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ABSTRACT

Structural health monitoring by estimation of stiffness and mass parameters changes using dynamic characteristics has attracted much attention in recent decades. The used dynamic characteristics to locate and quantify structural damages are natural frequencies, mode shapes, mode shape curvature, modal strain energy, frequency response function (FRFs) and so forth. Using FRF data has the advantage of avoiding modal analysis errors indirectly extracted from the measured FRF data. Moreover, past studies showed that strains are more sensitive to localized damage compare to displacement. So, in this study FRF of strain data are utilized to identify unknown structural parameters using a sensitivity-based model updating approach. In this paper a quasi-linear sensitivity equation which diminishes the adverse effects of incompleteness of FRFs data is proposed for model updating. The efficiency of the proposed method is validated through a numerical 2D-frame example using FRF of strain data considering the adverse effects of noise and incompleteness of measurements. The results indicate that this method can locate and quantify the severity of damage precisely.

KEYWORDS : *model updating, quasi-liner sensitivity equation, incomplete measurement, FRF of strain data.*

INTRODUCTION

The main purpose of structural health monitoring is to detect damages of existing structure in order to prevention of their deterioration. Identifying structural parameters to monitor their changes has attracted much attention in recent decades. These changes would affect the static and dynamic characteristics of structure such as static displacement, strain and curvature as well as natural frequencies, mode shapes, mode shape curvature, modal strain energy, frequency response function (FRFs) and so forth. Past research studies showed that strains are more sensitive to localized damage compare to displacements. Furthermore, strain gages are usually less expensive than accelerometers and can be embedded inside structural elements to measure the response of inaccessible parts of structures.

Panedy et al. [1] used mode shape curvature (strain) to locate structural damage. Strain data was approximated using central difference method. Wahab and Roeck [2] used modal curvature data for damage detection of a real prestressed concrete bridge. Sampaio et al. [3] applied the curvature-based method to FRF for damage detection. Shi et al. [4] used the ratio of change in modal strain energy as a sensitive parameter to detect damage of elements. Qiao et al. [5] demonestrated that the curvature-based method exhibited better performance than the other dynamic-based damage detection approaches in composite plates. Catbas et al. [6] investigated the application of the flexibility-based curvature to detect damage of a steel grid structure experimentally.

FRF data have been utilized directly to detect structural damage by many researchers [7-10]. Using FRF data instead of other dynamic characteristics data extracted indirectly from the measured FRF data has the advantage of avoiding modal analysis errors. In addition, applying FRF data on a broadband frequency range lessen the problem of not enough data in modal based model updating approaches. To eliminate incomplete measurement difficulty, Lin et al. [11] used several approaches by model reduction, data expansion, and replacing the unmeasured response with their analytical values. Despite the opportunity gained by using model reduction and data expansion to deal with incomplete measurement, these methods lead to convert model updating to an nonlinear optimization process [12]. In sensitivity-based model updating method a linear sensitivity equation to achieve accurate results is highly desired. However, obtaining this type of sensitivity matrix requires complete measurement which is impractical. Esfandiari et al. [13] approximated unmeasured FRF data using analytical mode shapes of intact structure and measured natural frequencies of damaged structure. Hence, by utilizing this approximation, they proposed a quasi-linear sensitivity equation. Sanayei et al. [12] demonstrated the validity of above-proposed method using experimental data.

The measured axial or flexural strain data of structure can be used in dynamic state by presenting them in frequency domain as FRF data. Maia et al. [14] detected structural damage using the curvatures of FRF data. Other researchers also utilized the advantages of both FRF data and curvature to detect damage [3,6]. Esfandiari et al. [15] used strain-based FRF data as a sensitive parameter to structural damage and introduced a quasi-linear sensitivity equation to identify damage. Li [16] reviewed various types of strain-based methods for damage identification and indicated that the strain-based FRF methods perform better than others due to its avoidance of numerical modal extraction.

In this paper, a new robust method for detecting structural damages using updated structural stiffness and mass parameters is developed. The method correlates the sensitivity of strain-based FRF data due to damage to structural parameters changes at the elemental level. The proposed sensitivity equation diminishes negative effects of unmeasured FRF of strain data utilizing approximation of them by appropriately shifting analytical FRF data of intact structure. The efficiency of the proposed method is validated through a numerical frame example considering noisy data.

1 THEORETICAL BACKGROUND

For an n DOF structure, the equation of motion is:

$$M\ddot{x} + C\dot{x} + Kx = f(t) \tag{1}$$

Where M, K and C are n×n matrices of mass, stiffness and damping, respectively. f(t) and x(t) are n×1 vectors of applied force and displacement, respectively. For a harmonic excitation, the applied force and displacement response vectors can be expressed as:

$$f(t) = F(\omega)e^{j\omega t} \quad and \quad x(t) = X(\omega)e^{j\omega t} \tag{2}$$

Hence,

$$(-\omega^2 M + j\omega C + K)X(\omega) = F(\omega) \tag{3}$$

Equation (3) can be written as:

$$X(\omega) = H(\omega)F(\omega) \tag{4}$$

Where H(ω) is an n×n frequency response function matrix defined as:

$$H(\omega) = \left(-\omega^2 M + j\omega C + K\right)^{-1} \tag{5}$$

In a damage state, properties and response of the structure change, So Equation (4) can be expressed as:

$$X_d(\omega) = X(\omega) + \delta X(\omega) = H_d(\omega)F(\omega) \tag{6}$$

Considering the effect of damage on the three matrices of mass, stiffness and damping and defining their changes by δM, δK and δC, respectively, $H_d(\omega)$ can be written as,

$$H_d(\omega) = \left(-\omega^2\left(M + \delta M\right) + j\omega\left(C + \delta C\right) + K + \delta K\right)^{-1} \tag{7}$$

By expanding Equation (6) and subtracting Equation (4), yielding

$$\delta X\left(\omega\right) = -H_d(\omega)\left(-\omega^2 \delta M + j\omega\delta C + \delta K\right) X\left(\omega\right) \tag{8}$$

The analytical relation between nodal displacements of element in local coordinates $\overline{x}_i$ and its strains ε_i is :

$$\varepsilon_i = B_i \overline{x}_i \tag{9}$$

B_i is the strain-displacement transformation matrix. For an axial element, B_i, defined as

$$B_i = \frac{1}{L_i}\left[-1 \quad 0 \quad 1 \quad 0\right] \tag{10}$$

Which L_i is the length of ith element. For a beam-column element, B_i is defined as,

$$B_i = \begin{bmatrix} \frac{-1}{L_i} & \frac{-6z\xi}{L_i^2} & \frac{-z\left(6\xi - 2\right)}{2L_i} & \frac{1}{L_i} & \frac{6z\xi}{L_i^2} & \frac{-z\left(6\xi + 2\right)}{2L_i} \end{bmatrix} \tag{11}$$

Where z is the distance from the neutral axis and ξ is the local coordinate parameters, described as:

$$\xi = 2\overline{u}/L_i - 1 \tag{12}$$

In the above definition, $\overline{u}$ is the distance from the first node in the local coordinates. By converting Equation (9) in the global coordinates using transformation matrix T_i and assembling for all elements,

$$\varepsilon\left(\xi\right) = \sum_{i=1}^{ne} B_i(\xi) T_i x_i = B(\xi) x \tag{13}$$

Using Equation (13), Equation (8) can be rewritten in the strain-based from, as follow

$$\delta\varepsilon\left(\omega, \xi\right) = H_{\varepsilon d}(\omega, \xi)(-\omega^2 \delta M + j\omega\delta C + \delta K) X(\omega) \tag{14}$$

Where the strain-based FRF is :

$$H_{\varepsilon d}(\omega, \xi) = B(\xi) H_d(\omega) \tag{15}$$

And $\delta\varepsilon(\omega, \xi)$ is ne×1 vector of the residual between the analytical and the measured strain data. In Equation (14) damping matrix is not considered to be updated, because it is nearly impossible to model damping properly in finite element model. Additionally, the effect of damping on structure response rapidly decreases by moving away from the resonance frequencies. Therefore, in this study, just stiffness and mass matrices are considered to be updated and the effect of damping is ignored at the selected points away from resonances. The variation of stiffness and mass matrix can be expressed as the sum of the partial differential of them to each element parameter to be updated as follows:

$$\delta K = \sum_{i=1}^{ns} \frac{\partial K}{\partial P_i^S} \delta P_i^S \tag{16}$$

$$\delta M = \sum_{j=1}^{nm} \frac{\partial M}{\partial P_j^M} \delta P_j^M \tag{17}$$

In Equations (16) and (17) ns and nm are total number of updating parameters in stiffness and mass matrices, respectively. δP_i^S and δP_j^M are the ith and jth updating parameters related to elemental stiffness and mass characteristics, respectively. By substituting Equation (16) and (17) into Equation (14),

$$\delta\varepsilon\left(\omega, \xi\right) = \sum_{i=1}^{ns} \left\{-H_{\varepsilon d}(\omega, \xi)\left(\frac{\partial K}{\partial P_i^S}\right) X\left(\omega\right)\right\} \delta P_i^S + \sum_{j=1}^{nm} \left\{-H_{\varepsilon d}(\omega, \xi)\left(-\omega^2 \frac{\partial M}{\partial P_j^M}\right) X\left(\omega\right)\right\} \delta P_j^M \tag{18}$$

Equation (18) can be framed in the following matrix form,

$$\delta\varepsilon(\omega, \xi) = \begin{bmatrix} S^S & S^M \end{bmatrix} \begin{bmatrix} \delta P^S \\ \delta P^M \end{bmatrix} = S(\omega, \xi)\delta P \tag{19}$$

Where, S^S and S^M are the stiffness and mass parameters sensitivity matrices and S(ω,ξ) is the total sensitivity matrix. δP^S and δP^M are the vectors of stiffness and mass parameters changes and δP is the vector of all stiffness and mass parameters changes.

As can be seen from Eq. (14), if the complete measurement of structural responses at all DOFs of the damaged structure were possible, the changes of frequency response function would be a linear function of the structural parameters changes. Due to the fact that the full measurement of all $H_{\varepsilon d}(\omega,\xi)$ entries is impractical, some researchers approximate unmeasured DOFs by using analytical values of $H_{\varepsilon}(\omega,\xi)$ instead of unmeasured values, or use model reduction and data expansion that increase the order of equations. In this study another approximation which converts the equation to a quasi-linear form is applied. The FRF of the damaged structure in the vicinity of the resonances, used for model updating, is reconstructed by shifting the FRF value of the intact structure. The amount of the shift depends on the difference between the natural frequencies of the intact structure and the damaged structure at each resonance. Figure 1 illustrates the proposed concept. In this figure, the FRF of the intact structure and the damaged structure are indicated by H and H_d, respectively, and the reconstructed FRF of the damaged structure is shown by H_T.

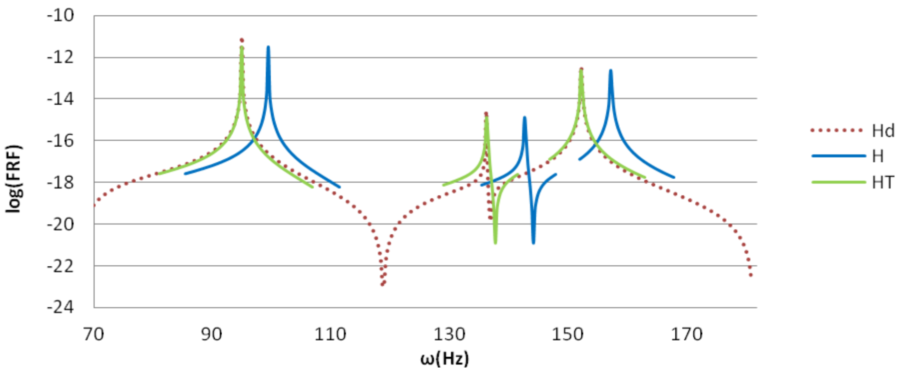


Figure 1: A template of proposed approximation performance

Several methods can be used to solve Equation (19). Though, in this study, Least Square method (LS) is employed. The quality of predicted results depends on the selected frequency range, excitation locations, sensor locations, the quality of the measured FRF data, accuracy of mathematical model, weighting technique applied to the set of equations and etc.

The weighting technique applied to this study is to normalize each row of the sensitivity matrix and corresponding output residual using the norm of that row. It is worth noting that sensitivity matrix is formed at some selected frequency points. The frequency points with high sensitivity which are located near resonances are selected for updating. Also, two rules are considered to avoid updating process errors. The first rule excludes frequencies very close to resonances which are affected by damping from total candidate frequencies. Therefore, a small zone of 2 Hz around the resonances is not considered for model updating. The second rule indicates that the frequencies between corresponding resonances of the intact and damaged structures should not be selected. The aforementioned rule avoids non-smooth behavior of optimization process [12].

2 NUMERICAL EXAMPLE

In order to prove the validity and the robustness of the proposed method, a one story one bay 2D frame structure is considered as a numerical example. The 2D frame model as shown in Figure 2 consists of 21 two-node frame elements and 22 nodes. The mechanical properties of elements are presented in Table 1.

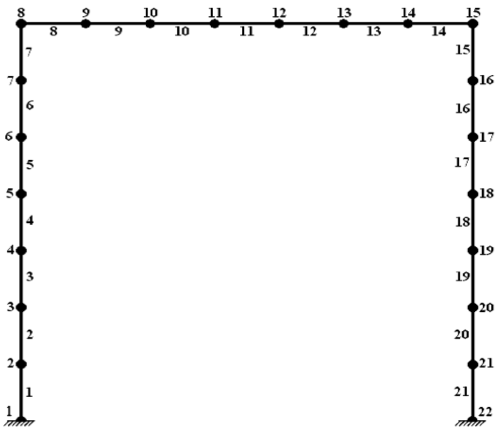


Figure 2: 2D frame structure

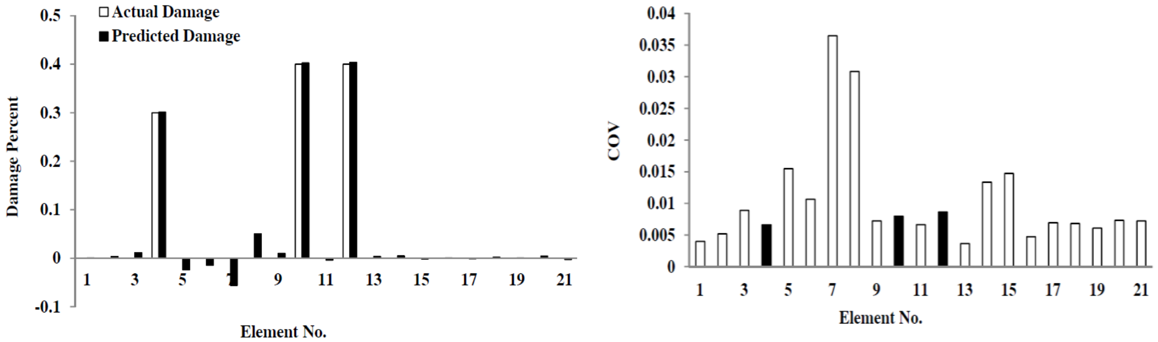
The measured strain value by a strain gage consists of both axial and flexural strains. In this study, damage is considered as the changes in both axial and flexural rigidities. Therefore, the unknown structural parameters are flexural rigidity, EI, and axial rigidity, EA. Although, in practice, exciting a structure in axial direction is difficult which leads to low contribution of axial rigidity to the sensitivity matrix. Therefore, only the flexural rigidity of elements are updated. To investigate precision of the method, several damage scenarios which differ in location, severity and number of damaged elements are considered.

Table 1: Properties of elements.

Density (kg/m^3)	**Young modulus (GPa)**	**Length (cm)**	**Cross-sectional area (cm^2)**	**Moment of inertia (cm^4)**
7800	200	10	1.84	0.2685

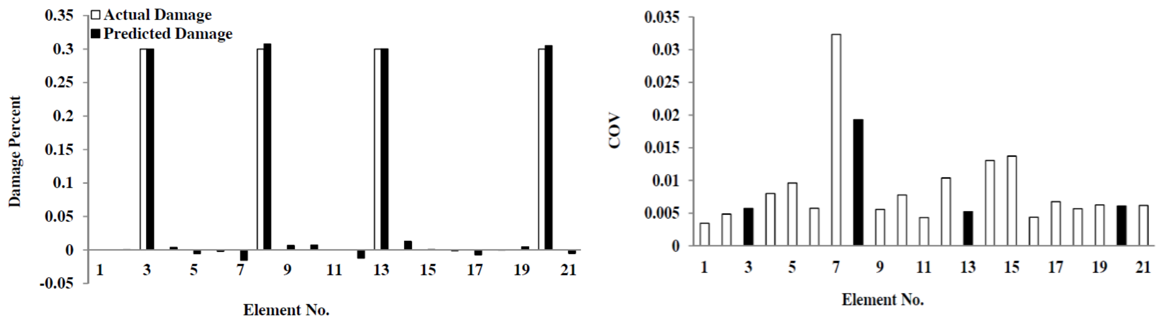
In order to simulate errors appeared in the measured strain-based FRF data (measurement errors), 10 percent uniform random error is added to the extracted FRF of strain data from the FE model of the damaged structure. The measured natural frequencies of a light damped structure can be assumed as low noise contaminated or even noise-free using the available accurate measurement equipments. Therefore, in this study the measured natural frequencies are assumed to be noise-free. Additionally, it is assumed that the first 8 natural frequencies of the damaged structure are measureable. The FRF of the strain data is extracted from the damaged structure which is excited by a single load perpendicular to the structure at the node numbers 4, 7, 12, 17 and 19. The strain values are measured at element numbers 2, 6, 9, 13, 16 and 19.

The reliability of the method against measurement errors is investigated by considering 50 sets of random noise-contaminated data. The averages of the predicted parameters with their coefficient of variation (COV) are plotted in Figures 3 to 5. Low COV of the estimated parameters shows a robust and low-scatter prediction. The results shown in Figures 3 to 5 prove the ability of the proposed method in identifying correctly both the damage location and severity using incomplete noise contaminated strain-based FRF data.



a) b)

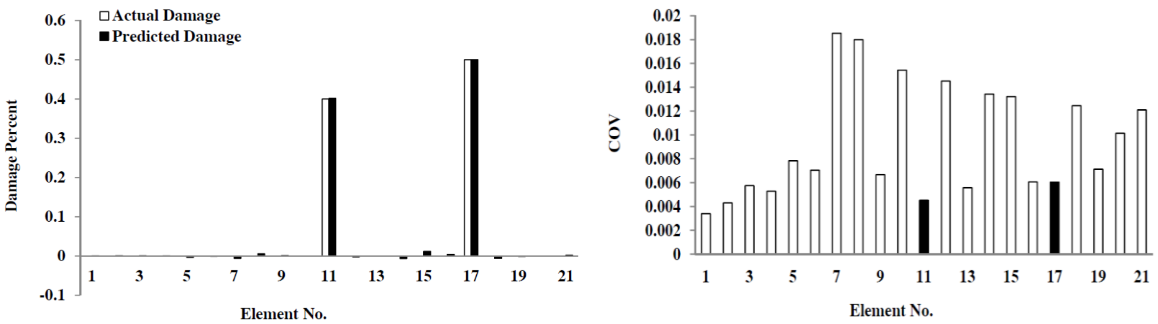
Figure 3: a) Actual and predicted stiffness parameters change for case 1 and b) COV of predicted parameters



a) b)

Figure 4: a) Actual and predicted stiffness parameters change for case 2 and b) COV of predicted parameters

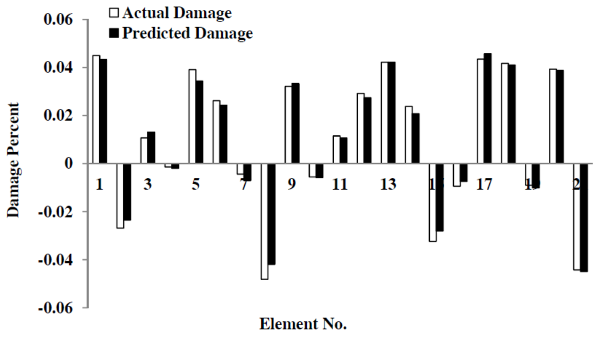
The appeared false low changes in elements which are not actually damaged are due to measurement errors, ignoring axial rigidity changes and incomplete measurement. These errors can be reduced by appropriately selection of excitation locations, measurement locations and excitation frequencies.



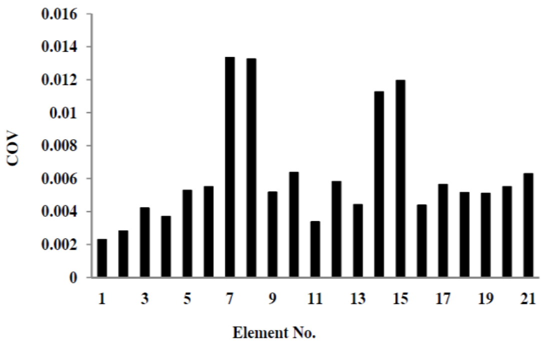
a) b)

Figure 5: a) Actual and predicted stiffness parameters change for case 3 and b) COV of predicted parameters

Another issue studied numerically in this paper is the overall deterioration of structure which is simulated by defining damage in all elements of the frame structure. For this purpose a 5 percent random changes is defined in all elements stiffness parameters as a damage scenario (case 4). The comparison between the actual damage and predicted damage and the obtained COV for this case is shown in Figure 6. It should be noted that for the cases with low damage the adverse effects of measurement noise is more apparent. However, presented results in Figure 6 indicate the robustness of the presented method against measurement errors.



a)



b)

Figure 6: a) Actual and predicted stiffness parameters change for case 4 and b) COV of predicted parameters

CONCLUSION

In this study, a new robust structural damage detection method using FRF of strain data and natural frequencies is proposed. The method contains a quasi-linear sensitivity equation which correlates FRF of strain data changes to structural parameter changes. The side effect of incomplete measurement decreases using a new proposed approximation of FRF of the damaged structure. Each row of equation is normalized using the second norm of corresponding row of the sensitivity matrix to increase the stability of method. The exciting frequency ranges are selected deliberately for model updating which improve the parameters prediction. Finally, the changes of structural parameters are found by implementing least square method to solve the sensitivity equation. The methods are verified by applying to a numerical 2D frame example using simulated incomplete noisy FRF of strain data to indentify unknown flexural stiffness parameters. The results of the example for various damage cases show the feasibility and confidence of the method to detect both damage location and severity.

REFERENCES

[1] A. K. Pandey, M. Biswas, and M. M. Samman. Damage detection from changes in curvature mode shapes. *Journal of Sound and Vibration, 145 (2), 321–332*, 1991.

[2] Wahab, M.M.A. and Roeck, G.D. Damage detection in bridges using modal curvatures: application to a real damage scenario. *Journal of Sound and Vibration*, *226(2), 217–235*, 1999.

[3] R. P. C. Sampaio, N. M. M. Maia, J. M. M. Silva. Damage detection using the frequency-response function curvature method. *Journal of Sound and Vibration, 226 (5), 1029–1042*, 1999.

[4] Z. Y. Shi, S. S. Law, and L. M. Zhang. Structural Damage Detection from Modal Strain Energy Change. *Journal of Engineering Mechanics, 126(12), 1216–1223*, 2000.

[5] P.H. Qiao, K. Lu, W. Lestari, J. Wang. Curvature mode shape-based damage detection in composite laminated plates. *Composite Structures, 80(3), 409–428*, 2007.

[6] F. N. Catbas, M. Gul, J. L. Burkett. Damage assessment using flexibility and flexibility-based curvature for structural health monitoring. *Smart Materials and Structures, 17(1), 15–24*, 2008.

[7] U. Lee and J. Shin. A frequency response function-based structural damage identification method. *Computers and Structures, 80, 117–132*, 2002.

[8] N. M. M. Maia, J. M. M. Silva, and E. A. M. Almas. Damage detection in structures: from mode shapes to frequency response function methods. *Mechanical Systems and Signal Processing, 17(3), 489–498*, 2003.

[9] X. Liu, N. A. J. Lieven, and P. J. Escamilla-Ambrosio. Frequency response function shape-based methods for structural damage localization. *Mechanical Systems and Signal Processing, 23, 1243-1259*, 2009.

[10] J. Li, S. S. Law, and Y. Ding. Substructure damage identification based on response reconstruction in frequency domain and model updating. *Engineering Structures, 41, 270–284*, 2012.

[11] R. M. Lin, M. K. Lim, and J. H. Ong. Improving finite element models in the higher frequency range using modified frequency response function sensitivity method. *Finite Elem Anal Des, 15, 157–175*, 1993.
[12] M. Sanayei, A. Esfandiari, A. Rahai, and F. Bakhtiari-Nejad. Quasi-linear sensitivity-based structural model updating using experimental transfer functions. *Structural Health Monitoring,* 0(0), 1–15, 2012.
[13] A. Esfandiari, F. Bakhtiari-Nejad, A. Rahai, and M. Sanayei. Structural model updating using frequency response function and quasi-linear sensitivity equation. *Journal of Sound and Vibration, 326(3–5), 557–573*, 2009.
[14] N. M. M. Maia, J. M. M. Silva, and R. P. C. Sampaio. Localization of damage using curvature of the frequency response functions. in *XV International Modal Analysis Conference Orlando, USA, 942-946*, 1997.
[15] A. Esfandiari, M. Sanayei, and F. Bakhtiari-Nejad. Finite element model updating using frequency response function of incomplete strain data. *AIAA Journal, 48(7)*, *1420-1433*, 2010.
[16] Y.Y. Li. Hypersensitivity of strain-based indicators for structural damage identification: A review. *Mechanical Systems and Signal Processing, 24, 653-664*, 2010.