Modeling and control of pedestrian behaviors: An environment optimization approach

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New Trends in Optimal Control
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Choice of the behavior

Background assumptions

no panic – known environment – given target

- **Basic**[1]: Path is computed once at the initial time, assuming that the environment is empty.
- **Rational**[2]: Path is recomputed continuously, taking into account the *current* pedestrian distribution.
- **Highly rational**[3]: Path is computed once, taking into account the pedestrian distribution at *current and later* time. Highly rational crowd $\rightarrow$ Nash equilibrium.
- θ**-rational**: Path is recomputed continuously, taking into account the pedestrian distribution at current and later time up to a time $t+\theta$.

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A multi-purpose approach

First goal

Creating a unique model such that the degree of rationality can be freely tuned.

We consider a macroscopic first-order model based on a nonlinear nonlocal anisotropic conservation law

$$\frac{\partial}{\partial t}\rho(x,t) + \mathrm{div}\big(\rho(x,t)v[\rho](x,t)\big) = 0$$

with

$$v[\rho](x,t) = v_b^* + v_i$$

and

$$v_i = v_i[\rho(\cdot,t)](x) = \int_{\mathcal{S}(x)} \mathcal{F}(y-x)\rho(y,t)dy$$

This model is 2D and fundamental-diagram free

Basic behavior + minimum time problem

Perceived dynamics

$$\dot{y}(t) = v_b(t), \qquad t > 0.$$

At the initial time we solve once

Eikonal equation

$$\begin{cases} |\nabla\phi(x)| - 1 = 0, & x \in \Omega \\ \phi(x) = 0, & x \in \mathit{Target} \end{cases}$$

Then, the optimal behavioral velocity field is taken in feedback form as

$$v_b^*(x) = -\frac{\nabla\phi(x)}{|\nabla\phi(x)|}, \qquad x \in \Omega.$$

v_b^* is time-independent coherently with the fact that pedestrians are insensitive to the evolution of the crowd.

Rational behavior + minimum time problem

Perceived dynamics

$$\dot{y}(t) = v_b(t) + v_i[\rho(\cdot,\tau)](y), \quad t > \tau.$$

At any given time $t = \tau$ we solve

Modified eikonal equation

$$\begin{cases} |\nabla\phi_\tau(x)| - v_i[\rho(\cdot,\tau)](x) \cdot \nabla\phi_\tau(x) - 1 = 0, & x \in \Omega \\ \phi_\tau(x) = 0, & x \in \mathit{Target} \end{cases}$$

Then, the optimal behavioral velocity field is taken in feedback form as

$$v_b^*(x) = -\frac{\nabla\phi_\tau(x)}{|\nabla\phi_\tau(x)|}, \qquad x \in \Omega.$$

At any fixed time, the HJB equation is independent of the conservation law for ρ.

Highly rational behavior + minimum time problem

Perceived dynamics

$$\dot{y}(t) = v_b(t) + v_i[\rho(\cdot,t)](y), \quad t > 0.$$

We solve once the following forward-backward equation

Mean-field equation

$$\begin{cases} \partial_t \rho(t,x) + \mathrm{div}\Big[\rho(x,t)\big(v_b^*(x,t) + v_i[\rho(\cdot,t)](x)\big)\Big] = 0 & \color{red}{t \rightarrow} \\ \max\limits_{v_b \in \overline{B_1(0)}} \{-(1,\ v_b + v_i[\rho(\cdot,t)](x)) \cdot \nabla_{x,t}\phi(x,t) - 1\} = 0 & \color{red}{\leftarrow t} \\ v_b^*(x,t) \in \arg\max\limits_{v_b \in \overline{B_1(0)}} \{-(1,\ v_b + v_i[\rho(\cdot,t)](x)) \cdot \nabla_{t,x}\phi(t,x) - 1\} \end{cases}$$

The HJB and the CL are fully coupled.

θ-rational behavior + minimum time problem

Perceived dynamics

$$\dot{y}(t) = v_b(t) + v_i[\rho^\theta(\cdot, t)](y),$$

$$\rho^\theta(x,t) := \rho(x,t), t \in [\tau, \tau+\theta], \qquad \rho^\theta(x,t) := \rho^\theta(x, \tau+\theta), t > \tau+\theta.$$

At any given time $t = \tau$ we solve

Mean-field equation /2

$$\begin{cases} \partial_t \rho^\theta + \mathrm{div}\big[\rho^\theta \big(v_b^{*,\theta} + v_i[\rho^\theta]\big)\big] = 0 \quad \text{in} \quad (\tau, \tau+\theta) \times \Omega \qquad t \rightarrow \\ \max\limits_{v_b \in \overline{B_1(0)}} \Big\{ - \Big(1,\ v_b + v_i[\rho^\theta(\cdot,t)](x)\Big) \cdot \nabla_{x,t} \phi^\theta(x,t) - 1 \Big\} = 0 \qquad \leftarrow t \\ v_b^{*,\theta}(x,t) \in \arg \max\limits_{v_b \in \overline{B_1(0)}} \Big\{ - \Big(1,\ v_b + v_i[\rho^\theta(\cdot,t)](x)\Big) \cdot \nabla_{x,t} \phi^\theta(x,t) - 1 \Big\} \end{cases}$$

Inducing unconscious rationality

Second goal

Forcing people to behave more rationally than they would naturally do

By means of the models described above we can describe

The natural behavior

The expected behavior according to their real limited predictive ability (f.e., the basic one)

The target behavior

A particularly efficient behavior one would like they to assume (f.e., the rational or highly rational one)

Key idea

But people are hardly controlled... So we want to get the target behavior still *keeping* the natural one

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Environmental optimization

Environmental control

We assume that one can introduce in the domain additional controlled obstacles, hoping they improve the dynamics (Braess' paradox)

Environmental cost

The natural behavior in the new environment should be as close as possible to the target behavior in the original environment in terms of

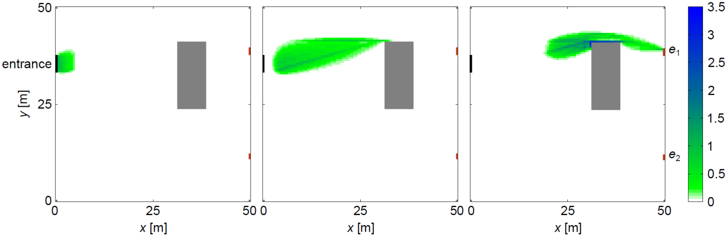
- evacuation times;
- exits usage (if more than one);
- maximal densities (related to overcompression and then injuries);
- ...

Minimization strategy (rectangular obstacles)

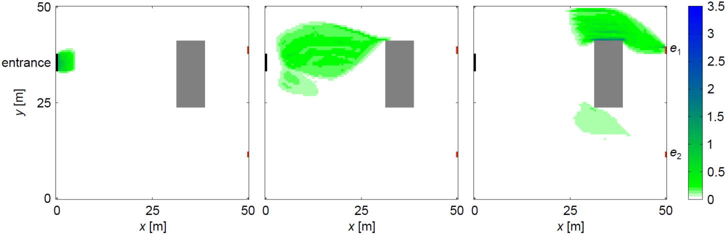
Exhaustive search, compass search + simulated annealing.

Room with fixed obstacle

natural = basic [$t_{\text{evac}} = 120.6$]

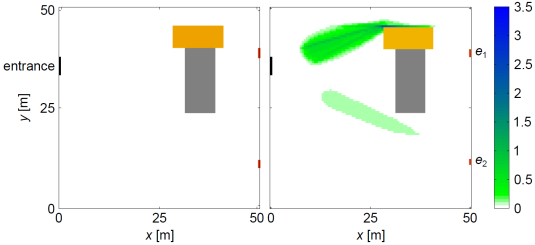


target = rational [$t_{\text{evac}} = 95.8$]

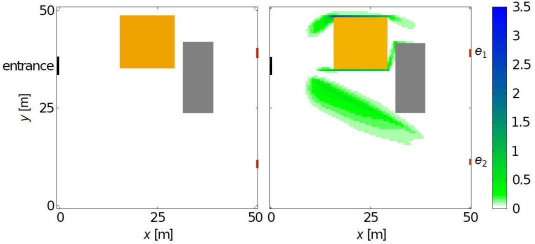


Room with fixed obstacle

natural behavior with manual guess [$t_{\mathrm{evac}} = 125.5$]

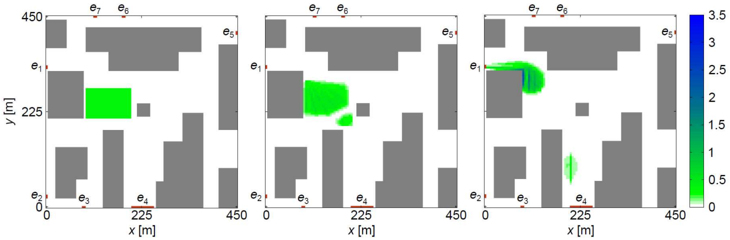


natural behavior with best obstacle [$t_{\mathrm{evac}} = 99.0$]

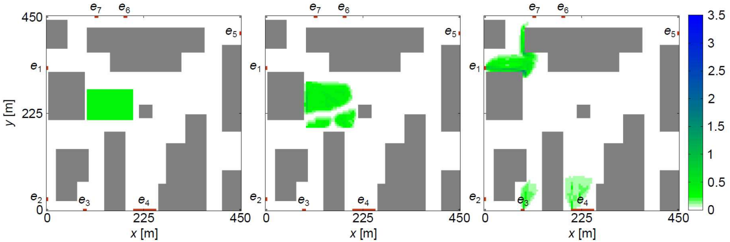


Sapienza campus

natural = basic [92%, 0, 8%, 0]

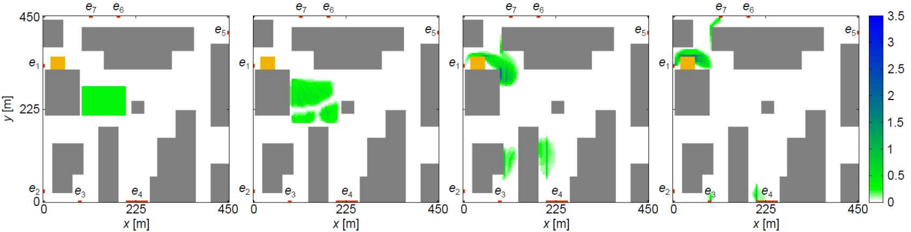


target = rational [61%, 10%, 15%, 14%]



Sapienza campus

natural behavior with best obstacle [60%, 12%, 18%, 10%]



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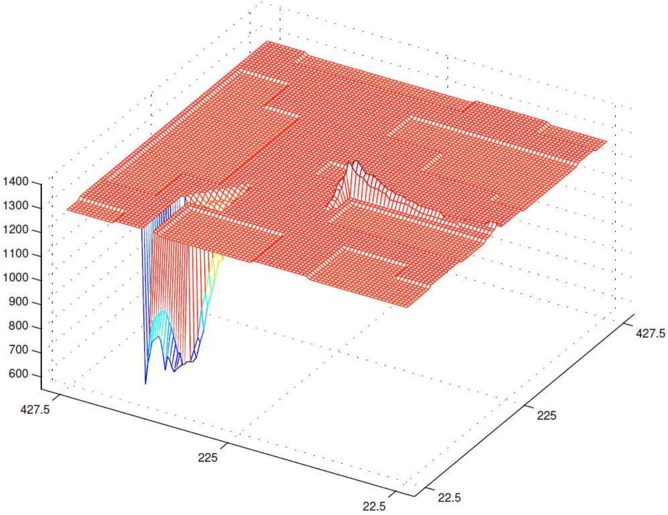
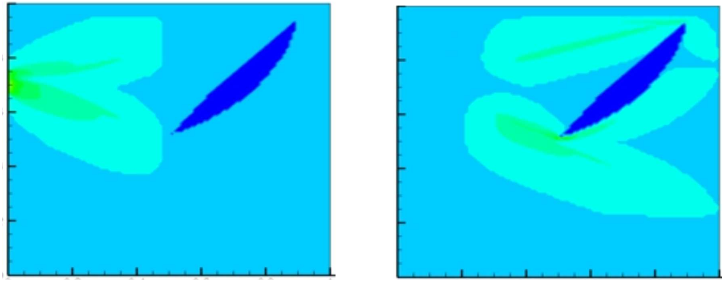


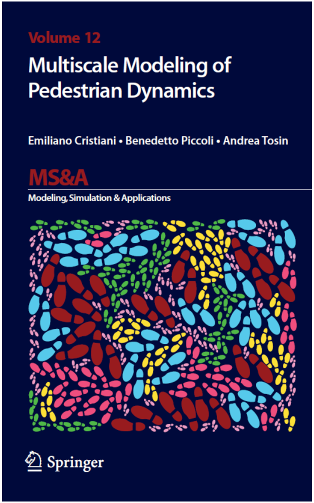
Current work

Obstacles with free shape (with D. Peri)



Coming soon

Multiscale Modeling of Pedestrian Dynamics



Thank you

THANK YOU

(pay attention when you leave the room, I'm watching you...)