# An Experimental Study of the Pseudo-Local Flexibility Method for Damage Detection of Hyper-Static Beams

Ting-Yu Hsu, Shen-Yuan Shiao, Wen-I Liao, Chin-Hsiung And Loh

# AN EXPERIMENTAL STUDY OF THE PSEUDO-LOCAL FLEXIBILITY METHOD FOR DAMAGE DETECTION OF HYPER-STATIC BEAMS

**Ting-Yu Hsu[1], Shen-Yuan Shiao[1], Wen-I Liao[2] and Chin-Hsiung Loh[3]**

[1] *NCREE, 200, Sec. 3, Xinhai Rd. Taipei 10668, Taiwan, R.O.C.*
[2] *NTUT, 1, Sec. 3, Zhongxiao E. Rd., Taipei 10608 Taiwan, R.O.C.*
[3] *NTU, 1, Sec. 4, Roosevelt Road, Taipei, 10617 Taiwan, R.O.C*

dysheu@ncree.narl.org.tw

**ABSTRACT**

Many vibration-based structural damage detection techniques which perform damage diagnosis of a structure based on structural dynamic characteristic parameters have been proposed in the last two decades. One of the promising approaches proposed recently is the local flexibility method. The local flexibility method, which is founded on virtual forces that cause nonzero stresses in a local part of the structure, can estimate damage locations and local stiffness variations of beam structures. It does not require a finite element model of the beam structure. The structural modal parameters identified from the ambient vibration signals both before and after damage are the key information for the local flexibility method. The number of modes necessary for the local flexibility method is usually quite small, especially for a simply supported beam where only the first mode could be sufficient. However, for a hyperstatic beam, the number of modes required for estimation of the damage could be much higher. This makes the feasibility of the local flexibility method much lower because in practice only the first few modes could be identified with high quality using ambient vibration signals. Therefore, in this study, non-local virtual forces which cause concentrated stresses in a local part and nonzero stresses in the other parts of a structure are employed. The theoretical basis of the proposed method which uses non-local virtual forces is derived. The proposed method is validated with a continuous steel beam experiment. The results illustrate that the non-local virtual forces can determine the local variations of stiffness more accurately with less identified modes. Therefore, the feasibility of the proposed method is higher because limited number of high quality modes can be identified in real world application.

**KEYWORDS :** *flexibility matrix, local flexibility method, damage detection, beam structures.*

## 1 INTRODUCTION

Many vibration-based structural damage detection techniques which perform damage diagnosis of a structure based on structural dynamic characteristic parameters have been proposed in the last two decades. Toksoy and Aktan (1994) first tried to detect damage locations based on structural flexibility matrices of a beam structure. However, the damage detection algorithms based on flexibility matrices lack a solid theoretical background until the damage location vector method which can locate damage was developed by Bernal (2002). Reynders and De Roeck (2010) further developed the local flexibility method (LFM) with a robust theoretical background to not only detect damage locations but also damage extents.

The LFM utilizes flexibility matrices of a beam structure constructed by modal parameters. Combined the flexibility matrices with designated local virtual forces which cause stress fields restricted within a local region of the beam structure, the damage extent of the local region can be estimated. The LFM does not require a finite element model of the beam structure. The structural modal parameters identified from the ambient vibration signals both before and after damage is the key information for the LFM. The number of modes necessary for the LFM is usually quite small, especially for a simply supported beam where only the first mode could be sufficient. However, for a hyperstatic beam or more complex structures, the number of modes required for estimation of the damage could be much more. This makes the feasibility of the LFM much lower because in practice only the first few modes could be identified with high quality using ambient vibration signals. Therefore, in this study, non-local virtual forces which cause concentrated stresses in a local part and nonzero stresses in the other parts of a structure are employed. The proposed method is named as the pseudo local flexibility method (PLFM) and is explained and validated in the following sections. Firstly, the theoretical basis of the proposed method which uses non-local virtual forces is derived. Next, the effects of the number of modes on damage detection results of damage scenarios of both numerical and experimental hyperstatic beams are studied. Finally, the results are discussed and the advantage of the proposed PLFM is highlighted.

## 2 METHODOLOGY – THE PSEUDO LOCAL FLEXIBILITY METHOD

The PLFM considers a structure with volume $\Omega$ and boundary $\boldsymbol{\Gamma}$ which is subjected to the Dirichlet boundary conditions along part of the boundary (see figure 1). A first load configuration $f^1$ is applied at a limited number of $r$ DOFs where response can be measured. The first load configuration for the PLFM is chosen such that the induced stress field $\boldsymbol{\sigma}^1$ consists of concentrated stresses in the local volume $\Omega_p$ and also small stress outside $\Omega_p$. Note that $f^1$ is assumed only causes non-zero stress within $\Omega_p$ for the LFM. Based on the virtual work principle with the body force neglected:

$$\int_{\Gamma} \boldsymbol{t}^T \delta x d\Gamma = \int_{\Omega} \boldsymbol{\sigma}^T \delta \boldsymbol{\varepsilon} d\Omega \tag{1}$$

where $t$ is the vector with applied tractions, $\boldsymbol{\sigma}$ is the corresponding stress vector, $\boldsymbol{\delta}_x$ is a virtual displacement field that obeys the Dirichlet boundary conditions and $\boldsymbol{\delta}_{\boldsymbol{\varepsilon}}$ is the corresponding virtual strain vector. If the virtual displacement field is chosen as the one that is induced by the first load configuration $f^1$ while the forces and the stresses are due to the second load configuration $f^2$ which obeys the boundary condition of the system, one has that:

$$\sum_{j=1}^{r} f_j^2 x_j^1 = \int_{\Omega_p} (\boldsymbol{\sigma}_p{}^2)^T \boldsymbol{\varepsilon}_p{}^1 d\Omega_p + \int_{\Omega_q} (\boldsymbol{\sigma}_q{}^2)^T \boldsymbol{\varepsilon}_q{}^1 d\Omega_q \tag{2}$$

where $x^1_j$ is the displacement at DOF $j$ corresponding to the first load configuration. Assume that the structure is linear elastic and that $\boldsymbol{\sigma}^1$ is proportional to $\boldsymbol{\varepsilon}^1$ with stiffness constant K. If the virtual work is calculated both before and after damage has occurred, one has that:

$$\frac{\sum_{j=1}^{r} f_j^2 x_j^1}{\sum_{j=1}^{r} f_j^2 x_{jd}^1} = \frac{\int_{\Omega_p} (\boldsymbol{\sigma}_p{}^2)^T \frac{\boldsymbol{\sigma}_p{}^1}{K_p} d\Omega_p + \int_{\Omega_q} (\boldsymbol{\sigma}_q{}^2)^T \frac{\boldsymbol{\sigma}_q{}^1}{K_q} d\Omega_q}{\int_{\Omega_p} (\boldsymbol{\sigma}_{pd}{}^2)^T \frac{\boldsymbol{\sigma}_{pd}{}^1}{K_p + \Delta K_p} d\Omega_p + \int_{\Omega_q} (\boldsymbol{\sigma}_{qd}{}^2)^T \frac{\boldsymbol{\sigma}_{qd}{}^1}{K_q + \Delta K_q} d\Omega_q} \tag{3}$$

Assume that the stress $\boldsymbol{\sigma}^1$ and $\boldsymbol{\sigma}^2$ are concentrated within the local volume $\Omega_p$, and the stress $\boldsymbol{\sigma}^1$ and $\boldsymbol{\sigma}^2$ outside the local volume are small, hence the strain energy outside the local volume is much smaller than the strain energy within the local volume. We can neglect the strain energy outside the local volume and have:

$$\frac{\sum_{j=1}^{r} f_j^2 x_j^1}{\sum_{j=1}^{r} f_j^2 x_{jd}^1} \cong \frac{\int_{\Omega_p} (\boldsymbol{\sigma}_p{}^2)^T \frac{\boldsymbol{\sigma}_p{}^1}{K_p} d\boldsymbol{\Omega}_p}{\int_{\Omega_p} (\boldsymbol{\sigma}_{pd}{}^2)^T \frac{\boldsymbol{\sigma}_{pd}{}^1}{K_p + \Delta K_p} d\boldsymbol{\Omega}_p} \tag{4}$$

Assume that $K$ and $\Delta K$ are constant within the local volume $\Omega_p$, which means only lump estimation of $K$ and $\Delta K$ within the local volume $\Omega_p$ can be achieved. Then they can be moved outside the integration as:

$$\frac{\sum_{j=1}^{r} f_j^2 x_j^1}{\sum_{j=1}^{r} f_j^2 x_{jd}^1} \cong \frac{\frac{1}{K_p}\int_{\Omega_p} (\boldsymbol{\sigma}_p{}^2)^T \boldsymbol{\sigma}_p{}^1 d\boldsymbol{\Omega}_p}{\frac{1}{K_p + \Delta K_p}\int_{\Omega_p} (\boldsymbol{\sigma}_{pd}{}^2)^T \boldsymbol{\sigma}_{pd}{}^1 d\boldsymbol{\Omega}_p} \tag{5}$$

Furthermore, we assume that stress will not change too much after the damage is introduced, and then the integration both in the numerator and denominator are approximately the same and can be canceled out. Equation (5) becomes:

$$\frac{\sum_{j=1}^{r} f_j^2 x_j^1}{\sum_{j=1}^{r} f_j^2 x_{jd}^1} \cong \frac{K_p + \Delta K_p}{K_p} \tag{6}$$

For a beam structure, following similar derivation procedures, the damage detection equation can be obtained to estimate the rigidity reduction ratio within the local volume, denoted as $R$, if the shear deformation can be neglected:

$$\frac{\sum_{j=1}^{r} f_j^2 x_j^1}{\sum_{j=1}^{r} f_j^2 x_{jd}^1} \cong \frac{EI_p + \Delta EI_p}{EI_p} \equiv R \tag{7}$$

where $EI_p$ is the bending rigidity within the local volume $\Omega_p$. The virtual displacement vector $x^1$ under the first load configuration $f^1$ can be obtained using the following equation:

$$\boldsymbol{x}^1 = \boldsymbol{H}\boldsymbol{f}^1 \tag{8}$$

where $H$ is the flexibility matrix. With the assumption of lumped and approximately equally distributed mass, the flexibility matrix can be estimated using the identified unscaled modal parameters as (Reynders & De Roeck, 2010)

$$\boldsymbol{H} \cong \boldsymbol{H}^n = -\boldsymbol{\Phi}\boldsymbol{\Lambda}_c{}^{-1}(\boldsymbol{\Lambda}_c{}^H\boldsymbol{\Phi}^H\boldsymbol{\Phi}\boldsymbol{\Lambda}_c + \boldsymbol{\Phi}^H\boldsymbol{\Phi})^{-1}\boldsymbol{\Lambda}_c{}^H\boldsymbol{\Phi}^H \tag{9}$$

where $\boldsymbol{\Phi}$ is the matrix of mode shapes, $\boldsymbol{\Lambda}$c is the diagonal matrix of system poles and $H$ is the Hermitian transpose. If only the first $n$ modes are available, then the flexibility matrix is truncated, denoted as Hn. Note that, contradict to the stiffness matrix, the contribution of the modes in the flexibility matrix is proportional to the inverse of square of the system poles. The influence of the higher modes on the flexibility matrix is much smaller than the one of lower modes. As a result, the number of truncated modes needed to approximate a non-truncated flexibility matrix is much smaller than the ones needed to approximate a non-truncated stiffness matrix. This benefits the practical cases where only limited number of lower modes can be identified with acceptable accuracy, especially using ambient vibration signals.

The calculation of both the LFM and PLFM includes: using the identified modal parameters to construct the truncated flexibility matrix by Equation (9); and then multiply the truncated flexibility matrix by the first load configuration to obtain the virtual displacement, as shown in Equation (8); and finally, multiply the virtual displacement by the second load configuration both before and after damage, as shown in Equation (7). Actually, the truncated flexibility matrix and the second load configuration $f^2$ for LFM and PLFM are identical. The only difference of the calculation between LFM and PLFM is the first load configuration $f^1$.

Based on the theory above, the three main sources of theoretical error when estimating R using PLFM are: (1) the truncation error of flexibility matrix due to limited number of modes in Equation (9), denoted as $ER_H$ ; (2) the assumption that stress does not change due to damage for hyper-static structures in Equation (5), denoted as $ER_\sigma$ ; and (3) the neglect of virtual strain energy within $\Omega_q$ in Equation (3), denoted as $ER_E$. Note that for the LFM only $ER_H$ and $ER_\sigma$ exist. The key point of the success of PLFM is that if the non-local first load configuration f1 can greatly reduce $ER_H$ and at the same time induce $ER_E$ which is tolerable, then the total theoretical error of PLFM could be smaller than the one of LFM. That is, the release of the constraint in LFM that $f^1$ only causes non-zero stress within $\Omega_p$ may benefit the damage detection results. Further discussion will be provided in the following sections.

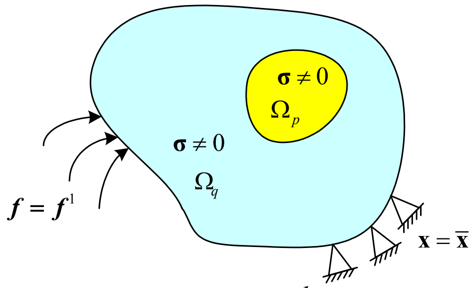


Fig. 1: A structure subjected to the first load configuration $\boldsymbol{f}^1$ that causes concentrated stress within the local region $\boldsymbol{\Omega}_{\mathbf{p}}$.

## 3 EXPERIMENTAL STUDIES

A continuous steel beam of 2.4m length is designed and constructed to verify the proposed approach as shown in the left figure of Figure 2. The beam cross section is rectangular with 5mm depth and 40mm wide. The beam is supported at points 1, 5 and 13 with hinges and rollers as shown in Figure 3. Due to the size of the specimen is quite small, the beam is excited by impact forces simulated using human fingers at several different points. The vertical acceleration response is measured at all the nodes except node 1, 5 and 13 because these nodes are supports.

Without loss of generality, the “damage” is simulated by increasing rigidity by attaching a small plate of length 50mm with the same cross section as the beam as shown in the right figure of Figure 2. Three damage cases are considered as shown in Figure 3. The first damage case simulates the damage close to the middle of the long span, and the second case simulates the damage close to one of the support. The third case simulates two damages locations at both the short span and the long span.

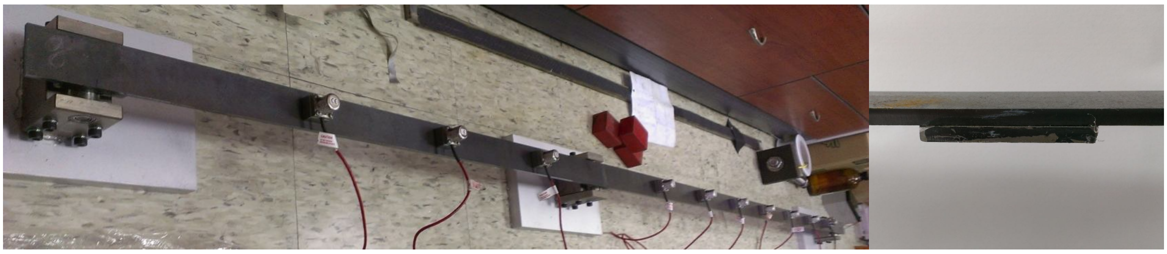

Figure 2. (Left) A continuous beam specimen with accelerometers; (Right) A typical example to increase the rigidity of the continuous beam by attaching a steel plate to simulate “damage”

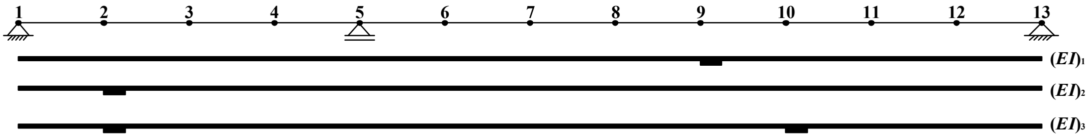


Figure 3. Measurement points and locations of stiffener of the three “damaged” cases of the continuous beam specimen.

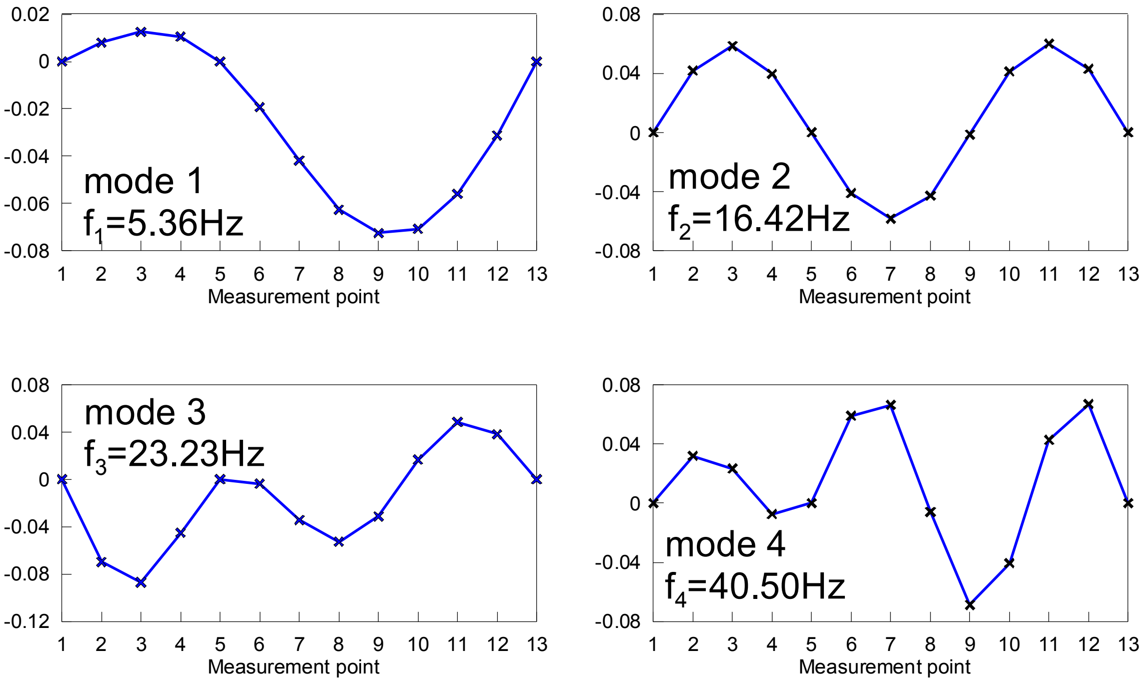


Figure 4. Typical mode shapes of the intact continuous beam specimen

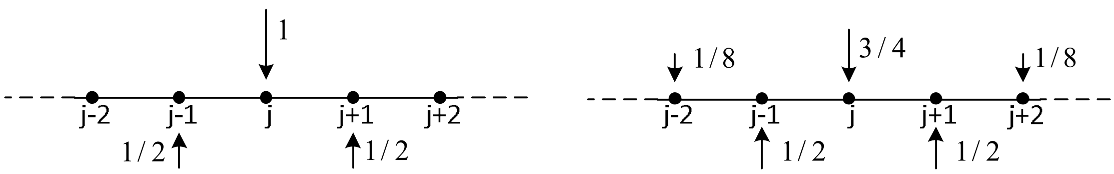


Fig. 5: A load configuration of a hyper-static beam structure that causes concentrated virtual stresses within the region between node j-1 to node j+1 (left figure) and that causes non-zero virtual stresses only within the region between node j-2 to node j+2 (right figure).

Totally 4 modes with good quality can be identified from the output only acceleration measurement using stochastic subspace identification algorithm (Van Overschee & De Moor 1996). The typical identified modal frequencies and mode shapes when the beam is intact are shown in

Figure 4. At each measurement point, the force configurations in the left part and right part of Figure 5 are applied as $\boldsymbol{f}^1$ for the PLFM and LFM, respectively. The corresponding displacements $\boldsymbol{x}^1$ at the measured DOFs are calculated using the truncated flexibility matrices constructed by different number of modes and $\boldsymbol{f}^1$. For both methods, the force configuration in Figure 5 is applied as $\boldsymbol{f}^2$. The rigidity reduction ratio is estimated using Equation 7 using these identified modal parameters. The reference value of rigidity reduction ratio is calculated using the flexibility matrix of a numerical beam model constructed by 48 beam elements.

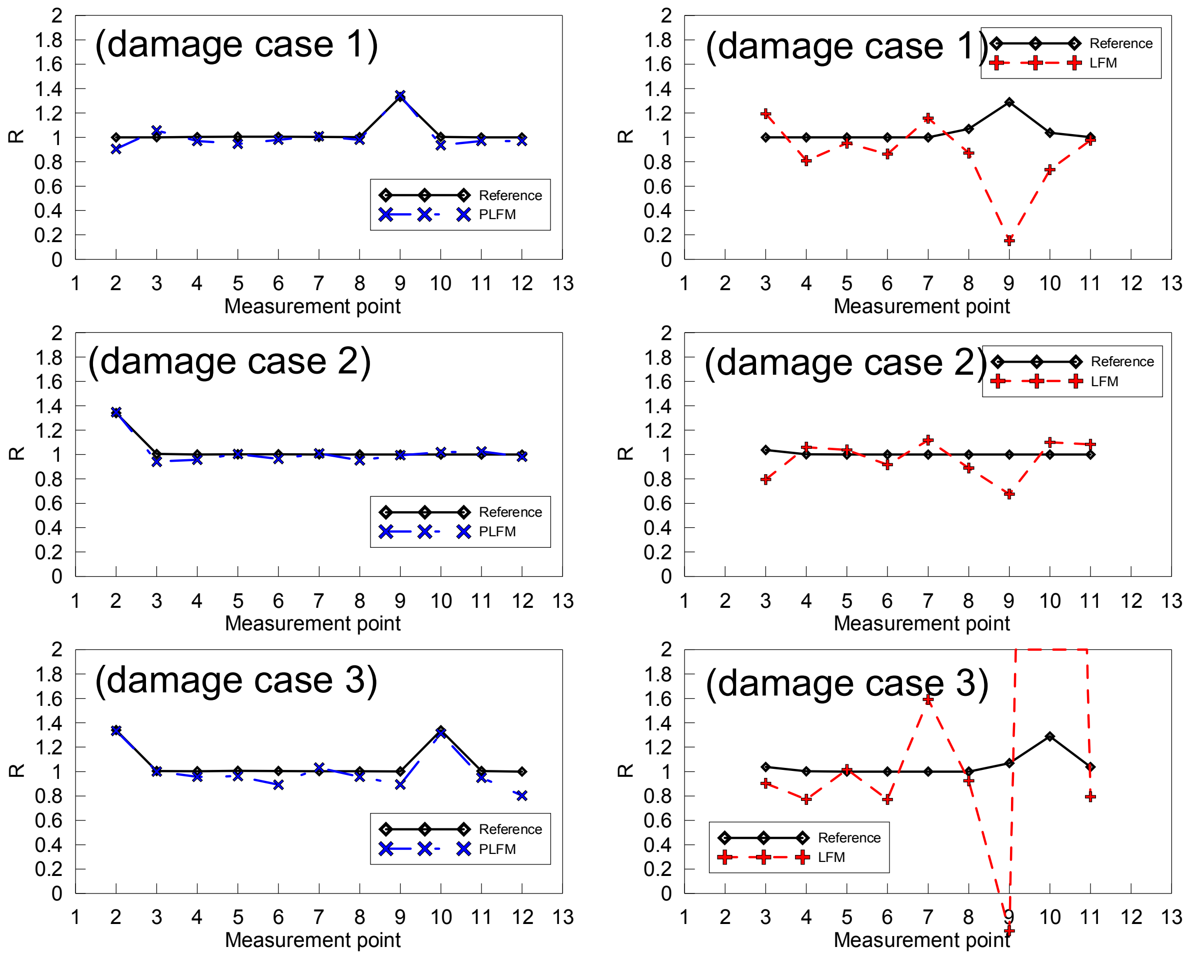


Figure 6. Estimated rigidity reduction ratio using the first 2 unscaled modes using (left) PLFM; (right) LFM.

For the proposed PLFM, the estimated rigidity reduction ratio at each measurement point using the first 2 modes is already quite close to the reference values for all the three damage cases, as shown in the left part of Figure 6. Using more modes does not improve the results in all the cases, probably because the quality of the higher modes is not as good as the lower ones. On the other hand, for the original LFM, the estimated rigidity reduction ratio at most of the measurement points using the first 2 modes is not as close to the reference values as the one using the PLFM, as shown in the right part of Figure 6. Some of the estimated rigidity reduction ratios are extremely big or small which implies unreliable results are obtained. The estimated rigidity reduction ratio using the LFM becomes much better if all the first 4 modes are employed, as shown in Figure 7. However, it is evident that the estimated rigidity reduction ratio using the PLFM is much closer to the reference values than the one using the LFM, even only the first 2 modes are employed using the PLFM. This confirms again the superiority of the PLFM over the LFM, especially when fewer modes can be identified from measured signals.

Another interesting finding is that the increase of rigidity at node 2 in damage case 2 and damage case 3 cannot be easily discovered if the LFM is used, as shown in the reference value in Figure 6. The reference value at node 2 of these cases is only 1.038, which is very close to 1 and such a small change is not easy to be detected. The sensitivity to damage of the PLFM is higher than the one of the LFM probably because of two reasons. The first reason is because the non-zero stress induced by the first virtual load configuration using the LFM is between node 1 and node 5, while the region of concentrated stress induces by the first virtual load configuration using the PLFM is between node 1 and node 3. As discussed in Equation 4, only lump reduction ratio of the stiffness/rigidity can be estimated within the local region, therefore the same extent of damage will induce higher lump reduction ratio if smaller local region is valid. That is, the PLFM is more sensitive to damage especially when the damage size or extent is smaller.

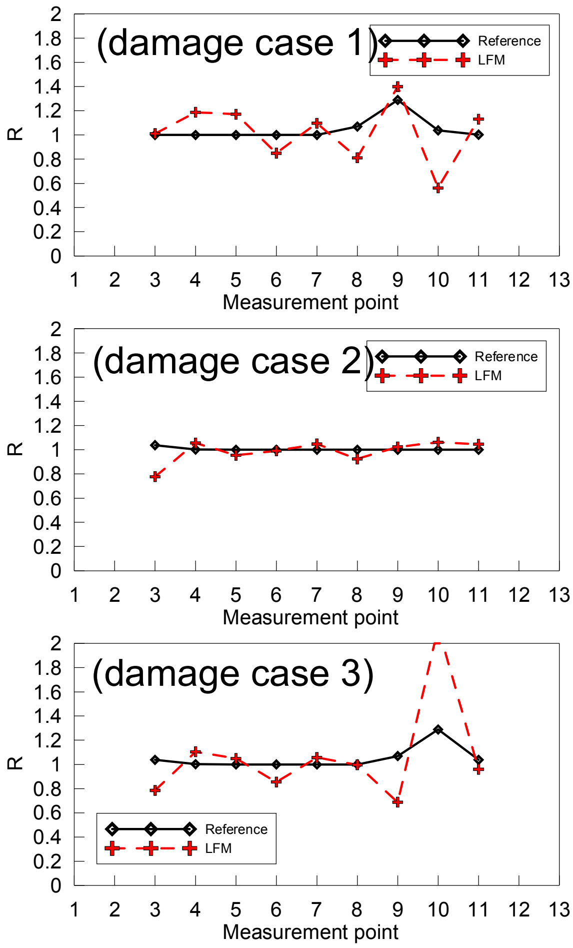


Figure 7. Estimated rigidity reduction ratio using the first 4 unscaled modes using LFM: (a) damage case 1; (b) damage case 2; (c) damage case 3

Another reason is that the location of damage within the local region also affects the value of lump reduction ratio, due to the non-uniform moment diagram induced by the applied virtual forces. The moment close to the center of the virtual forces is higher than the moment at the other parts in the local region. As a result, the closer the damage is to the center of the local region, the higher the lump reduction ratio will be. In this case, the non-zero stress induced by the first virtual load configuration using the LFM is between node 1 and node 5, while the increase of rigidity is at node 2. On the other hand, the region of concentrated stress induces by the first virtual load configuration using the PLFM is between node 1 and node 3, while the increase of rigidity is at the middle point. Therefore, the lump reduction ratio of the PLFM is much higher than the one of the LFM. This implies that when the damage is close to support of a hyperstatic beam, the damage is more possible to be discovered using the PLFM. This reveals another advantage of the PLFM because the PLFM is more sensitive to the damage close to the support than the LFM.

## 4 CONCLUSIONS AND DISCUSSIONS

In this paper, a pseudo local flexibility method (PLFM) to localize and quantify the damage of a hyperstatic beam structure is proposed. The PLFM breaks the constraint of the original local flexibility method (LFM) that only virtual forces which cause nonzero local stress within a small part of a structure can be applied. That is, non-local virtual forces which cause a global but concentrated stress filed of a structure is valid for the PLFM. The release of the constraint on the virtual forces makes the virtual forces configuration simpler. The results of experimental studies of hyperstatic beams show that fewer modes are required for the PLFM to estimate the damage location and extent with acceptable accuracy. Note that the complexity of the intact and damaged structures increase the number of modes required achieving acceptable damage detection results. Nevertheless, the feasibility of the PLFM is higher due to limited number of high quality modes can be identified in real world application, especially for ambient vibration measurement where fewer modes can be identified due to lack of information of the excitation.

It is also found that the simplification of the first virtual force configuration in PLFM makes the local region possible to be smaller. For the LFM, the local region is between 5 nodes while the local region for the PLFM is only between 3 nodes. Because only lump reduction ratio of the stiffness/rigidity can be estimated within the local region, the same extent of damage will induce higher lump reduction ratio if smaller local region is valid. That is, the PLFM is more sensitive to damage especially when the damage size or extent is smaller. Furthermore, due to the moment close to the center of the virtual forces is higher than the moment at the other parts in the local region, the closer the damage is to the center of the local region, the higher the lump reduction ratio will be. Therefore, when the damage is close to support, the damage is more possible to be discovered using the PLFM. This reveals another advantage of the PLFM because the PLFM is more sensitive to the damage close to the support than the LFM.

## ACKNOWLEDGEMENTS

We thank E. Reynders & G. De Roeck who provided valuable assistance to our research. This research was supported in part by the National Science Council of the Republic of China under the grant NSC 101-2625-M-492-007 and NSC 102-2622-E-006-032.

## REFERENCES

[1] D. Bernal. Load Vectors for Damage Localization. *Journal of Engineering Mechanics,* 128(1): 7-14, 2002.

[2] E. Reynders, G. De Roeck. A Local Flexibility Method for Vibration-based Damage Localization and Quantification. *Journal of Sound and Vibration*, 329(12): 2367-2383, 2010.

[3] T. Toksoy, AE. Aktan. Bridge-condition assessment by modal flexibility. *Experimental Mechanics*, 34(3): 271–278, 1994.

[4] P.Van Overschee, B. De Moor. *Subspace Identification for Linear Systems: Theory - Implementation – Applications*. Kluwer Academic Publishers: Dordrecht, The Netherlands, 1996.