Dispersion-Corrected Stabilization Diagrams for Model Order Assessment in Structural Identification

Vasilis K. Dertimanis, Eleni Chatzi

DISPERSION–CORRECTED STABILIZATION DIAGRAMS FOR MODEL ORDER ASSESSMENT IN STRUCTURAL IDENTIFICATION

Vasilis K. Dertimanis, Eleni Chatzi

Institute of Structural Engineering, D-BAUG, ETH Zürich, Stefano–Franscini–Platz 5, 8093 Zürich, Switzerland

v.derti@ibk.baug.ethz.ch

ABSTRACT

This study aims at overcoming some of the inconsistencies of stabilization diagrams, by applying a novel dispersion analysis framework that relies on the effective modal decomposition of the vibration output's zero-lag covariance matrix, under the assumption of broadband random excitation. A new metric is introduced, which expresses a certain part of the total stochastic vibration energy, and is attributed to each vibration mode. It is then shown how this metric can be integrated into a stabilization diagram, in order to enrich the displayed information and facilitate the model order selection process. In this respect, a vibration mode is identified as structural when it appears stabilized in both its frequency and its dispersion metric. The method's performance is assessed through structural identification problems from both simulated and experimental vibration data, for which subspace and prediction-error based methods are utilized.

KEYWORDS : *Structural identification, model order selection, stabilization diagrams, dispersion analysis.*

INTRODUCTION

Parametric, time–domain methods have been widely used in structural identification problems and continue to gain increased attention in both an experimental and operational setting [1–3]. In contrast to their non–parametric counterparts, where structural estimation is mainly based on information extracted from graphical representations, such as the frequency response function, time–domain methods lead to the establishment of a closed–form mathematical description of the structure in either state–space, or transfer function representation.

Irrespectively of the adopted model representation and the applied method for the estimation of its parameters, a critical issue that is intimately related to structural identification is that of (correctly) estimating the number of vibration modes. To this end, stabilization diagrams (SDs) have proven effective in demanding structural problems [4], on the basis of the principle that physical eigenmodes tend to appear at a certain frequency, irrespectively of the order of the time–domain model. Yet, the current setting of SDs is amenable to quite a few inconsistencies, including sensitivity to noise, frequency splitting, and stabilization of spurious modes, that in many cases alter the final decision. Certain clearing tools [5, 6] may facilitate the decision process, especially when very large structures are considered, however the inherent discrepancies (e.g., the stabilization of spurious modes) can still be present.

Current tools that distinguish structural from extraneous modes are, in general, method specific. Among others, the use of modal amplitude coherence that is employed within the context of ERA [7] has been reported to be problematic [8], while conventional dispersion analysis [9] is limited to transfer function representations and it is based on a "sensitive", residue–based calculation. In an effort to enhance the effectiveness of the model order selection process in a more general framework, Goethals and De Moor [10] have proposed a method based on a measure of the error when an associated mode is removed from the model. This method has been recently extended by Reynders and De Roeck [11] to the modal transfer norm metric and it can be considered as global, in the sense that it can be applied to both state–space and transfer function models. A similar global perspective is employed in Dertimanis [12], who extends the conventional dispersion analysis scheme to cover state–space models as well [13].

This study aims at overcoming some of the aforementioned inconsistencies of SDs by incorporating a certain degree of sophistication to them. To achieve this, a novel dispersion analysis framework is applied, which relies on the effective modal decomposition of the vibration output's zero-lag covariance matrix. This decomposition attributes an additional quantitative measure to every vibration mode, apart from the natural frequency, the damping ratio and the mode shape, which expresses a certain part of the total stochastic vibration energy and can be used in both forward and inverse setting. A corresponding *modal dispersion metric* is then integrated to the SDs in either pure, or normalized form and provides further information not only on the presence of a structural mode, but also about its estimated significance. In this respect, a vibration mode is considered structural when it appears stabilized in both frequency and dispersion.

The proposed method is characterized by global applicability, in that it can be applied to both state-space and transfer function models, serving thus also as a common measure of effectiveness among diverse estimation methods. Its performance is investigated through induced structural identification problems from simulated and experimental vibration data. In the former, a five degree–of–freedom (DOF) lumped–mass system serves as a test case and the PO–MOESP method [14] is employed for the estimation of state–space models using noise–corrupted acceleration and displacement data. In the latter, a suspended steel subframe flexible structure is considered [15] and the two–stages least–squares (2SLS) method is applied to the estimation of VARMAX models using applied forces and measured accelerations.

1. THE DISPERSION ANALYSIS FRAMEWORK

1.1 The structure in discrete–time state–space format

A structural system with n DOFs can be represented by a second–order vector differential equation as

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{D}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{P}\mathbf{f}(t) \tag{1}$$

in which $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are the real $[n\times n]$ mass, viscous damping and stiffness matrices, $\mathbf{q}$(t) is the $[n\times 1]$ vibration displacement vector, $\mathbf{f}$(t) is the $[p\times 1]$ vector of excitations and $\mathbf{P}$ is a $[n\times p]$ "coordinates" matrix. By defining a $[2n\times 1]$ state vector as $\mathbf{x}(t)=[\mathbf{q}^T(t)\;\;\dot{\mathbf{q}}^T(t)]^T$, a discrete–time representation of Equation 1 in state–space is given by

$$\mathbf{x}[t+1]=\mathbf{A}_d\mathbf{x}[t]+\mathbf{B}_d\mathbf{f}[t] \tag{2a}$$

$$\mathbf{y}[t]=\mathbf{C}_d\mathbf{x}[t]+\mathbf{D}_d\mathbf{f}[t] \tag{2b}$$

where $\mathbf{A}_d=e^{\mathbf{A}_cT_s}$ (T_s (s) denotes the sampling period), $\mathbf{B}_d=\begin{bmatrix}\mathbf{A}_d-\mathbf{I}\end{bmatrix}\mathbf{A}_c^{-1}\mathbf{B}_c$, $\mathbf{C}_d=\mathbf{C}_c$, $\mathbf{D}_d=\mathbf{D}_c$ and

$$\mathbf{A}_c=\begin{bmatrix}\mathbf{O}_n & \mathbf{I}_n\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D}\end{bmatrix}\;[2n\times 2n],\quad \mathbf{B}_c=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{P}\end{bmatrix}\;[2n\times p] \tag{3}$$

$$\mathbf{y}(t)=\mathbf{q}(t)\quad:\;\mathbf{C}_c=\begin{bmatrix}\mathbf{I}_n & \mathbf{O}_n\end{bmatrix}\;[n\times 2n]\qquad \mathbf{D}_c=\mathbf{0} \tag{4a}$$

$$\mathbf{y}(t)=\dot{\mathbf{q}}(t)\quad:\;\mathbf{C}_c=\begin{bmatrix}\mathbf{O}_n & \mathbf{I}_n\end{bmatrix}\;[n\times 2n]\qquad \mathbf{D}_c=\mathbf{0} \tag{4b}$$

$$\mathbf{y}(t)=\ddot{\mathbf{q}}(t)\quad:\;\mathbf{C}_c=\begin{bmatrix}-\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D}\end{bmatrix}\;[n\times 2n]\qquad \mathbf{D}_c=\mathbf{M}^{-1}\mathbf{P} \tag{4c}$$

The above expressions for the discrete–time state and input matrices correspond to the assumption of constant inter–sample behaviour of the input signal (e.g., zero–order hold principle).

1.2 Derivation of Modal Dispersions

Under the assumption that the structural excitation can be modeled as a zero–mean Gaussian white noise process with covariance matrix

$$\mathbf{\Gamma}_{ff}[h] = \mathbf{\Sigma}_{ff}\delta[h] \tag{5}$$

where $\delta[h]$ denotes Kronecker's Delta function and h the time lag, it can be proved [12] that a modal decomposition of the state vector's covariance matrix is

$$\mathbf{\Gamma}_{xx}[h] \equiv E\left\{\mathbf{x}[t+h]\mathbf{x}^T[t]\right\} = \sum_{k=1}^{2n} \mathbf{P}_k \lambda_k^h \tag{6}$$

where λ_k is the kth eigenvalue of $\mathbf{A}_d$ (distinct eigenvalues have been assumed) and is $\mathbf{P}_k$ given by

$$\mathbf{P}_k = \mathbf{G}_k \mathbf{\Sigma} \sum_{m=1}^{2n} \frac{\mathbf{G}_m^T}{1-\lambda_k\lambda_m} \tag{7}$$

In Equation 7, $\mathbf{\Sigma} = \mathbf{B}_d \mathbf{\Sigma}_{ff} \mathbf{B}_d^T$ and $\mathbf{G}_k$'s are the projectors of the spectral decomposition of $\mathbf{A}_d$ [16],

$$f(\mathbf{A}_d) = \sum_{k=1}^{2n} \mathbf{G}_k f(\lambda_k) \tag{8}$$

for which $\mathbf{G}_k^2 = \mathbf{G}_k$, $\mathbf{G}_i\mathbf{G}_j = \mathbf{O}$, for $i \neq j$ and $\sum_k \mathbf{G}_k = \mathbf{I}$. In Equation 8, f denotes a function that is defined for every λ_k. Using Equation 6 it is possible to derive a corresponding expression for the output vector $\mathbf{y}[t]$, by just exploring the output equation. It can be easily deduced that $\mathbf{\Gamma}_{yy}[h]$ can be expressed as

$$\mathbf{\Gamma}_{yy}[h] \equiv E\left\{\mathbf{y}[t+h]\mathbf{y}^T[t]\right\} = \sum_{k=1}^{2n} \mathbf{Q}_k \lambda_k^h \tag{9}$$

where the matrices $\mathbf{Q}_k$ are calculated in respect to the type of the vibration output: in the displacement/velocity case $\mathbf{Q}_k = \mathbf{C}_d \mathbf{P}_k \mathbf{C}_d^T$, whereas in the acceleration case the expressions are more complicated. See Dertimanis [12] for further details.

Having established a modal decomposition for the covariance matrix of the output vector under broadband stochastic excitation, it follows that at zero lag

$$\mathbf{\Gamma}_{yy}[0] = \sum_{k=1}^{2n} \mathbf{Q}_k = \sum_{k=1}^{n} \mathbf{Q}_k + \mathbf{Q}_k^* \tag{10}$$

with the asterisk denoting complex conjugate. Taking under consideration that $\mathbf{\Gamma}_{yy}[0]$ corresponds to the multivariate equivalent of variance and it can, thus, be associated to the stochastic vibration energy of the output series, a *modal dispersion matrix* can be defined as

$$\mathbf{E}_k = \mathbf{Q}_k + \mathbf{Q}_k^* \tag{11}$$

in order to assess the contribution of the kth mode to the total vibration energy. Correspondingly, the *normalized modal dispersion matrix* is defined as a matrix $\mathbf{\Delta}_k$ with elements

$$[\mathbf{\Delta}_k]_{ij} = \frac{[\mathbf{E}_k]_{ij}}{\left[\sum_{m=1}^{n} \mathbf{E}_m\right]_{ij}} 100\% = \frac{[\mathbf{E}_k]_{ij}}{\left[\mathbf{\Gamma}_{yy}[0]\right]_{ij}} 100\% \tag{12}$$

2. APPLICATION TO SYSTEM IDENTIFICATION

Essentially, the dispersion analysis framework previously outlined attributes an additional quantitative index to each vibration mode, when a structure is excited by a broadband random force. To see that this is indeed the case, a *modal dispersion metric* (MDM) can be defined from the modal dispersion matrices as

$$\delta_{E,k,2} = \|\mathbf{E}_k\|_2 \quad \text{or} \quad \delta_{\Delta,k,2} = \|\boldsymbol{\Delta}_k\|_2 \tag{13a}$$

$$\delta_{E,k,\infty} = \|\mathbf{E}_k\|_\infty \quad \text{or} \quad \delta_{\Delta,k,\infty} = \|\boldsymbol{\Delta}_k\|_\infty \tag{13b}$$

using the L_2 or L_∞ norms, where $k = 1,2,\ldots,n$. Equations 13 imply that every vibration mode can now be characterized by four quantities: the natural frequency, the damping ratio, the mode shape and the MDM. This feature can be used in both forward (e.g., for model reduction) and inverse setting.

In the latter case, the MDM can be very easily incorporated into SDs. While several alternatives can be potentially examined, two implementations are currently discussed and assessed in Section 3.. The first implementation simply refers to modify the SD in a way that renders the estimated dispersion of each mode. This can be done by either introducing a third dimension to the original two–dimensional format of the SDs, or by attributing a certain color range to the modal dispersion band and, accordingly, coloring each identified mode. In this way, mode stabilization can not only refer to the value of a specific frequency, but also to its attributed color.

Since the MDM that accompanies a specific mode may be expanded over a wide range of numerical values, a second implementation can be accomplished by introducing a *normalized modal dispersion metric* (nMDM) as

$$\bar{\delta}_{\natural,k,\#} = \frac{\delta_{\natural,k,\#}}{\max\limits_k(\delta_{\natural,k,\#})} \tag{14}$$

for $\natural = E,\Delta$ and $\# = 2,\infty$. This metric is always a number between 0 and 1, with the latter value corresponding to the mode of the highest contribution to the stochastic vibration energy of the structure. The use of this normalization to the SDs enables direct extraction of the most important vibration mode. In addition, a threshold can be further employed so as to prevent modes with negligible nMDM, compared to the one of the most important mode, to appear to a SD.

While the process described herein stems from the original state–space representation, it can be extended to cover transfer function representations as well. In specific, assuming a model of the form,

$$\mathbf{y}[t] = \mathbf{K}(q)\cdot\mathbf{f}[t] + \mathbf{L}(q)\cdot\mathbf{e}[t] \tag{15}$$

where $\mathbf{K}(q)$ and $\mathbf{L}(q)$ are multivariate transfer functions that describe the output–to–input and the output–to–noise dynamics, respectively, the dispersion analysis framework can be applied to the noise–free part. Since $\mathbf{K}(q)$ can be factorized as a product of two matrix polynomials of order p, the noise–free part of $\mathbf{y}[t]$ can be written recursively as

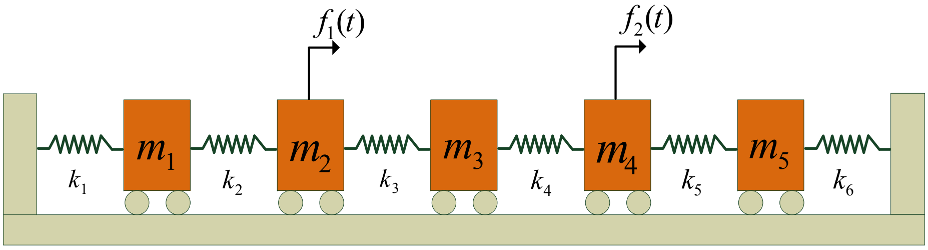
$$\mathbf{y}_s[t] + \sum_{i=1}^{p} \mathbf{V}_i \cdot \mathbf{y}_s[t-i] = \mathbf{W}_0 \cdot \mathbf{f}[t] + \sum_{j=1}^{p} \mathbf{W}_j \cdot \mathbf{f}[t-j] \tag{16}$$

where the $\mathbf{V}_i$'s and $\mathbf{W}_j$'s are matrices of appropriate sizes. A state–space realization of Equation 16 in the form of Equation 2 is then given in Dertimanis [12].

3. APPLICATIONS

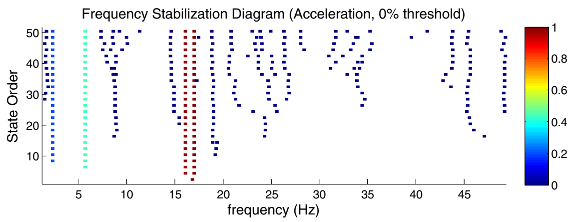
3.1 Simulated structure

The proposed method is now applied to the identification problem of the two–input, five–output structural system illustrated in Figure 1. To each vibration mode, 1% damping has been added using the

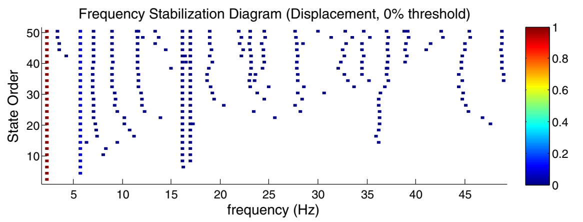


(a)

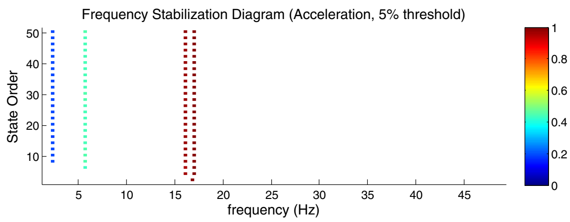
Figure 1 : Five DOF structural system with $m_i = 0.1$ Kg, $i = 1,\ldots,5$, $k_1 = k_3 = k_5 = 500$ N/m, $k_2 = 10$ N/m and $k_4 = k_6 = 100$ N/m.



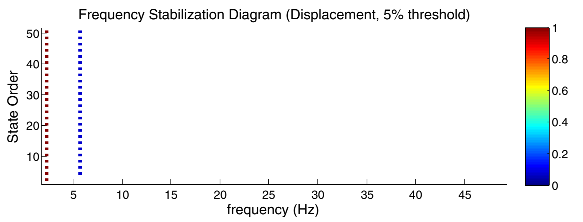
(a)



(b)



(c)



(d)

Figure 2 : Identification Results for the five DOF lumped-mass structure. The left column corresponds to acceleration and the right column to displacement structural response, respectively.

procedure described in Clough and Penzien [17, Sec. 12.5], before the structure is brought into the state–space format, for both acceleration and displacement output (two distinct cases). Accordingly, discretization is performed using the zero order hold at $T_s = 0.01$ s and the discrete–time state–space models (for acceleration and displacement outputs, respectively) are excited by a zero–mean bivariate Gaussian process of covariance matrix

$$\mathbf{\Sigma}_{ff} = \begin{bmatrix} 0.968 & -0.006 \\ -0.006 & 0.999 \end{bmatrix}$$

In each case, the resulting structural responses are zero–mean subtracted and noise–corrupted at 10% noise–to–signal ratio, forming a final two input–five output data set of $N = 5000$ data per channel. Identification is performed by the PO–MOESP [14] method (Hankel matrix with 50 block rows is chosen), for even state orders between 2 and 50.

The results are shown at Figure 2 and Table. 1. It is obvious that the modified SDs covey significantly enriched in comparison to the conventional ones. Focusing on the acceleration case, four areas exhibit stabilization in both the frequency and the nMDM. When no threshold is activated (Figure 2(a)), many frequencies appear naturally, some of which tend to stabilize. These are however attributed by negligible nMDMs and can be safely discarded as extraneous. On the contrary, when

Table 1 : Theoretical and estimated nMDMs for the acceleration and the displacement cases (simulated structure; PO–MOESP; 10% added white measurement noise). The estimated data have been extracted from the state–space model of the same order to the theoretical one.

		Acceleration				Displacement			
		Theoretical		Estimated		Theoretical		Estimated	
Mode	f_n (Hz)	$\delta_{\Delta,n,2}$	$\bar{\delta}_{\Delta,n,2}$	$\delta_{\Delta,n,2}$	$\bar{\delta}_{\Delta,n,2}$	$\delta_{\Delta,n,2}$	$\bar{\delta}_{\Delta,n,2}$	$\delta_{\Delta,n,2}$	$\bar{\delta}_{\Delta,n,2}$
1	2.299	244.20	0.200	226.35	0.191	502.26	1.000	502.02	1.000
2	5.671	559.59	0.453	533.04	0.451	42.59	0.085	37.11	0.074
3	11.366	12.25	0.010	–	–	0.14	0.000	–	–
4	16.111	1198.90	0.980	1157.60	0.979	1.20	0.002	1.17	0.002
5	16.973	1223.50	1.000	1182.40	1.000	1.19	0.002	2.02	0.005

a 5% threshold is employed (Figure 2(c)), the SD becomes very clear and the fundamental frequencies of the structure are identified along with their corresponding contribution. One structural mode remains unidentified, as it does not appear in the SDs. Yet, as Table 1 reveals, this is the third mode, which is characterized by negligible dispersion. Similar performance is observed in the displacement case, where now it is the first mode that dominates the stochastic response. The two trailing vibration modes have been identified and stabilized in both frequency and nMDM (Figure 2(b)), but when a 5% threshold is employed they are also vanished, since they are now characterized by negligible dispersion (Table 1).

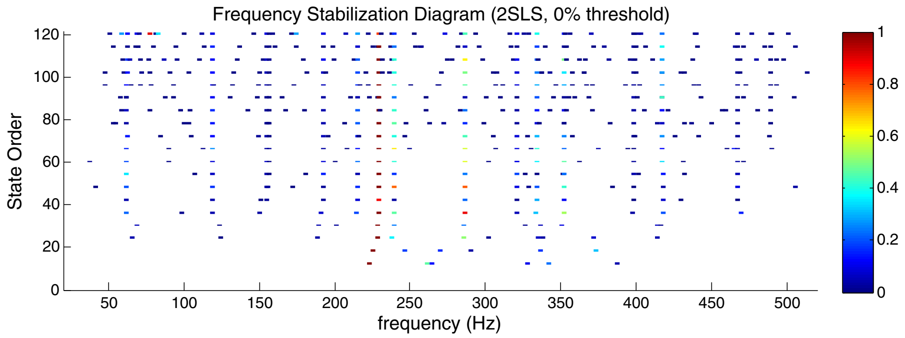
3.2 Experimental structure

The structural identification problem of a suspended steel subframe flexible structure described in [18] is considered as a second application study. The data set (8523 samples per channel, sampling frequency $F_s = 1024$ Hz) is available from the SISTA Identification Database [19] and it consists of 2 force input signals and 28 vibration acceleration responses around the structure. The input–output data set used for the identification tasks consists of the first 4000 samples of the input channels and of six output channels (node numbers 1,2,12,13,16 and 26). The 2SLS method for the estimation of $\mathrm{VARMAX}(k,k+1,k,0)$ models is employed, for $k = 1,2,\ldots,20$. Consistency to previously published results for the structure [18] forces the analysis to concentrate to modes within the $[25\ 512]$ Hz band.

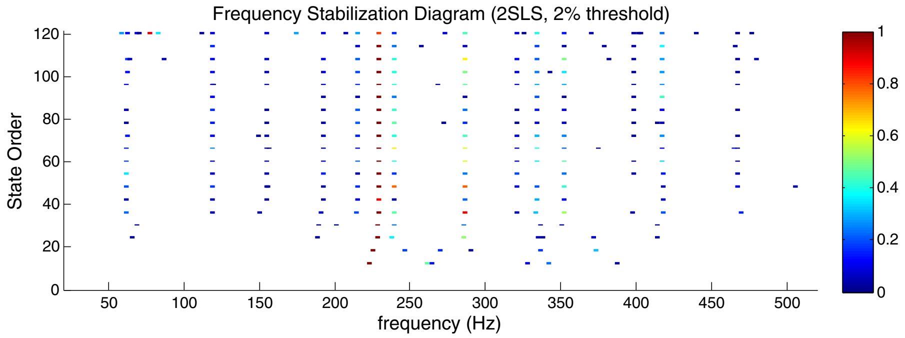
Figure 3 and Table 2 display the identification results. The upper SD (0% threshold) reveals the existence of many structural frequencies and indicates a dominated mode at around 230 Hz. Yet, at higher state orders the diagram is starting to become fuzzy. The application of a 2% threshold improves things a lot, leading to the identification of 14 structural modes, presented in Table 2, and the selection of $\mathrm{VARMAX}(10,11,10,0)$ (state order 60) as a final model. It is however evident that the dispersion stabilization of some modes is not as consistent, an issue that requires further investigation.

CONCLUSION

This paper presented a novel methodology for the performance improvement of SDs, through the incorporation of additional information to their conventional form. To this end, and starting from a state–space representation, a dispersion analysis framework was applied and adapted, which takes advantage of a specific modal decomposition of the output covariance matrix. It was shown how a newly



(a)



(b)

Figure 3 : Identification Results for the steel subframe structure. State order equals $6k$.

introduced index, the MDM, can be associated to a specific structural vibration mode, which quantifies the modal contribution to the stochastic structural response, under the assumption of broadband random excitation. A normalized version of the MDM was then integrated into the SD, aiming at facilitating the selection process by overcoming problems like the stabilization of extraneous frequencies, yet also by directly providing insights about the significance of each stabilized mode. The promising results encourage further investigation towards this path, especially in respect to operational modal analysis and output–only identification, as well as automated processes that are currently areas of very active research.

REFERENCES

[1] E. Antonacci, A. De Stefano, V. Gattulli, M. Lepidi, and E. Matta. Comparative study of vibration-based parametric identification techniques for a three-dimensional frame structure. *Structural Control and Health Monitoring*, 19(5):579–608, 2012.

[2] A. Bayraktar, A. C. Altunişik, B. Sevim, and T. Türker. Seismic response of a historical masonry minaret using a finite element model updated with operational modal testing. *Journal of Vibration and Control*, 17(1):129–149, 2011.

[3] E. Reynders. System Identification Methods for (Operational) Modal Analysis: Review and Comparison. *Archives of Computational Methods in Engineering*, 19(1):51–124, 2012.

[4] J. Lau, J. Lanslots, B. Peeters, and H. Van der Auweraer. Automatic Modal Analysis: Reality or Myth? In *Proceedings of the 25th International Modal Analysis Conference, IMAC-XXV*, Orlando, USA, 2007.

[5] D. Giraldo, J. M. Caicedo, W. Song, B. Mogan, and S. J. Dyke. Modal identification through ambient vibration: A comparative study. In *Proceedings of the 24th International Modal Analysis Conference, IMAC-XXIV*, St Louis, USA, 2009.

[6] H. Van Der Auweraer and B. Peeters. Discriminating physical poles from mathematical poles in high order systems: Use and automation of the stabilization diagram. In *Proceedings of the 21st IEEE*

Table 2 : Estimated structural modes and modal dispersions for the steel subframe structure.

Mode	f_n (Hz)	ζ_n $(\%)$	$\delta_{\Delta,n,2}$	$\delta_{\Delta,n,\infty}$	$\bar{\delta}_{\Delta,n,2}$	$\bar{\delta}_{\Delta,n,\infty}$
1	62.250	0.655	39.4391	60.5168	0.0239	0.0315
2	119.006	0.170	318.0198	347.2865	0.1925	0.1806
3	154.655	0.089	25.8525	28.9100	0.0157	0.0150
4	192.480	0.257	46.2405	53.2194	0.0280	0.0277
5	214.976	0.128	246.2037	275.4982	0.1490	0.1433
6	229.445	0.155	1651.8599	1922.6608	1.0000	1.0000
7	239.328	0.259	130.6726	229.2881	0.0791	0.1193
8	286.749	0.185	109.3640	200.7942	0.0662	0.1044
9	321.128	0.159	23.5248	34.5900	0.0142	0.0180
10	333.951	0.156	223.2171	252.8298	0.1351	0.1315
11	352.215	0.248	39.2654	75.0049	0.0238	0.0390
12	398.011	0.148	25.3417	32.4184	0.0153	0.0169
13	417.490	0.107	161.3172	201.4029	0.0977	0.1048
14	467.418	0.212	39.3773	42.8258	0.0238	0.0223

Instrumentation and Measurement Technology Conference, Como, Italy, 2004.

[7] J.-N. Juang and R. S. Pappa. An eigensystem realization algorithm for modal parameter identification and model reduction. *Journal of Guidance, Control, and Dynamics*, 8(5):620–627, 1985.

[8] A. Florakis, S. D. Fassois, and F. M. Hemez. MIMO LMS-ARMAX identification of vibrating structures-Part II: A critical assessment. *Mechanical Systems and Signal Processing*, 15(4):737–758, 2001.

[9] S. D. Fassois. MIMO LMS-ARMAX identification of vibrating structures-Part I: The method. *Mechanical Systems and Signal Processing*, 15(4):723–735, 2001.

[10] I. Goethals and B. De Moor. Model reduction and energy analysis as a tool to detect spurious modes. In *Proceedings of the 2002 International Conference on Noise and Vibration Engineering, ISMA*, Leuven, Belgium, 2002.

[11] E. Reynders and G. De Roeck. Reference-based combined deterministic-stochastic subspace identification for experimental and operational modal analysis. *Mechanical Systems and Signal Processing*, 22(3):617–637, 2008.

[12] V. K. Dertimanis. On the use of dispersion analysis for model assessment in structural identification. *Journal of Vibration and Control*, 19(15):2270–2284, 2013.

[13] V. K. Dertimanis and D. V. Koulocheris. *VAR based state-space structures: realization, statistics and spectral analysis*, volume 85 of *Lecture Notes in Electrical Engineering*, pages 301–315. Springer, 2011.

[14] M. Verhaegen. Identification of the deterministic part of MIMO state space models given in innovations form from input-output data. *Automatica*, 30(1):61–74, 1994.

[15] M. Abdelghani, M. Verhaegen, P. Van Overschee, and B. De Moor. Comparison study of subspace identification methods applied to flexible structures. *Mechanical Systems and Signal Processing*, 12(5):679–692, 1998.

[16] C. D. Meyer. *Matrix Analysis and Applied Linear Algebra*. Society for Industrial and Applied Mathematics, Philadelphia, USA, 2000.

[17] R. W. Clough and J. Penzien. *Dynamics of Structures*. Computers & Structures, Inc., Berkeley, USA, 3rd edition, 2003.

[18] M. Abdelghani, C. T. Chou, and M. Verhaegen. Using subspace methods in the identification and modal analysis of structures. In *Proceedings of the 15th International Modal Analysis Conference, IMAC-XV*, Orlando, USA, 1997.

[19] B. L. R. De Moor. DaISy: Database for the Identification of Systems. url-http://homes.esat.kuleuven.be/ smc/daisy/. Accessed 31/03/2014, [Used dataset: Experiment on a Steel Subframe Flexible structure, section Mechanical Systems, 96–013].