Simple Techniques to Analyze Vibration Records from Buildings

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ABSTRACT

In parallel to rapid developments in recording, sensor, and communication technologies, the number of structures installed with vibration monitoring systems is increasing rapidly all over the world. The scientific field dealing with the analyses of structural monitoring data is known as System Identification, and involves a large number of advanced techniques requiring expertise and background for usage. Since analysis of digital data is typically not covered in standart structural engineering curriculum, a large amount of data from monitored structures are not being analyzed properly and correctly. This paper shows that there are simple techniques to analyze vibration data from structures. For building-type structures, it is possible to identify basic dynamic characteristics of a structure by using only Fourier transforms and band-pass filters. We show that, by using these two tools, we can identify modal frequencies and damping ratios, mode shapes, interstory drifts, torsional vibrations, rocking vibrations, and soil-structure interaction.

KEYWORDS : *System identification, vibration records, structural monitoring.*

INTRODUCTION

In parallel to recent developments in sensor, recording, and communication technologies, the number of structures installed with vibration monitoring networks are increasing rapidly all over the world. However, the development of simple tools and techniques for structural engineers to analyze and interpret the data from these networks are not keeping up pace with the data accumulation. One of the reasons for this is that signal processing and signal analysis are not a part of typical Civil/Structural Engineering curriculum. Data from most of structural monitoring networks are recorded and archived but not adequately analyzed and interpreted. This will eventually create a problem, as the owners and financiers of these networks will start asking what they are getting out of their support.

This paper presents simple techniques to analyze vibration data from structural monitoring systems in multi-story building-type structures. We show that one can identify all the basic dynamic characteristics of a structure by using only Fourier transforms and band-pass filters. They include modal frequencies and damping ratios, mode shapes, interstory drifts, torsional vibrations, rocking vibrations, and the soil-structure interaction. More advanced techniques can be found in Safak et. al. (2010).

1 FOURIER TRANSFORMS AND BAND-PASS FILTERS

The vibration data collected from structural monitoring systems are in discrete-time domain. We can express the same information in frequency domain by taking the Fourier transform of the recorded signals.

If *x(t)* denotes a *N-point* discrete-time vibration record with sampling interval Δ, we express its discrete Fourier transform, *X(f)*, by the following equation:

$$X(f_k) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x(t_j)\, e^{-i2\pi kj/N}$$

$$t_j = j\Delta\,; \quad f_k = \frac{k-1}{N\Delta}\,; \quad j = 0,\cdots,(N-1); \quad k = 1,\cdots,(N/2+1); \quad i = \sqrt{-1}$$

The frequency array f_k is unique and set by the number of points in the record, *N*, and the sampling interval, *Δ* as defined in the equation above. The maximum frequency information that can be extracted from the signal is $f_{max}=1/(2\Delta)$, which is known as the *Nyquist* frequency.

As seen from the equation, *X(f)* is a complex-valued quantity. Its amplitude and phase, $A(f_k)$ and $\phi(f_k)$, are defined by the following equations

$$A(f_k) = \sqrt{X(f_k)X^*(f_k)} \quad \text{and} \quad \phi(f_k) = \tan^{-1}\left(\frac{\mathrm{Im}[X(f_k)]}{\mathrm{Re}[X(f_k)]}\right)$$

where $X^*(f_k)$ is the complex-conjugate of $X(f_k)$, and Im[…] and Re[…] denote the complex and real parts. Taking a Fourier transform is equivalent to expressing the signal as a sum of sinusoids. $A(f_k)$ denotes the amplitude of the sinusoid at frequency f_k in the sum, and $\phi(f_k)$ its phase *(i.e.,* the time shift of the sinusoid with respect to *t=0* axis).

There is a one-to-one correspondence between the time-domain and frequency-domain representations. Note that an *N-point* time-domain signal can be represented identically in the frequency domain only by *N/2+1* points. The reason for the reduction in the number of points is that because each point in the frequency domain has actually two components, the real component and an imaginary component. Although the Fourier transform equation above gives *N* frequency points, the values between *N/2+2* to *N* are the complex conjugate of the points from *N/2* to 2.

Band-pass filters are used to eliminate unwanted frequency components (i.e., sinusoids) outside a specified frequency band. For example, if we want to determine the modal response at a particular mode of the structure from the data, we filter the records using a narrow-band-pass filter around the corresponding modal frequency. The filtered signals represent the response at that mode. Band-pass filters can be expressed by the following equation:

$$y(t_k) = \sum_{r=1}^{m} a_r y(t_{k-r}) + \sum_{s=0}^{n} b_s x(t_{k-s})$$

a and *b* are the filter coefficients, and *m* and *n* are the filter orders. These parameters are different for different frequency bands and filter types. Most data processing software, such as MATLAB (MathWorks, 2014), include routines to calculate these parameters for a specified band-pass filter. An important point when using band-pass filters is that we do not want to change the phase content of the original signal when band-pass filtering. To ensure this, we either have to use zero-phase filters (e.g., Ormsby filters) or filter the signal twice, first forward and next backward (e.g., Butterworth filters).

2 MODAL FREQUENCIES AND DAMPING RATIOS

The simplest way to identify modal frequencies is to inspect Fourier Amplitude Spectra (FAS) of the records. FAS reach peak values at frequencies corresponding to modal frequencies. For cases where we have the records of the response (i.e., the output) and the excitation (i.e., the input), we calculate the input-to-output transfer function, which is the output/input ratio of the FAS. Earthquake excitations are in this category, because we generally record ground level accelerations along with structural response. For cases where we do not have the excitation record (e.g., wind-induced vibrations), we assume that excitation has a wide frequency band, and therefore the peaks in the FAS of the response corresponds to modal frequencies.

Vibration records almost always contain some noise due to imperfections in the recording instruments and other effects in the recording environment, such as temperature, wind, microtremors, etc. Noise creates spurious peaks in FAS that can be misinterpreted as modes, especially in low-amplitude high-noise ambient vibration signals. To minimize the effects of noise, FAS should be smoothed by using running *Frequency Smoothing* windows. Guidelines for the selection of optimal smoothing can be found in Safak (1997).

For low-amplitude vibration records, such as ambient vibrations of structures or ground surface, it is preferable to use the auto-correlations of the records instead of the original records in the Fourier analysis. Taking the auto-correlations improve the signal-to-noise ratios and do not alter the frequency content of the signals, and thus permits more accurate detection of resonant frequencies in noisy signals.

There are several simple techniques to determine modal damping ratios. The two most widely used ones are the half-power bandwidth method and the logarithmic decrement method. In the half-power bandwidth method, we first plot the FAS or the transfer function of the modal response (which is determined by narrow-band-pass filtering the records around the corresponding modal frequency). We then determine the two frequencies, f_a and f_b, that the line 0.707 x (Maximum amplitude of FAS) intersects with the FAS. The corresponding damping ratio, ξ, is determined from the equation:

$$\xi = \frac{f_b - f_a}{f_b + f_a}$$

The logarithmic decrement method is based on the decay of the amplitudes of the auto-correlation function, $R(\tau)$, with increasing time lag, and is particularly convenient for ambient vibration data. The auto-correlation functions should be calculated separately for each mode, i.e., after narrow-band-pass filtering the records around the modal frequency. The damping is determined by fitting a straight line to the peaks of the logarithm of the auto-correlation function, and calculated from the following equation:

$$R(\tau) = C \cdot \exp(-\xi\omega_0\tau) \quad \Rightarrow \log_e\left[R(\tau)\right] = -\xi\omega_0\tau + \log_e(C) \tag{1}$$

where $R(\tau)$ is the auto-correlation at time lag τ, ω_0 is the modal frequency, and C is a constant.

3 IDENTIFICATION OF MODE SHAPES

Provided that all records are time-synchronized, mode shapes can be identified by inspecting the modal displacement time histories Modal displacements are determined first by narrow-band-pass filtering the accelerations around each modal frequency and then double integrating the filtered accelerations. By plotting modal displacement time histories together (e.g. from top to bottom for a multi-story building) we can identify the direction and the amplitude of each measuring point at that mode. The process is schematically shown in Figure 1 below.

In multi-story buildings, if the peaks and valleys of modal displacements all align (i.e., occur at the same time in all floors), this indicates that the damping in the building is mass and/or stiffness proportional and the building has real modes. If they do not align, this is a sign of non-proportional damping and complex modes (i.e., modal response of different floors have phase differences). This is usually the case in very flexible structures, such as skyscrapers or suspension bridges.

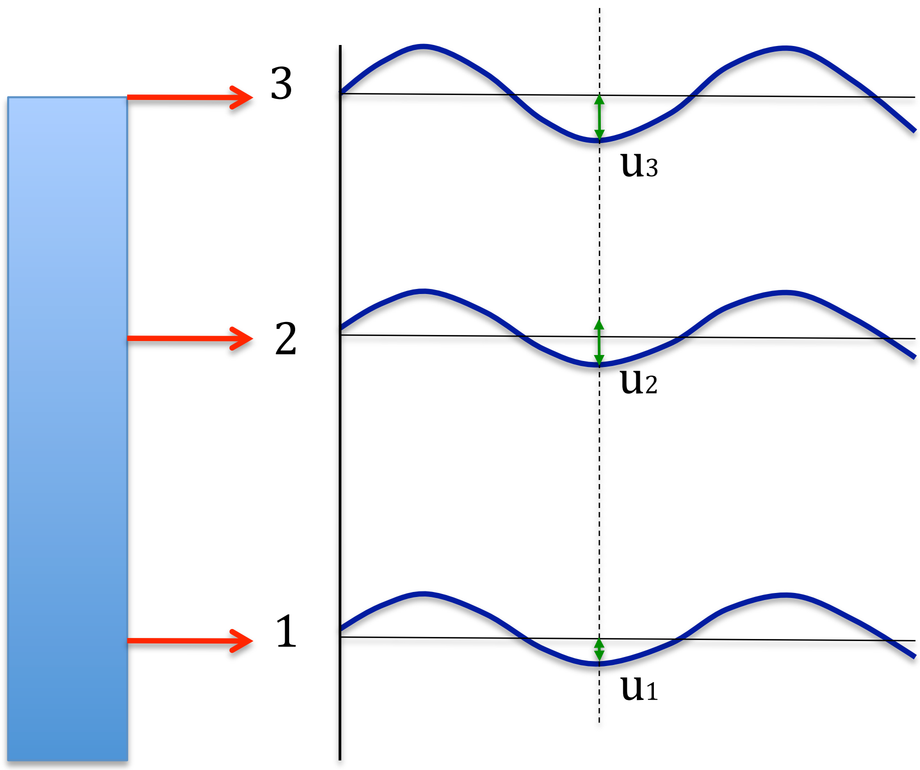

Figure 1 : Identification of mode shapes from modal displacements.

4 CALCULATION OF INTER-STORY DRIFTS

In multi-story buildings, relative displacements of two adjacent floors divided by the story height are called the *Interstory Drift* (ID). ID is one of the key design parameters and damage indicators. Calculation of IDs from recorded motions involves first the calculation of displacements from recorded accelerations by double integration, and then taking the differences of the displacements of the adjacent floors. Integration and difference operations are very sensitive to the noise level in the records, and for low signal-to-noise ratios (SNR) can lead to erroneous drift values (Safak, 2004; Safak, 2007).

To minimize the noise effects, we need to calculate IDs separately for each mode and then combine to get the total ID. We first narrow-band-pass filter the acceleration time histories around each modal frequency and then double integrate the filtered accelerations to get modal displacements. Since SNR around modal frequencies are much higher than at other frequencies, the effects of noise at filtered signals will be much less. We take the difference of the modal displacements to get IDs for each mode, and then combine modal IDs to get the total IDs. It is important that all the records are time synchronized, and the phase of the signals are not altered during band-pass filtering.

5 CALCULATION OF ROTATIONAL VIBRATIONS

Rotational vibrations in buildings are typically identified from the difference of two parallel horizontal accelerometers. Again, as in drift calculations, in order to minimize the noise effects when taking the difference, we first band-pass filter the records around the torsional frequencies. Torsional frequencies are determined by investigating the modal displacement time histories of the two parallel sensors. The modes at which the displacements go in opposite direction indicate a torsional mode. We calculate the torsional time histories by taking the difference of the signals after they are band-pass filtered around torsional frequencies.

In order to separate the torsional contribution to lateral vibrations, we need to determine the center of rotation at each floor. This can be accomplished by investigating the cross-correlation of the calculated torsional time histories with the recorded translational time histories. Two methods, one in frequency domain and the other in time domain are given in Safak and Celebi (1990a) and Safak and Celebi (1990b), respectively.

6 IDENTIFICATION OF ROCKING VIBRATIONS

Rocking vibrations are the rigid body vibrations of a structure with respect to its foundation. Such vibrations are fairly common in tall buildings (Safak and Celebi, 1991). We need to measure vertical motions at three or more corners of the foundation to identify rocking motions. The common approach to determine rocking motions is to take the difference of vertical records. Since vertical records typically have much smaller amplitudes than horizontal ones, the SNR is very low, and therefore the noise induced errors in the difference signal can be very large. We again follow the band-pass filtering approach, i.e., take the difference after band-pass filtering the records around the rocking frequency. The rocking frequency is identified by looking at the FAS of vertical records at the foundation, as well as the FAS of the horizontal records at upper stories. The frequency common in all FAS indicate the rocking frequency.

7 DETECTION AND IDENTIFICATION OF SOIL-STRUCTURE INTERACTION

Soil-Structure Interaction (SSI) refers to the influence of soil flexibility surrounding the building's foundation during its vibrations under earthquake loads. The primary effects of SSI are to reduce the natural frequency of the structure and to filter out the high frequency components in the base excitation. It can be shown that the presence of SSI can be detected by comparing the dominant frequencies of the base-to-roof transfer function and the FAS of roof records (Safak 1995; 2009).

The dominant frequency of the base-to-roof transfer function (i.e., roof FAS divided by base FAS) shows the frequency of the structure for a fixed-base (i.e., no SSI) case, whereas the dominant frequency of the roof FAS shows the frequency of the structure with flexible base (i.e., with SSI). If these two frequencies are identical or very close, this indicates no SSI effects. If they are different, there is SSI. The frequency with SSI is always smaller than the fixed-base frequency. Once these frequencies are identified, the other parameters of SSI can be approximated from the records. More detail can be found in Safak (1995).

CONCLUSION

There are simple techniques to analyze vibration data from structures. For building-type structures, it is possible to identify basic dynamic characteristics of a structure by using only Fourier transforms and band-pass filters. We show that by using these two tools, we can identify modal frequencies and damping ratios, mode shapes, interstory drifts, torsional vibrations, rocking vibrations, and soil-structure interaction.

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