old, current and (perhaps) future advances to homogenization in random environments

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NETCO June 2014

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is about the study of
the effective (averaged) macroscopic behavior
of phenomena (equations) depending on many scales and
taking place at the microscopic level in self-averaging environments

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the issues

- reduce the complexity of the problem
- pass from microscopic to macroscopic – bridge scales rigorously
- identify effective (averaged) equations

general problems

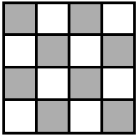
$$F(D^2u_\varepsilon, Du_\varepsilon,\ x/\varepsilon) = 0 \quad \text{and} \quad -\varepsilon\,\mathrm{tr}\,A(Du_\varepsilon, x/\varepsilon, \omega)D^2u_\varepsilon + H(Du_\varepsilon, x/\varepsilon, \omega) = 0$$

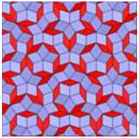
mathematical questions

- qualitative
  - do the u_ε's have a limit $\bar{u}$?
  - does $\bar{u}$ satisfy a translation invariant equation $\overline{F}(D^2\bar{u}, D\bar{u}) = 0$ and $\overline{H}(D\bar{u}) = 0$?
- quantitative
  - rates of convergence
  - numerical approximations
- are there any qualitative and quantitative differences between periodic and random environments (besides nontrivial technicalities)

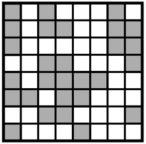
examples of “self averaging” environments

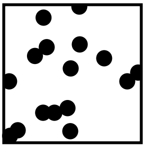
- periodic, quasiperiodic, almost periodic

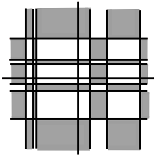




- random







FUNDAMENTAL DIFFERENCE

periodic/almost periodic	compact
random	not compact

the heuristics of the random media

equations appear with the same frequency no matter where they are located (stationarity) but they mix (ergodicity)

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if they do not mix there will be, at large scales, solutions of “only blues” and of “only reds”

but if the “blues” and the “reds” mix, the picture will become “uniformly purple”

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if we know a rate for the way they mix, it is possible to estimate, for a given small scale, how many solutions are close to the averaged one

random setting

in the family of all equations, $F(Q,p,x)$, satisfying some common uniform properties like uniform ellipticity, Lip continuity, coercivity, ... concentrate on a subfamily $F(Q,p,x,\omega)$ appearing with some "frequency" described by the fact that ω belongs to a probability space $(\Omega,\mathcal{F},\mathbb{P})$ such that

- if $F(Q,p,\cdot,\omega)$ is in the family and we change position to y, then $F(Q,p,\cdot+y,\omega)$ is also in the family, i.e., $F(Q,p,\cdot+y,\omega)=F(Q,p,\cdot,\tau_y\omega)$, and the frequency of appearance is independent of the change of the location, i.e., $\tau_y : \quad \Omega\to\Omega$ is measure preserving stationarity
- under translations, the operators "repeat themselves", i.e.,

  if $\mathbb{P}(A)>0$, then $\mathbb{P}(\cup_{y\in\mathbb{R}^d}\tau_y A)=1$ ergodicity

definitions

$(\Omega, \mathcal{F}, \mathbb{P})$ probability space endowed with a group $(\tau_y)_{y\in\mathbb{R}^d}$ of measure preserving transformations $\tau_y : \Omega \to \Omega$

$f : \mathbb{R}^d \times \Omega \to \mathbb{R}$ stationary iff its distribution is indep. of the location

$f : \mathbb{R}^d \times \Omega \to \mathbb{R}$ stationary iff $f(x+y, \omega) = f(x, \tau_y \omega)$

$(\tau_x)_{x\in\mathbb{R}^d}$ ergodic iff if $\tau_x A = A$ for all x, then $\mathbb{P}(A) = 0$ or 1

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think that Ω is the set of all equations F (and their translations) sharing some common properties

$\mathcal{S}$, $\mathcal{S}(r)$ the smallest σ-algebras generated by the measurable sets $\{F(\cdot, y, \cdot) : y \in Q_1\}$ and $\{F(\cdot, y, \cdot) : \mathrm{dist}(y, Q_1) \geqq r\}$ of Ω for $r > 0$ and Q_1 the unit cube in $\mathbb{R}^d$

$(\Omega, \mathcal{F}, \mathbb{P})$ is strongly mixing with rate g if

$$\sup_{\substack{A\in S \\ B\in S(r)}} |\mathbb{P}(A \cap B) - \mathbb{P}(A)\mathbb{P}(B)| \leq g(r)$$

and $g(r) \to 0$ as $r \to \infty$

some technical differences between periodic and random settings

- no compact Sobolev imbeddings — Poincare, isoperimetric inequalities

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integrating a mean 0 periodic function yields a periodic function

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integrating a mean 0 periodic function yields a periodic function

integrating a mean 0 stationary function does not yield a stationary function

instead it gives a function with strictly sublinear at infinity growth

recall ergodic theorem

$v : \Omega \to \mathbb{R}$ stationary $\Rightarrow$ $\lim_{|R|\to\infty} \fint_{B(0,R)} v(\tau_x\omega)dx = \mathbb{E}(f)$

v stationary, $\mathbb{E}(v) = 0$ and $u(x) = \int_0^x v(y)dy$ $\Rightarrow$ $u(x)/|x| \to 0$ as $|x| \to \infty$

references for periodic/almost periodic media

- linear non divergence form equations

too many to state and still many open problems

- nonlinear equations

a very incomplete list

Lions-Papanicolaou-Varadhan, Evans, Majda-S, Caffarelli, ... Imbert, Monneau —dislocation dynamics

Capuzzo-Dolcetta - Ishii, Caffarelli-S. – error estimates

Evans- Gomes, Fathi-Siconolfi—- connections to KAM

Tanaka, Barles - Da Lio -Lions -S., Choi-Kim —-Neumann conditions

Cardaliaguet -Lions-S; Caffarelli-Monneau —-moving fronts

Cardaliaguet —noncoercive HJ

Cardaliaguet-Nolen-S. — G -equation

Feldman — Dirichlet conditions

Jing- S. -Tran—time dependent HJ with linear growth

......

Ishii – almost periodic HJ equations

Lions-S – almost periodic degenerate Bellman

references for random media

- linear equations

Papanicolaou-Varadhan; Kozlov; Jikov; Yurinskii; Piatniski, ...

- nonlinear problems

variational

Dal Masso-Modica

nonvariational

Hamilton-Jacobi equations
S ; Rezakhanlou-Tarver; Lions-S. (correctors); Lions-S (new proof); Schwab (random in time); Davini -Siconolfi; Armstrong -S. (unbounded media); Armstrong - S. (L^∞ variational problems); Cardaliaguet-S. (G-equation); Nolen -Novikov (special case of G-equation); Armstrong-S. (quasiconvex and "metric approach"); Armstrong, Cardaliaguet -S. (rates); Armstrong, Tran and Yu (some special non convex problem), Ciomaga, S and Tran (noncoervive HJ), ...

degenerate Bellman equations
Lions-S.; Kosygina-Rezankhanlou-Varadhan; Lions-S. (new proof); Armstrong-S. (unbounded media and "metric approach"); Armstrong-Tran; Armstrong-Cardaliaguet (error estimates), Kosygina-Varadhan (random in time), ...

fully nonlinear second order

Caffarelli-S-Wang; Caffarelli-Mellet (obstacle problems); Caffarelli-S. (error estimates);

Armstrong-Smart (degenerate elliptic and error estimates); Otto et al. (qualitative theory)

coercive degenerate Bellman equations

$-\varepsilon \operatorname{tr} A(x/\varepsilon,\omega)D^2u_\varepsilon + H(Du_\varepsilon, x/\varepsilon,\omega) = 0$

A stationary, ergodic, degenerate elliptic

H stationary, ergodic, coercive, quasi-convex*, bounded for bounded gradients or unbounded with moment bounds

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Theorem (convergence)

the problem homogenizes almost surely to a Hamilton-Jacobi equation with an effective Hamiltonian $\overline{H}$ which has the same qualitative properties as H.

S., Rezakhanlou -Tarver, Lions-S., Kosygina, Rezakanlou -Varadhan, Kosygina -Varadhan, Schwab, Armstrong -S. (metric approach), Armstrong-Tran, Armstrong-Tran-Yu* a nonconvex example, Kosygina-Varadhan and Schwab (oscillations in time)

a qualitative difference between periodic and random environments?

$$\begin{cases} u_{\varepsilon,t} - \varepsilon \Delta u_\varepsilon + |Du_\varepsilon|^2 - V(x/\varepsilon, \omega) = 0 & \text{in } \mathbb{R}^d \times (0,\infty) \\ u_\varepsilon(\cdot, 0; \omega) = 0 & \text{in } \mathbb{R}^d \end{cases}$$

- V stationary ergodic and $0 \leq V \leq 1$
- homogenization $\Rightarrow$ $\lim_{\varepsilon \to 0} u_\varepsilon(\cdot, \cdot; \omega) = \bar{u}(\cdot, \cdot)$ in $C(\mathbb{R}^d \times (0,\infty))$ and *a.s.*
- $\bar{u}(x,t) = -t\bar{H}(0)$ solves

$$\begin{cases} \bar{u}_t + \bar{H}(D\bar{u}) = 0 & \text{in } \mathbb{R}^d \times (0,\infty) \\ \bar{u}(\cdot, 0) = 0 & \text{in } \mathbb{R}^d \end{cases}$$

- what do we know about $\bar{H}(0)$?

visible and invisible potentials

- invisible potentials $\Rightarrow$ $\bar{H}(0) = 0$

$\mathbb{P}\{\omega \in \Omega : \sup_{|y| \leq \rho} V(y, \omega) \leq \alpha\} > 0$ for every $\rho, \alpha > 0$

i.i.d. potentials, Poisson clouds BUT not periodic

- visible potentials $\Rightarrow$ $\bar{H}(0) < 0$

$\mathbb{P}\{\omega \in \Omega : \sup_{|y| \leq \rho} V(y, \omega) \leq \alpha\} = 0$ for (fixed) $\rho > 0$ and $0 < \alpha < 1$

periodic potentials

$\bar{H}(0)$ is the principal eigenvalue of $-\Delta + V$ with "periodic boundary confitions"

rates of convergence

(model) microscopic problem $u_\varepsilon + H(Du_\varepsilon + p) = V(x/\varepsilon, \omega)$ in $\mathbb{R}^d$

H coercive, convex $H(0) = 0$ and $0 = \inf V \leq V \leq C$

effective Hamiltonian $\overline{H}$ may have "blocking zones"

homogenization $\Rightarrow$ $u_\varepsilon \to \overline{u}$ and $\overline{u} + \overline{H}(D\overline{u} + p) = 0$ $\Rightarrow$ $\overline{u} = -\overline{H}(p)$

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- periodic environment $|u_\varepsilon + \overline{H}(p)| \leq C\varepsilon(|p| + 1)$

$\Rightarrow$ $\varepsilon^{1/3}$ rate of convergence for general problem Capuzzo-Dolcetta and Ishii

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- random environment rate is more complicated and it can be arbitrarily slow

$u_\varepsilon + H(Du_\varepsilon + p) = V(x/\varepsilon, \omega)$ in $\mathbb{R}^d$

H coercive, quasiconvex $H(0) = 0$ and $0 = \inf V \leq V \leq C$

$V(\cdot, \omega)$ independent at finite range D

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Theorem (rate of convergence Armstrong, Cardaliaguet and S.)

- $\limsup_{\varepsilon \to 0} \sup_{B_R} \dfrac{-u_\varepsilon(\cdot, \omega; p) - \overline{H}(p)}{C\varepsilon^{1/3} |\log \varepsilon|^{1/3}} \leq 1$

if $\overline{H}(p) > 0$, *then*

- $\limsup_{\varepsilon \to 0} \sup_{B_R} \dfrac{-u_\varepsilon(\cdot, \omega; p) - \overline{H}(p)}{C\overline{H}(p)^{-1} \varepsilon^{1/3} |\log \varepsilon|^{1/2}} \leq 1$
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if $\overline{H}(p) = 0$ *and* $\mathbb{P}\{\omega \in \Omega : V(0, \omega) > \lambda\} \geq \lambda^\theta$, *then*

- $\liminf_{\varepsilon \to 0} \inf_{B_R} \dfrac{-u_\varepsilon(\cdot, \omega; p) - \overline{H}(p)}{C\varepsilon^{\alpha} |\log \varepsilon|^{\beta}} \geq 1$

$\alpha = \min(1/6, d/d + \theta)$ $\beta = 1/4$

general rate $-\mathrm{O}(\varepsilon^{1/8} |\log \varepsilon|^{3/16}) \leq u^\varepsilon - u \leq \mathrm{O}(\varepsilon^{1/5} |\log \varepsilon|^{1/5})$

$H(Du_\varepsilon, x/\varepsilon) = 0$ - the cell problem

the formal expansion $u_\varepsilon = \bar{u} + \varepsilon w(x/\varepsilon)$ leads to $H(D_x\bar{u} + D_y w, x/\varepsilon) = 0$

and the cell problem

- for each $p \in \mathbb{R}^d$ there exists a unique constant $\bar{H}(p)$ such that

$$H(Dw + p, y) = \bar{H}(p) \text{ in } \mathbb{R}^d$$

has a solution (corrector) w satisfying $w(y)/|y| \to 0$ as $|y| \to \infty$

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solution of the cell problem and perturbed test function $\Rightarrow$ homogenization

do correctors exist?

• coercive Hamiltonians Lipschitz bounds

YES in periodic media and NO in random media

the problem is not estimates but strictly sublinear growth

• non coercive Hamiltonians no Lipschitz bounds

everything depends on the particular setting

the approximate cell problem $\delta w_\delta + H(Dw_\delta + p, y) = 0$ in $\mathbb{R}^d$

coercivity $\Rightarrow$ Lip. bounds

periodic and almost periodic media $\Rightarrow$ $\delta w_\delta \to -\bar{H}$ uniformly in $\mathbb{R}^d$ and this proves homogenization

random media (after homogenization is proved)

$$\delta w_\delta \to -\bar{H} \text{ uniformly in balls of radius } 1/\delta$$

• new approach based on identifying quantities that control the behavior of solutions and are amenable to the ergodic and subadditive ergo theorems

$\overline{F}(\cdot)$ is the ergodic constant and $F_L(\cdot,\omega)$ is an L-periodic “approximation” of $F(\cdot,\omega)$ with ergodic constant $\overline{F}_L(\cdot,\omega)$

periodic approximations of the ergodic constants

$\overline{F}(\cdot)$ is the ergodic constant and $F_L(\cdot, \omega)$ is an L-periodic "approximation" of $F(\cdot, \omega)$ with ergodic constant $\overline{F}_L(\cdot, \omega)$

- questions
  - is it true that, as $L \to \infty$ and a.s., $\overline{F}_L(\cdot, \omega) \to \overline{F}(\cdot)$?
  - is there a rate of convergence?
- problems
  - how to define the periodic approximations?
  - how to pass to the large period limit?
  - how to connect the ergodic constants?

$$-\varepsilon \mathrm{tr}(A(x/\varepsilon,\omega)D^2u^\varepsilon) + H(Du^\varepsilon, x/\varepsilon,\omega) = 0$$

$A = A(y,\omega)$, $H = H(p,y,\omega)$ stationary ergodic, H convex and coercive in p and A the "square" of a Lipschitz matrix $\Rightarrow$ homogenization

correctors do not exist in general and the ergodic constant $\overline{H}$ is identified by

$$\sup_{y\in B_{c/\delta}} |\delta v^\delta(y,\omega) + \overline{H}(p)| \to 0$$

v^δ solves $\delta v^\delta - \mathrm{tr}(A(x,\omega)D^2v^\delta) + H(Dv^\delta + p, x,\omega) = 0$ in $\mathbb{R}^d$

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if A and H are replaced by $L-$ periodic $A_L(\cdot,\omega)$ and $H_L(\cdot,\omega)$, the effective Hamiltonian $\overline{H}_L(\cdot,\omega)$ is the unique constant for which

$$-\mathrm{tr}(A_L(x,\omega)D^2\chi) + H_L(D\chi + p, x,\omega) = \overline{H}_L(p,\omega) \text{ in} \mathbb{R}^d$$

has a a continuous, $L-$periodic solution χ

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it is possible to choose periodic A_L and H_L so that, as $L\to+\infty$, $\overline{H}_L(p,\omega)$ converges to $\overline{H}(p)$ locally uniformly in p and a.s. in ω

one direction of the convergence is based on the homogenization, while the other relies on the construction of subcorrectors to the periodic cell problem using approximate correctors for the original system

if a rate is known for the homogenization, then there is a rate for the a.s. convergence of the $\overline{H}_L(p,\omega)$'s to $\overline{H}(p)$

- it is necessary to take special care in choosing the A_L and H_L

taking $A_L = A$ and $H_L = H$ in $[-L/2, L/2)^d$ and extending them periodically cannot work

(i) for the L-problems to be well defined A_L and H_L need to be continuous

(ii) viscous HJB equations are very sensitive to large values of the Hamiltonians and, as a consequence, the H_L's must be substantially smaller than H at places where H and H_L differ

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$A \equiv 0$ and $H(p, x, \omega) = |p|^2 - V(x, \omega)$ with V bounded stationary ergodic $\Rightarrow$

$$\overline{H}(0) = \inf_{x \in \mathbb{R}^d} V(x, \omega)$$

if $H_L(p, x, \omega) = |p|^2 - V_L(x, \omega)$, with V_L $L-$periodic, then $\overline{H}_L(0, \omega) = \inf_{x \in \mathbb{R}^d} V_L(x, \omega)$

V_L cannot just be any regularized truncation of $V(\cdot, \omega)$, since it must satisfy the condition

$$\inf_{x \in \mathbb{R}^d} V_L(x, \omega) \to \inf_{x \in \mathbb{R}^d} V(x, \omega)$$

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define V_L in $[-(L-1)/2, (L-1)/2)^d$ as a smooth interpolation between $V_L = V$ in $[-(L-1)/2, (L-1)/2)^d$ and $V_L = 0$ on $\partial[-L/2, L/2)^d$ and extend V_L periodically

such approximation cannot always work because it implies $\overline{H}_L(0, \omega) \leq 0$ whatever the map V is

the approximation for Hamilton-Jacobi equations

H stationary ergodic, convex and coercive $\quad (C_1^{-1}|p|^\gamma - C_1 \leq H(p,x,\omega) \leq C_1|p|^\gamma + C_1)$

$\Rightarrow$ homogenization and $C_1^{-1}|p|^\gamma - C_1 \leq \overline{H}(p) \leq C_1|p|^\gamma + C_1$

the approximation for Hamilton-Jacobi equations

H stationary ergodic, convex and coercive $(C_1^{-1}|p|^\gamma - C_1 \leq H(p,x,\omega) \leq C_1|p|^\gamma + C_1)$

$\Rightarrow$ homogenization and $C_1^{-1}|p|^\gamma - C_1 \leq \overline{H}(p) \leq C_1|p|^\gamma + C_1$

$H_{L,\eta}(p,x,\omega) = (1-\zeta_\eta(x/L))H(p,x,\omega) + \zeta_\eta(x/L)H_0(p)$

$H_0(p) = C_2^{-1}|p|^\gamma - C_2$ is chosen so that $H_0(Dv^\delta + p) \leq \overline{H}(p)$ and $H_0(p) \leq \overline{H}(p)$

$\zeta_\eta : \mathbb{R}^d \to [0,1]$ is 1-periodic such that

$\zeta_\eta = 0$ in $Q_{1-2\eta}$, $\zeta_\eta = 1$ in $Q_1 \setminus Q_{1-\eta}$, $\|D\zeta_\eta\| \leq c/\eta$ and $\|D^2\zeta_\eta\| \leq c/\eta^2$

the approximation for Hamilton-Jacobi equations

H stationary ergodic, convex and coercive $\quad (C_1^{-1}|p|^\gamma - C_1 \le H(p,x,\omega) \le C_1|p|^\gamma + C_1)$

$\Rightarrow \quad$ homogenization and $C_1^{-1}|p|^\gamma - C_1 \le \overline{H}(p) \le C_1|p|^\gamma + C_1$

$H_{L,\eta}(p,x,\omega) = (1-\zeta_\eta(x/L))H(p,x,\omega) + \zeta_\eta(x/L)H_0(p)$

$H_0(p) = C_2^{-1}|p|^\gamma - C_2$ is chosen so that $H_0(Dv^\delta + p) \le \overline{H}(p)$ and $H_0(p) \le \overline{H}(p)$

$\zeta_\eta : \mathbb{R}^d \to [0,1]$ is 1-periodic such that

$\zeta_\eta = 0$ in $Q_{1-2\eta}$, $\zeta_\eta = 1$ in $Q_1 \backslash Q_{1-\eta}$, $\|D\zeta_\eta\| \le c/\eta$ and $\|D^2\zeta_\eta\| \le c/\eta^2$

Theorem

$\limsup_{L\to+\infty} \overline{H}_{L,\eta}(p,\omega) \le \overline{H}(p) \le \liminf_{L\to+\infty} \overline{H}_{L,\eta}(p,\omega) + C(|p|^\gamma + 1)\eta$
independence at finite range $\Rightarrow$ $\lim_{n\to+\infty} \overline{H}_{L_n,\eta_n}(p) = \overline{H}(p)$ *with rate*

it does not seem possible to let $\eta \to 0$ simultaneously with $L \to +\infty$

upper bound uses homogenization and that $H = H_{L,\eta}$ in $Q_{L(1-2\eta)}$

lower bound uses choice of H_0 and the periodic corrector to construct an approximate sub corrector of the random problem

non coercive problems - periodic setting

a periodic, $|\{a < 0\}| = \theta$ and $\begin{cases} u_t^\varepsilon + a(\frac{x}{\varepsilon})|Du^\varepsilon| = 0 & \text{in} \quad \mathbb{R}^d \times (0,\infty) \\ u^\varepsilon = u_0 & \text{on} \quad \mathbb{R}^d \times \{0\} \end{cases}$

then

$$\boxed{u^\varepsilon \rightharpoonup \theta u_0 + (1-\theta)\bar{u} \ \text{ in } \ L^\infty_{\text{loc}}(\mathbb{R}^d \times (0,\infty)) \text{ weak } *}$$

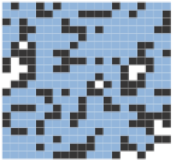
where

$$\begin{cases} \bar{u}_t = \overline{H}(D\bar{u}) & \text{in } \mathbb{R}^N \times (0,\infty) \\ \bar{u} = u_0 & \text{on } \mathbb{R}^N \times \{0\} \end{cases}$$

$\overline{H}(p) = \lim_{\delta \to 0} \bar{H}_\delta(p)$, $\qquad \overline{H}_\delta$ effective Hamiltonian for $\max(a,\delta)$

Cardaliaguet-Lions-Souganidis

classical Bernoulli percolation on the lattice $\mathbb{Z}^d$



each cube is painted white or black with probability p or $1-p$

when p is above some critical threshold $p > p_c$, there exists almost surely only one unbounded region (infinite white cluster)

$F(\omega)$(:= union of the black cubes) consists of mutually disjoint open connected black clusters $U_i^+(\omega)$

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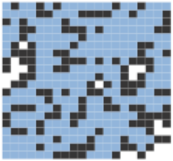

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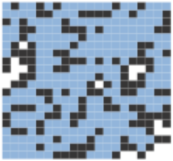

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$$u_{\varepsilon,t} + a(x/\varepsilon,\omega)|Du_\varepsilon| = 0 \qquad a(x,\omega) = \begin{cases} \min(d(x,\partial U_i^+(\omega)),1/2) & \text{if } x \in U_i^+(\omega) \\ -\min(d(x,\partial U_i^-(\omega)),1/2) & \text{if } x \in U_i^-(\omega) \end{cases}$$

the u^ε's converge weakly to $\bar{u} = (1-\theta)u_0 + \theta u_\infty$

u_∞ the solution corresponding to $\overline{H}_\infty$ associated to the infinite cluster and $\theta = \mathbb{P}[\mathcal{U}_\infty]$

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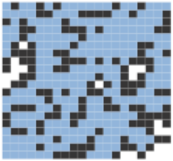

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when the configuration is a periodic checkerboard there is always trapping

a general result

$u_{\varepsilon,t} + a(x/\varepsilon,\omega)|Du_\varepsilon| = 0$ a bdd, Lip., stationary ergodic

for each ω $U_0(\omega) = \{a(\cdot,\omega) = 0\}$ and there exist connected disjoint sets $U_i(\omega)$ st

$\{a(\cdot,\omega > 0\} = \cup_{i\in I^+} U_i(\omega)$ and $\{a(\cdot,\omega < 0\} = \cup_{i\in I^-} U_i(\omega)$

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a.s. in ω each unbounded $U_i(\omega)$ is an infinite cluster, i.e., for $\delta > 0$ and each unit vector e there exist $R = R(\omega,\delta,e) > 0$ and random intervals (s_j,t_j) such that $|t_j - s_j| < R$ and
$I_e^\delta(\omega) := \{t > 0 : te \notin U_i^\pm(\omega)\} = \cup(s_j,t_j)$

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for each $i \in I^\pm$ let $\overline{H}_i$ be the effective Hamiltonian for the homogenization in $\varepsilon U_i(\omega)$

$$\boxed{u_\varepsilon \to \bar{u} = \theta_0 + \Sigma_{i\in I^\pm}\theta_i\bar{u}_i \text{ in } L^\infty_{\text{loc}}(\mathbb{R}^d \times (0,\infty)) \text{ weak } *}$$

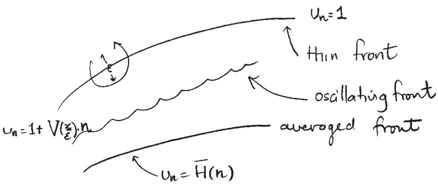
$\bar{u}_i$ is the solution to $\bar{u}_{i,t} + \overline{H}_i(D\bar{u}_i) = 0$ $\bar{u}(\cdot,0) = u_0$

$\overline{H}_i = 0$ if $U_i(\omega)$ is bounded

turbulent combustion I: the *G*- equation

a sharp interface model in turbulent combustion

thin front moving with normal velocity $V = 1 - < V(x/\varepsilon), n >$



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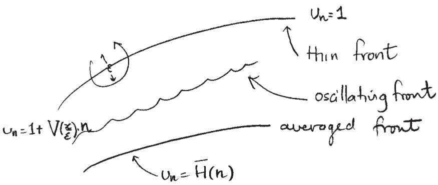

the G-equation is $\quad u_{\varepsilon,t} = |Du_\varepsilon| + < V(x/\varepsilon), Du_\varepsilon >$

"averaged" thin front moves with normal velocity $V = \overline{H}(n)$

is there enhancement?

non-coersive G-equation

$u_{\varepsilon,t} = |Du_\varepsilon| + < V(x/\varepsilon), Du_\varepsilon >$ in $\mathbb{R}^d \times (0, \infty)$

- $\|V\| < 1 \implies$ problem is coercive and homogenizes
- $\|V\| \geq 1 \implies$ problem is not coercive

homogenization may not be possible even in periodic setting

the u_ε's may not converge locally uniformly

the vector field "traps points"

to homogenize need an assumption on V (other than coercivity) that allows the controls to overcome the traps

the G-equation in periodic setting Nolen, Cardaliaguet and S.

$u_{\varepsilon,t} = |Du_\varepsilon| + < V(x/\varepsilon), Du_\varepsilon >$ in $\mathbb{R}^d \times (0, \infty)$

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$c_I \| \operatorname{div} V \|_{L^d(Q)} \leq 1$ (c_I is the isoperimetric constant in the cube Q)

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- enhancement

$\bar{H}(p) \;\geq |p|(1 - c_I \|\mathrm{div}\, V\|_{L^d(Q)}) + < \mathbb{E}(x\,\mathrm{div}\, V), p >$

if $\mathrm{div}\, V = 0$, $\bar{H}(p) = |p|$ if and only if $< V(x), p >= 0$ for all x

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the condition on V allows to obtain, in place of Lip bounds, a uniform control on the oscillation of the solutions to the approximate cell problem

$$\delta v_\delta = |Dv_\delta + p| + < V \cdot (Dv_\delta + p) >$$

the G-equation in random media Cardaliaguet-S.

$u_{\varepsilon,t} = |Du_\varepsilon| + < V(x/\varepsilon,\omega), Du_\varepsilon > \quad \text{in } \mathbb{R}^d \times (0,\infty)$

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V bounded, Lipshitz, stationary ergodic, $\mathbb{E}V = 0$ and $\operatorname{div} V = 0$

- homogenization

there exists a positively homogeneous (of degree one), Lip. continuous, convex, nonnegative effective $\overline{H}$ such that $\bar{H}(p) \geq |p|$

- enhancement

$\overline{H}(p) = |p|$ if and only if $< V(x, \omega), p >= 0$ for all x and a.s. in ω

new (old) strategy

$u_{\varepsilon,t} = |Du_\varepsilon| + < V(x/\varepsilon), Du_\varepsilon >$ in $\mathbb{R}^d \times (0,\infty)$ $\quad u_\varepsilon(\cdot,0) = u_0$

$u_\varepsilon(x,t,\omega) = \sup_{\alpha\in\mathcal{A}} u_0(X_t^{x,\alpha,\varepsilon,\omega}) = \sup_{y\in\mathcal{R}^\varepsilon(x,t,\omega)} u_0(y)$

for each $\alpha \in \mathcal{A} = L^\infty((0,\infty); B_1)$ and $x \in \mathbb{R}^d$, $X_t^{x,\alpha,\varepsilon,\omega}$ solves

$\dot{X}(s) = V(X(s)/\varepsilon,\omega) + \alpha(s) \quad X(0) = x$

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the long time behavior of $\mathcal{R}$ is determined by the minimal time function

$\theta(x,y,\omega) = \min\{t \geq 0 : y \in \mathcal{R}(x,t,\omega)\}$

natural setting for the subadditive ergodic theorem

$\theta(x, y, \omega) = \min\{t \geq 0 : y \in \mathcal{R}(x, t, \omega)\}$

- subadditivity and stationarity $\theta(0, (n+m)v, \omega) \leq \theta(0, nv, \omega) + \theta\,(0, mv, \tau_{nv}\,\omega)$
- no integrability, in general —– example with $\mathbb{E}[\theta(0, x, \cdot)] = \infty$

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- weak controlability pointwise bound

for all $\varepsilon \in (0, \varepsilon_0)$ and a.s. in ω, there exists $T(\omega, \varepsilon) > 0$ such that

$\theta(x, y, \omega) \leq T(\omega, \varepsilon) + \varepsilon|x| + (1+\varepsilon)|y - x|$ for all $x, y \in \mathbb{R}^d$

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there exists a positively homogeneous, convex $\bar{L} : \mathbb{R}^d \to \mathbb{R}$ such that $|q| \leq \bar{L}(q) \leq 1 + \|V\|$ and, a.s. in ω and for all $R > 0$,

$$\lim_{t\to\infty} \sup_{|p|\leq R, |x|\leq R} |t^{-1}\theta(tx, t(x+p), \omega) - \bar{L}(p)| = 0$$

$$\lim_{t\to\infty} t^{-1}\mathcal{R}(tx, t, \omega) = \{y \in \mathbb{R}^d : \bar{L}(x - y) \leq 1\}$$

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theorem $\Rightarrow$ $u_\varepsilon(x,t,\omega) \to \bar{u} = \sup\{u_0(y) : \bar{L}(x-y) \leq 1\}$

averaged Hamiltonian $\bar{H}(p) = \sup\{< p,q >: \bar{L}(q) = 1\}$

the way around the lack of integrability

for each $a \in B(0,1)$ find $\alpha \in \mathcal{A}$ so that the solution of the ode

$\dot{X}_t = V(X_t + \alpha(t) \quad X_0 = 0$ has, as $t \to \infty$ and a.s. in ω, the properties

(i) $t^{-1}X_t - a$ is small

(ii) $t^{-1}\theta(0, X_t, \omega)$ has a limit $\gamma(\omega)$

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(ii) follows from the subadditive theorem since $\theta(X_s, X_t, \omega) \leq M(t-s)$

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θ ergodicity and stationarity $\Rightarrow$

$\lim_{t\to\infty} t^{-1}\theta(0, ta, \omega)$ is independent of ω a.s.

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$u_{\varepsilon,t} = a(x/\varepsilon, t/\varepsilon)|Du_\varepsilon|$ in $\mathbb{R}^d \times (0,\infty)$ a is $\mathbb{Z}^{d+1}$ -periodic and $0 < \alpha \leq a \leq \beta$

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admissible curves $\mathcal{A}_{0,t} := \{\gamma : [0,t] \to \mathbb{R}^d : |\gamma'(r)| \leq a(\gamma(r), r)|$ ae in $[0,t]\}$

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controllability and subadditivity are a problem

the fix: forget about the space-time minimal time, treat the space-time periodicity separately and look at the spatial reachable sets directly the enlarged reachable set $\mathcal{R}_t(Q) = \cup_{x \in Q} \mathcal{R}_t(x)$

- controls the behavior of $\mathcal{R}_t(x)$
- is subadditive $\mathcal{R}_m(Q) \subset \mathcal{R}_k(Q) + \mathcal{R}_{m-k}(Q) - Q$

a subadditive theorem for convex sets $\Rightarrow$

there exists a compact, convex $D \subset \mathbb{R}^d$ such that $\lim_{m \to \infty} \frac{\mathrm{co}\mathcal{R}_m(Q)}{m} = D$

D is an upper bound for the long time behavior of $\mathcal{R}_(Q)$

in general is not possible to remove co from the ergodic limit of compact subadditive sets

use the space-time periodicity to design special paths to show that every $y \in D$ can be approximated by some $y_m \in \frac{\mathcal{R}_m(Q)}{m}$

fully nonlinear uniformly elliptic stationary ergodic

$$F(D^2 u_\varepsilon, x/\varepsilon, \omega) = 0 \text{ in } U \qquad u_\varepsilon = g \text{ on } \partial U$$

- homogenization with uniformly elliptic effective nonlinearity

F uniformly elliptic   Caffarelli-S.-Wang

F degenerate elliptic with $\mathbb{E}[\lambda^{-p}] < \infty$ for $p > d$ Armstrong-Smart

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- rate of convergence

strongly mixing media, i.e., at large distances the $F(\cdot,\cdot,x,\omega)$'s decorrelate

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- approximation of the ergodic constant by periodic ones

simpler than HJ-case; interpolation with any uniformly elliptic F_0 with the same ellipticity constants Cardaliaguet-S.

oscillatory boundary conditions

homogenize the Dirichlet problem $\begin{cases} F(D^2 u_\varepsilon) = 0 \text{ in } U \\ u_\varepsilon = g(\cdot/\varepsilon, \omega) \text{ on } \partial U \end{cases}$

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- nonlinear periodic setting Feldman

$\Gamma(U) := \{x \in \partial U : \nu_x \in \mathbb{R}\mathbb{Z}^d\}$

there exists $\beta_0(d, \lambda, \Lambda) > \max(0, \frac{\lambda}{\Lambda}(d-1) - 1)$ such that, if $\dim_{\mathcal{H}} \Gamma(U) < \beta_0$. there exists a continuous $\bar{g} = \bar{\mu}(g, \nu_x)$ such that the oscillatory Dirichlet problem homogenizes to one with Dirichlet condition $\bar{g}$

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β_0 determines the Hausdorff dimension of a subset of ∂U which is negligible

$\bar{\mu}$ is determined by homogenizing the Dirichlet problem with boundary a plane

- nonlinear random setting   Feldman-Kim-S.

assumptions on F

F uniformly elliptic with constants $0 < \lambda < \Lambda$ and 1-positively homogeneous

either $\lambda/\Lambda > d + 1/2d$ or F is convex or concave

assumptions on g

g stationary, bounded, Holder continuous and strongly mixing with rate ϕ st

$$\int_0^\infty \phi(r)^{1/2} r^{d-2} dr < \infty$$

homogenization of sets of exponentially small probability

isotropic diffusions in random environments — Sznitman-Zeitouni, Fehrman

$$\begin{cases} u_{\varepsilon,t} - \operatorname{tr} A(x/\varepsilon,\omega) D^2 u_\varepsilon + \frac{1}{\varepsilon} b(x/\varepsilon,\omega)\cdot u_\varepsilon = f(x, x/ep, \omega) \text{ in } \mathbb{R}^d \times (0,\infty) \\ u_\varepsilon = g \text{ on } \mathbb{R}^d \times \{0\} \end{cases}$$

assumptions

- A uniformly elliptic, A, b, f stationary
- finite range independence
- $(a(rx,\omega, b(rx,\omega))_{x\in\mathbb{R}^d}$ has the same law as $(ra(x,\omega)r^{\mathbb{T}}, rb(x,\omega))_{x\in\mathbb{R}^d}$ have the same law for any rotation r
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results

for η sufficiently small, there exists $\alpha > 0$ st

- homogenization with $f = f(x)$ to $\bar{u}_t - \alpha\Delta\bar{u} = f$ Sznitman-Zeitouni
- homogenization with $f = f(x, x/\varepsilon, \omega)$ to $\bar{u}_t - \alpha\Delta\bar{u} = \bar{f}$ Fehrman
- a.s. existence of invariant measure Fehrman
- Liouville property for strictly sublinear solutions Fehrman

some examples about non periodic perturbations of periodic environments — Lions-S.

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- uniformly elliptic

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