage detection scheme is unlikely to be of "first order" importance.

Replacement of utility with probability of detection simplifies the optimal sensor placement problem but the computational burden, except for small academic problems, remains excessive if the probabilities have to be estimated from simulations. The simplification typically adopted in practice is to define a cost function that takes a unique value for each sensor arrangement and to select the sensor positions by maximizing (minimizing) this function. Even under this simplification the resulting combinatorial optimizations are typically non-convex and solutions by exhaustive search are restricted to relatively small problems. This paper describes two sensor placement strategies: one takes the optimal distribution as that which maximizes the sensitivity of the non-centrality of a chi square distribution to the appearance of all damages of equal norm in parameter space [2,3] and the second takes the solution as the arrangement that maximizes a weighted version of the norm of the sensitivity of the output covariance to the possible damages [4]. The effectiveness of the two strategies is examined in a numerical example of a far coupled system with 15 DOF and 3 sensors.

2 STRATEGY I – MAXIMIZING THE NON-CENTRALITY IN SUBSPACE DETECTION

Let ζ_N be the residual vector of the orthogonality test used in subspace damage detection [5,6], this residual is asymptotically normally distributed with

$$\zeta_N \xrightarrow{d} \begin{cases} \mathcal{N}(0,\Sigma) & \text{under } \mathbf{H}_0 \\ \mathcal{N}(\mathcal{J}\,\delta\theta,\Sigma) & \text{under } \mathbf{H}_1, \end{cases} \tag{1a,b}$$

where $\mathcal{J}$ and Σ are the asymptotic sensitivity and covariance, θ are the parameters, $\mathbf{H}_0$ is the null hypothesis, i.e. that the structure is not damaged, and $\mathbf{H}_1$ is the alternative. Let $\widehat{\mathcal{J}}$ and $\hat{\Sigma}$ be consistent estimates of $\mathcal{J}$ and Σ. A decision between the hypotheses $\mathbf{H}_0$ and $\mathbf{H}_1$ can be made through a generalized likelihood ratio (GLR) test, amounting to

$$\chi_N^2 = \zeta_N^T \hat{\Sigma}^{-1} \widehat{\mathcal{J}} \left(\widehat{\mathcal{J}}^T \hat{\Sigma}^{-1} \widehat{\mathcal{J}} \right)^{-1} \widehat{\mathcal{J}}^T \hat{\Sigma}^{-1} \zeta_N, \tag{2}$$

which is compared to a threshold that is set up in the reference condition for a desired type I error. The variable χ_N^2 is asymptotically χ^2-distributed with $\mathrm{rank}(\mathcal{J})$ degrees of freedom and non-centrality parameter $\gamma = \delta\theta^T F \delta\theta$, where

$$F = \mathcal{J}^T \Sigma^{-1} \mathcal{J} \tag{3}$$

is the asymptotic Fisher information on θ_0 contained in ζ_N. For any damage distribution the sensor arrangement that offers maximum resolution is that which maximizes the non-centrality parameter γ. Let the dimension of the parameterization be *m*, i.e. $\delta\theta \in \mathbb{R}^m$. The key result, as shown in [2], is that for changes $\delta\theta$ of constant norm it holds that

$$\int_{\|\delta\theta\|=1} \gamma \, \mathrm{d}(\delta\theta) = \int_{\|\delta\theta\|=1} \delta\theta^T F \delta\theta \, \mathrm{d}(\delta\theta) = \frac{c_m}{m} \mathrm{tr}(F) \tag{4}$$

where c_m is area of the unit sphere in $\mathbb{R}^m$ and $\mathrm{tr}(\cdot)$ denotes the trace of a matrix. As can be seen, the mean value of the non-centrality parameter γ for changes in the system parameter vector of unit norm (or equal norm) is proportional to tr(F). The point, then, is to select the sensor placements to maximize the trace of the Fisher information. To calculate the Fisher information one needs the sensitivity of the residual with respect to the parameters and the covariance of the residual. Accepting that damage is a change on the parameters that describe the stiffness one has that the sensitivity matrix $\mathcal{J}$ can be written as

$$\mathcal{J} = \mathcal{J}_{\zeta,\vartheta} \mathcal{J}_{\vartheta,\mu} \mathcal{J}_{\mu,p} \tag{5}$$

where ϑ and μ are the collection of eigenvalues and eigenvectors of the discrete-time and continuous time systems, respectively, and $\mathcal{J}_{\mu,p}$ contains the sensitivity of μ with respect to the structural parameters p. As shown in detail in [2,3]

$$\mathcal{J}_{\zeta,\vartheta} = \left(\mathcal{O}_{p+1}(\vartheta)^{\dagger} \mathcal{H}_{p+1,q} \otimes S(\vartheta) \right)^T \mathcal{J}_{\mathcal{O},\vartheta} \tag{6}$$

where $\mathcal{O}_{p+1}(\vartheta)$ is the observability matrix in modal basis, $\mathcal{J}_{\mathcal{O},\vartheta}$ is the derivative of the vectorized observability matrix with respect to ϑ, and † denotes pseudo-inversion. Formulae for $\mathcal{O}_{p+1}(\vartheta)$ and $\mathcal{J}_{\mathcal{O},\vartheta}$ are given in [5,6]. The covariance of the residual function $\zeta_N = \sqrt{N} \mathrm{vec}(S(\vartheta)^T \hat{\mathcal{H}}_{p+1,q})$ can be obtained as

$$\Sigma = (I \otimes S(\vartheta)^T) \Sigma_{\mathcal{H}} (I \otimes S(\vartheta)) \tag{7}$$

where $\Sigma_{\mathcal{H}}$ is the covariance of the vectorized block Hankel matrix of output covariance that is used in the subspace identification scheme. For the optimal sensor placement ϑ, $\mathcal{O}_{p+1}(\vartheta)$, $\mathcal{O}'_{p+1}(\vartheta)$, $S(\vartheta)$, $\mathcal{J}_{\vartheta,\mu}$ and $\mathcal{J}_{\mu,p}$ are obtained using a FEM, and $\mathcal{H}_{p+1,q}$ and Σ are formed using output data generated from the FEM. Combining previous expressions one gets

$$\mathcal{J} = \left(\mathcal{O}_{p+1}(\vartheta)^{\dagger} \mathcal{H}_{p+1,q} \otimes S(\vartheta) \right)^T \mathcal{J}_{\mathcal{O},\vartheta} \mathcal{J}_{\vartheta,\mu} \mathcal{J}_{\mu,p} \tag{8}$$

We close by noting that since the pseudo-inverse of the observability matrix is needed in the sensitivity computation, sensor configurations leading to poorly conditioned observability matrices can be dismissed a priori.

2.1 Weighting

As noted previously, the sensor placement outlined in this section uses a cost function maximized over all damages for which the parameter vector change is of a given norm. The relative importance of a particular parameter, therefore, depends on its relative value. Normalization to attain a desired weighting is, however, always possible. It is important to note that since the optimization is done over all changes of equal norm, results are heavily weighted by multiple damage scenarios, a situation that is not in agreement with the expectation that the likely damage is local. One anticipates the importance of the noted discrepancy to increase as the size of the free parameter space increases. Some quantitative observations on this matter are given later in the paper.

3 STRATEGY II - COVARIANCE BASE OPTIMAL SENSOR PLACEMENTS

In this section we outline an approach, introduced by Parker in [4], in which sensors positions are obtained by maximizing a scalar function of the norm of the sensitivity of the output covariance to system changes. We considered two variants of the approach, one that severely penalized "blind spots" and another that does not.

3.1 Derivation

From the state recurrence in discrete time it follows that the covariance of the state satisfies the equation

$$Q_x = A_d Q_x A_d^T + G_d Q_\omega G_d^T \tag{9}$$

Replacing $A_d \cong I + A_c \Delta t$ and $G_d \cong G_c \Delta t$ in eq.9 it follows that

$$A_c Q_x + Q_x A_c^T + A_c Q_x A_c^T \Delta t + G_c Q_\omega G_c^T \Delta t = 0 \tag{10}$$

The covariance of the state in discrete time is related to the covariance of the state in continuous time by

$$Q_x = Q_x^{ct} \Delta t \tag{11}$$

Substituting eq.11 into eq.10 and looking at the limit when $\Delta t \to 0$ gives

$$A_c Q_x^{ct} + Q_x^{ct} A_c^T + G_c Q_\omega G_c^T = 0 \tag{12}$$

where it's worth emphasizing that while the covariance of the state is in continuous time the covariance of the process noise Q_ω is in discrete time. Consider now the output equation

$$y(t) = Cx(t) + D\omega(t) \tag{13}$$

The covariance of the output in continuous time is thus

$$Q_y^{ct} = C Q_x^{ct} C^T + \frac{1}{\Delta t} D Q_\omega D^T \tag{14}$$

Let the system parameter be designated by p. We assume that only stiffness terms are affected by damage so only the matrix A_c is a function of p. Differentiating eq.14 gives

$$\frac{\partial Q_y^{ct}}{\partial p} = C \frac{\partial Q_x^{ct}}{\partial p} C^T \tag{15}$$

and from eq.12 it follows that

$$\frac{\partial A_c}{\partial p} Q_x^{ct} + A_c \frac{\partial Q_x^{ct}}{\partial p} + \frac{\partial Q_x^{ct}}{\partial p} A_c^T + Q_x^{ct} \frac{\partial A_c^T}{\partial p} = 0 \tag{16}$$

which can be written as

$$A_c \frac{\partial Q_x^{ct}}{\partial p} + \frac{\partial Q_x^{ct}}{\partial p} A_c^T + T = 0 \tag{17}$$

where

$$T = \frac{\partial A_c}{\partial p} Q_x^{ct} + Q_x^{ct} \frac{\partial A_c^T}{\partial p} \tag{18}$$

Eq.12 is a Lyapunov equation that can be solved for the covariance of the state for any spatial distribution of the loading. This result is then used to evaluate eq.18 for the selected p and the result is used in eq.17, which is another Lyapunov equation, to solve for the derivative of the state covariance with respect to p. Substituting the derivative of the covariance of the state into eq.15 gives the derivative of the output covariance with respect to the selected structural parameter. Let the derivative of the output covariance be characterized by its largest singular value (i.e. by its 2-norm) and designate this as q, namely

$$q = \left\| \frac{\partial Q_y^{ct}}{\partial p} \right\|_2 \tag{19}$$

let the number of damage scenarios = n_d , the number of possible positions for the sensors = n_s and the number of sensor = m. Assume that the algorithm is applied to each possible scenario and the results (the vales of q) are placed in a matrix $\Theta \in \mathbb{R}^{z \times n_d}$ where the rows are the possible combinations of sensor positions and the columns are the damage scenarios. From the well-known expression for combinations one has

$$z = \binom{n_s}{m} = \frac{n_s!}{m!(n_s - m)!} \tag{20}$$

Assume temporarily that the sizes are such that the matrix Θ can be explicitly computed. We consider two cost functions, one is a weighted sum of the rows of Θ and the other is a weighted sum of the reciprocals. To be explicit, let the j[th] row of Θ be $\theta_j = \{ q_{j,1} \quad q_{j,2} \quad \dots \quad q_{j,n_d} \}$ and define

$\vartheta_j = \left\{ \frac{1}{q_{j,1}} \quad \frac{1}{q_{j,2}} \quad \cdots \quad \frac{1}{q_{j,n_d}} \right\}$. Let a vector of weight be $w = \{ w_1 \quad w_2 \quad \ldots \quad w_{n_d} \}$ then the two cost functions are:

$$J_1 = \theta_j \cdot w^T \tag{21}$$

and

$$J_2 = \vartheta_j \cdot w^T \tag{22}$$

where J_1 does not penalize "blind spots", while J_2 makes unfeasible any arrangements for which some damage is unobservable (unless, of course, the weights eliminate their influence). The sensor placement problem is then: for J_1: select the j where J_1 is largest and for J_2: select the j for which J_2 is smallest.

4 NUMERICAL EXAMPLE

The system in fig.1 has springs with equal stiffness k = 100, masses are m = [1 2 3 1 3 1 2 1 2 1 2 1 2 1 3] and damping is classical with each mode having 2% of critical. It is assumed that 3 sensors are available, leading to $15!/((15-3)!\cdot 3!) = 445$ possible sensor layouts. Sensor layouts are numbered consecutively, with layout #1 as {1,2,3}, then {1,2,4}, …, {1,2,15}, {1,3,4}, {1,3,5}, … until {13,14,15} which has number #455.

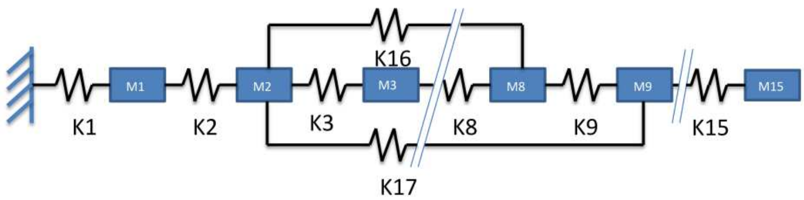

Fig.1 Mass-spring system of numerical example

To evaluate the performance of the damage detection for the different sensor layouts 200 Monte Carlo simulations of the system were made in the reference state and in each damaged states for each sensor layout, where damages were simulated by decreasing the stiffness of springs 11 to 15 (one at a time) by 5%. For each simulation, 30,000 data samples were generated from white noise excitation with 5% added output noise.

Strategy I

The average power of the test for each sensor arrangement is shown in Fig. 2 together with the value of the trace of the Fisher information computed by giving springs 11 to 15 ten times more weight than the other 12. As can be seen, there is good correlation between the two quantities, indicating that the influence of the multiple damages, which are not in the simulation, did not degrade the correlation much. As the number of springs that may be damaged increases the correlation will decrease (due to the increasing number of multiple damage patterns that are not part of the possible damages) and this is what the numerical results in fig.3 show.

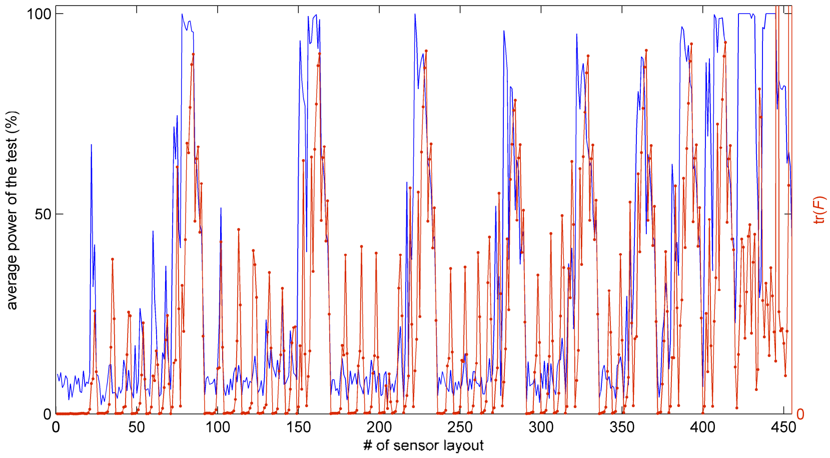

Fig.2 Average power of the test (blue) for all sensor layouts considering damage in springs k11-k15 (one at a time), using Monte Carlo simulations, and trace of the Fisher information (red).

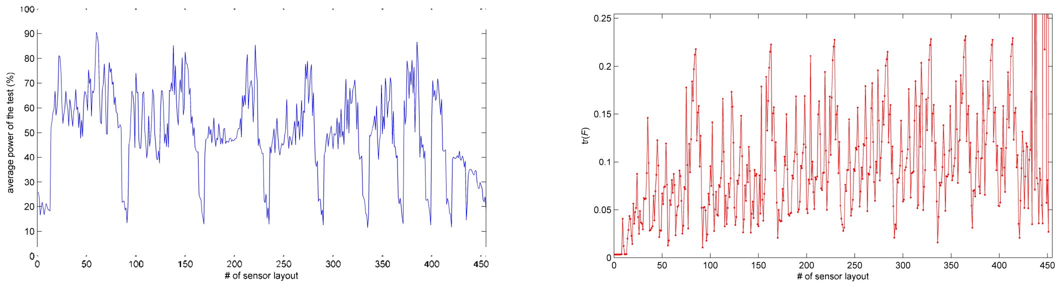

Fig.3 (left) average power of the test for all arrangements when damage takes place in any of the 17 springs (right) trace of the Fisher information when all parameters are weighted equally.

Strategy II

Fig.4 plots the indices of eq.21 and the reciprocal of eq.22 when possible damage is restricted to damage in springs 11 to 15. The sensor arrangement deemed optimal by either expression is position 441 which corresponds to sensors at {10,12,14}.

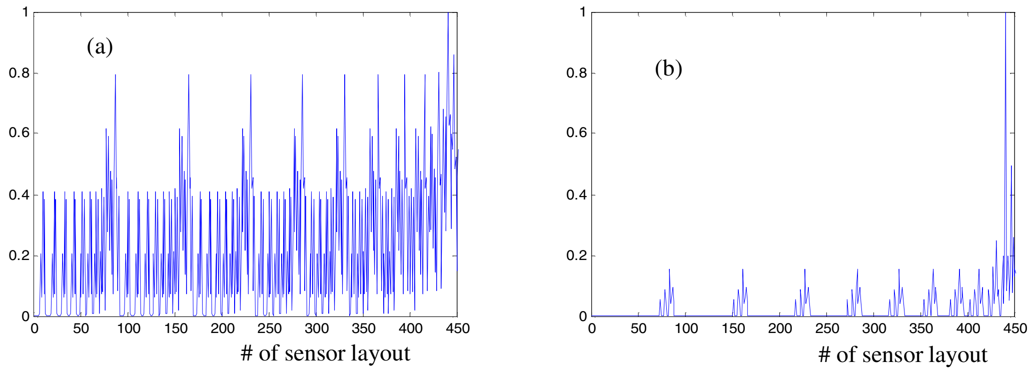

Fig.4 Results for strategy II for the conditions in fig.2, both normalized to maximum of unity (a) eq.21, (b) reciprocal of eq.22.

A comparison with the blue line in fig.2 shows that the correlation between sensor placements and performance is excellent.

5 CONCLUDING COMMENTS

Sensor placement strategy I is directly linked to the algorithm used to detect the damage so simulations are needed only to provide quantitative insight into how much the multiple damages patterns implicit in the optimization (and which are not in the simulations) affect results. While it is evident that the effect of the extra damage patterns on the optimization results increases as the number of free parameters increases, quantitative assertions of general validity are difficult. In the numerical example, which considered a set of five springs as candidates for damage, the correlation between the sensor placement obtained and the average power of the test (for single damage) was high. Strategy II requires that the damage scenarios be defined explicitly and, albeit not derived from the algorithm that was used to detect damage in the numerical simulation, the correlation between placements and performance was excellent.

6 REFERENCES

[1] M. van de Wal, B. de Jager. A review of methods for input/output selection. *Automatica* 37(4), 487-510, 2001.

[2] M. Basseville, A. Benveniste, G. Moustakides, A. Rougée. Optimal sensor location for detecting changes in dynamical behavior. *IEEE Transactions on Automatic Control* 32(12), 1067-1075, 1987.

[3] M. Döhler, K. Kwan, D. Bernal. Optimal sensor placement with a statistical criterion for subspace-based damage detection. *Proc. 31st International Modal Analysis Conference*, Garden Grove, CA, USA, 2013.

[4] D. Parker. *Multi-objective design optimization framework for structural health monitoring*. PhD thesis, Mississippi State University, 2011.

[5] M. Basseville, L. Mevel, M. Goursat. Statistical model-based damage detection and localization: subspace-based residuals and damage-to-noise sensitivity ratios. *Journal of Sound and Vibration* 275(3), 769-794, 2004.

[6] M. Döhler, L. Mevel, F. Hille. Subspace-based damage detection under changes in the ambient excitation statistics. *Mechanical Systems and Signal Processing*, 45(1):207-224, 2014.