

On the Optimization of Riemann-Stieltjes-Control-Systems with Application in Vehicle Dynamics

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On the Optimization of Riemann-Stieltjes-Control-Systems with Application in Vehicle Dynamics

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Content

Problem Description

Impulsive Multi-Body-Systems

Riemann-Stieltjes-Control-Systems

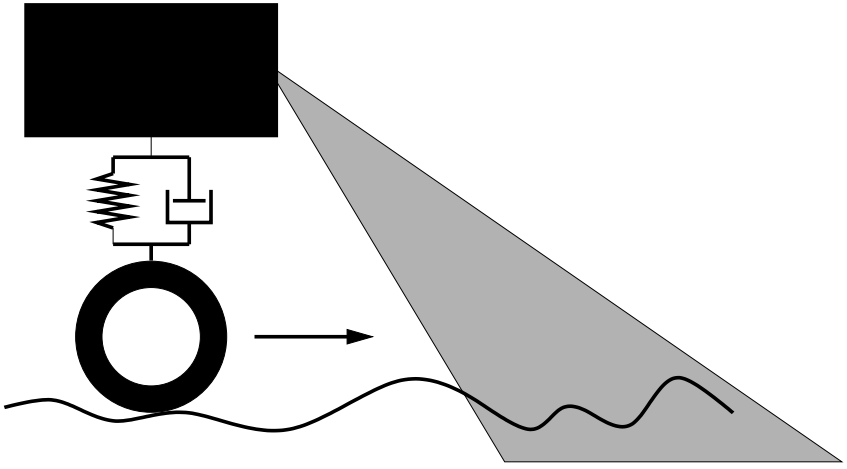
Optimization of Riemann-Stieltjes-Control-Systems

Numerical Examples

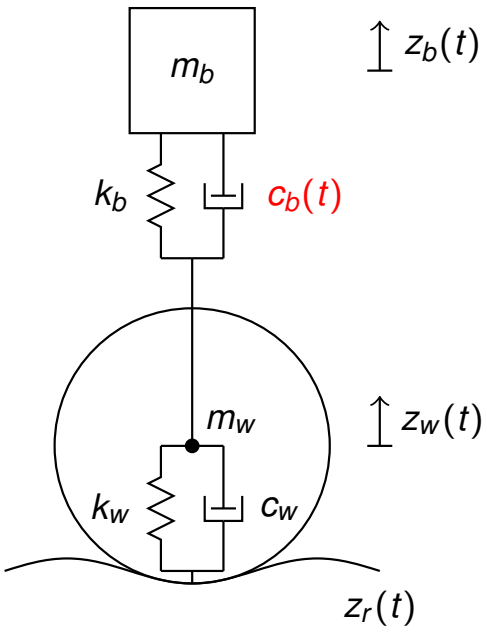
Proactive–Chassis–Control

Setting:

- Input: Vetical Road Model delivered e.g. by Preview Sensors
  $z_r(t)\ t \in [0, T]$
- Cost Function: Comfort and safety characteristics
  (acceleration body, road-to-wheel-force,...)
- Controls: Damping coefficients
  (electrorheological dampers, ...)



Classical Model



Classical quartercar model

- two masses (m_w, m_b)
- spring–damper connection between body (z_b) and wheel (z_w)
- spring–damper connection between wheel and road (z_r)
- k_b, k_w, c_w are constants
- $c_b \in [c_{min}, c_{max}]$ is the control

Model delivers twice continuously differentiable state

$\Rightarrow$ Mechanical optimal control problem (easy to solve)

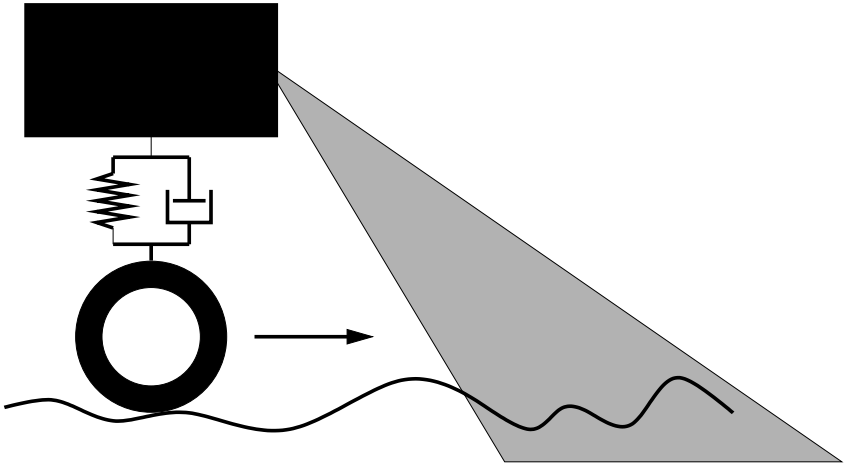
Optimal Control Problem (OCP)

$$\min_{u \in \mathbb{U}[0,T]} \quad J(u) = \int_0^T \mu_b \, F_{bw}^2 + \mu_c \, F_{wr}^2 \, dt$$

$$s.t. \qquad \begin{pmatrix} m_b \ddot{z}_b \\ m_w \ddot{z}_w \end{pmatrix} = \begin{pmatrix} F_{bw} \\ F_{wr} - F_{bw} \end{pmatrix}$$

$$z_b(0) = z_{b0}, z_w(0) = z_{w0}, v_b(0) = v_{b0}, v_w(0) = v_{w0}$$

$$z_r, v_r \in C^1[0, T]$$

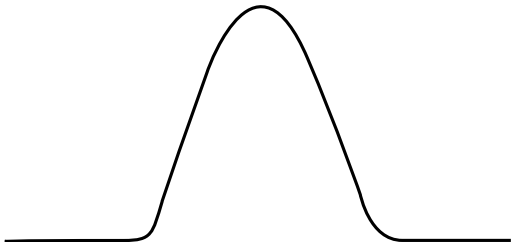


Singular events

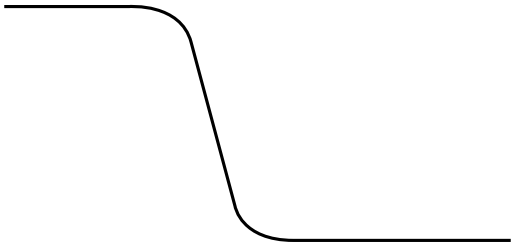
What if the road undergoes rapid changes?

Examples:

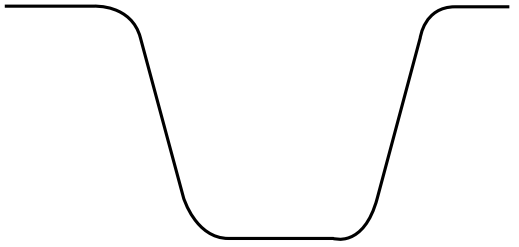
Speedbump



Step



Pothole

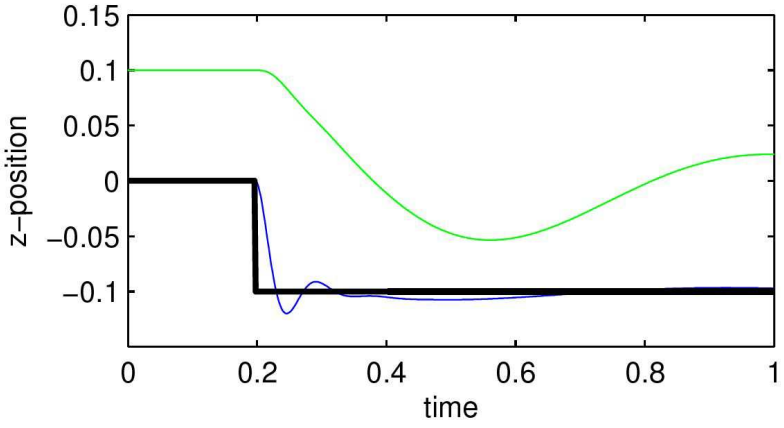


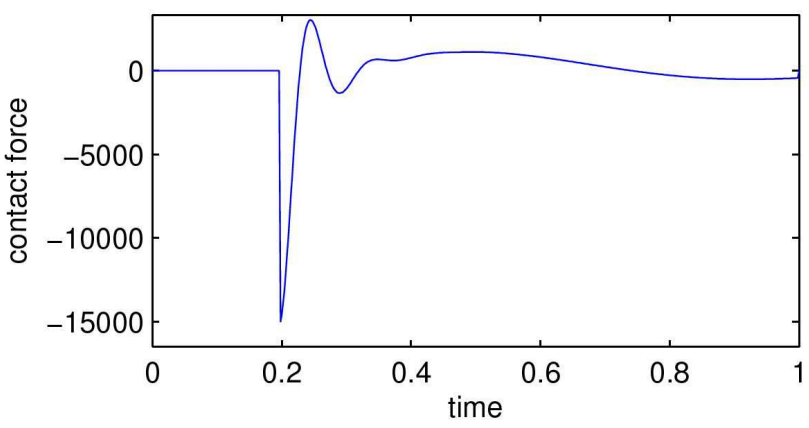
Definition (Singular Event)

A singular event is an individual, short and rapid change in the road surfaces, with a deflection amount that is considerably higher than the common road disturbances

Singular events

Behaviour of the classical model at singular events





Negative contact force at the transition

Problem: Model does not incorporate the possible occuring contact loss

Way out: Remove spring–damper connection between wheel and road.

Contact formulation

Possibilities:

- Full FEM-Formulation (pde-ode-coupling)
- Commercial Tire Libraries (Fraunhofer CDTire)
- Reduced Formulation using Hertzian stress (tyre deflection approximated by Hertzian stress)

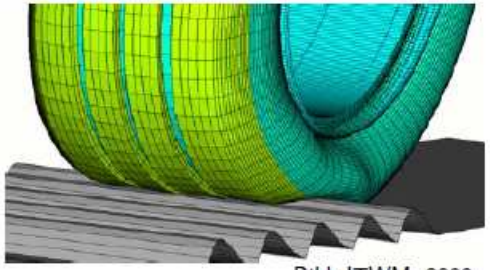
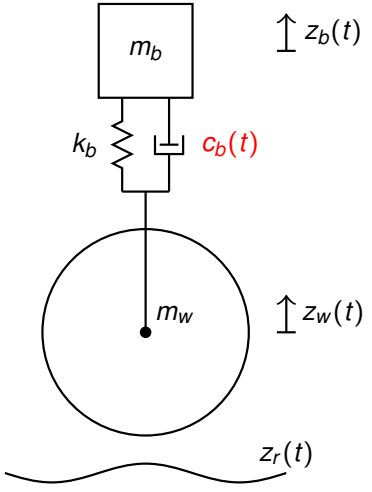


Bild: ITWM, 2009

or...

- Masspoint impact model using coefficients of restitution (COR)



Impact model with COR

Requirements

- Wheel above road

$$(z_w - z_r) \geq 0$$

- Contact force positive

$$F_i \geq 0$$

- Complementarity condition

$$(z_w - z_r)F_i = 0$$

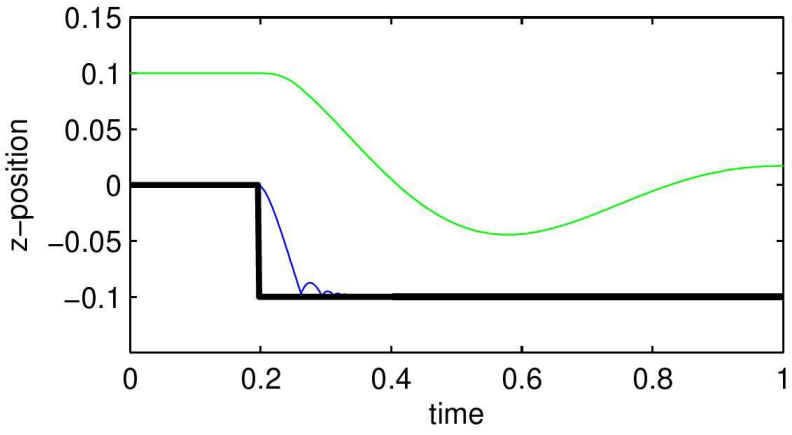
- Newtonian impact equation with coefficient of restitution ε

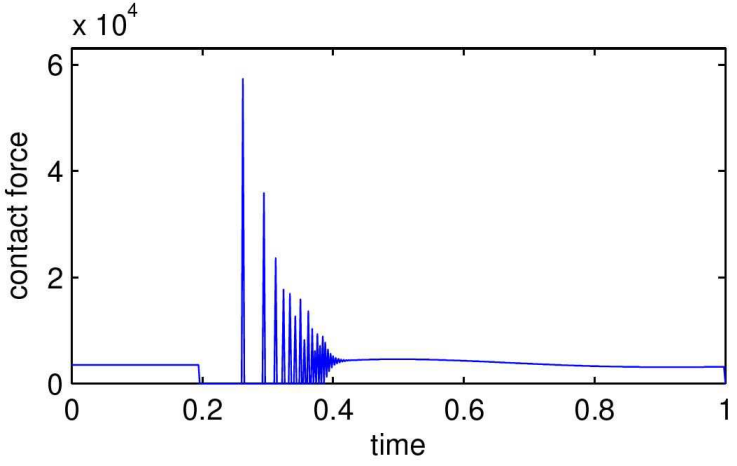
$$F_i(v_w^+ + \varepsilon v_w^-) = 0$$

Behaviour of a system with an impact at time τ, ($F_i(\tau) > 0$):

$$v(\tau^+) = -\varepsilon v(\tau^-) \quad \text{for} \quad z_w(\tau^-) = z_r(\tau^-), v_w(\tau^-) \leq 0$$

Impact model with COR





Challenges:

- Contact detection
- Incorporate friction
- Control optimization

Objective: Closed Formulation of impact model with COR to obtain sensitivity information

Impulsive Multi-Body-Systems

Regarding the system

$$M(q)\ddot{q} + b(q, \dot{q}) + g(q) = F_i(q)$$

with

- $q \in \mathbb{R}^n$ – generalized coordinates
- $\dot{q} \in \mathbb{R}^n$ – generalized velocities
- $b(q, \dot{q}) \in \mathbb{R}^n$ – coupling forces
- $g(q) \in \mathbb{R}^n$ – weight forces
- $F_i(q)$ – impuls and contact forces

Transformation to first order system with $x = [q, \dot{q}]^\top$:

$$\dot{x} = F(x) + F_i(x).$$

- $F(x)$ – continuous Forces of the system
- $F_i(x)$ – large impact Force for the short contact time

Impulsive update

Systemformulation using

- Continuous dynamic

$$\dot{x}(t) = F(x(t)) \quad \text{contact free}$$

- Jump condition for velocities

$$\Delta x(\tau) = S(x(\tau)) \geq 0 \quad \text{if contact occurs}$$

$\Rightarrow$ hybrid system formulation

Transformation into an integral equation

$$x(t) = x_a + \int_a^t F(x(\tau))d\tau + \int_a^t d\alpha(x)(\tau)$$

leads to a Riemann-Stieljes-Integral

Integral System Equation

Definition

Let $\alpha(x)(t) \in BV([a,b],\mathbb{R}^n)$, $F : \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ und $G : \mathbb{R}^{2n} \to (\mathbb{R}^{2n} \times \mathbb{R}^n)$. Let $x : [a,b] \to \mathbb{R}^n$ be the function that solves the Integral equation

$$x(t) = x_a + \int_a^t F(x(\tau))d\tau + \int_a^t G(x(\tau))d\alpha(x)(\tau)$$
$$x_a = x(a)$$

for $a \leq t \leq b$. This x is called an impulsive system.

- Without contact it holds $d\alpha(x)(\tau) = 0$
- At contact: Determine $G(x(\tau))$ and $d\alpha(x)(\tau)$, the impact condition holds.

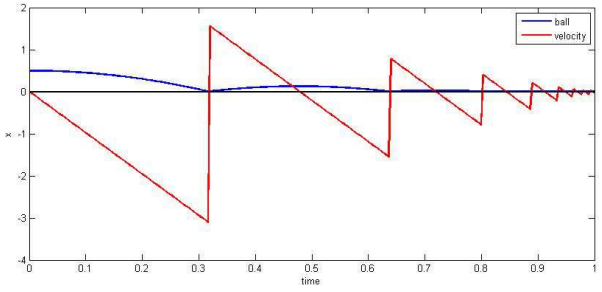
Theorem

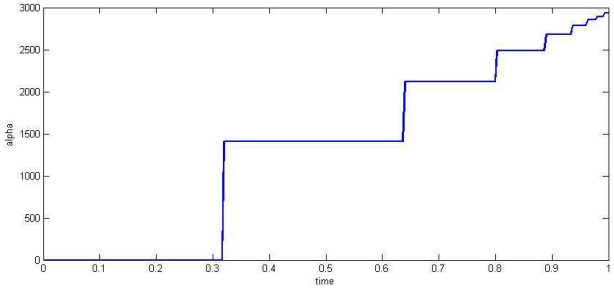
If G is continuous on $[a,b]$ and $\alpha \in BV([a,b],\mathbb{R}^n)$, the Riemann-Stieltjes-Integral exists.

Example: Jump condition – 1D

For the 1D problem with horizontal contact surface the jump condition writes as:

$$d\alpha(x)(\tau) = \begin{cases} 0 & x(\tau) > 0 \\ 0 & x(\tau) = 0 \ \& \ v(\tau) \geq 0 \\ (1+\varepsilon)v(\tau^-) & \text{otherwise} \end{cases} \qquad G := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$





Remark

Let α be defined as above, then it holds $\alpha \in BV([a,b],\mathbb{R}^n)$ [vgl. Natanson 1977]

Generalization

To Do: Determination of α for 3D contacts with Coulomb-Friction

General contacts means:

- Multi-body-system with n bodies and given transformation into state space $\xi_i(q), i = 1, \ldots, n$
- Parametrized contact surfaces $c_j, j = 1, \ldots, l$ with admissable region ν_j.

Then at contact point it holds:

$$v_{ij}^{c+} = E_{ij} v_{ij}^{c-} \quad \text{with} \quad E = diag(e_{ij}^n, e_{ij}^t, e_{ij}^{t'})$$

Given: COR in normal direction e_{ij}^n

Umknown: COR in tagential directions $e_{ij}^t, e_{ij}^{t'}$

Update with friction

Collecting active Contact points, one derives:

Contact Optimization Problem (COP)

$$\min_{e_{ij}^k,\,(i,j)\in I_A,\,k\in t,t'} \quad K^+(E) = \dot{q}(\tau^+)^\top M \dot{q}(\tau^+)$$

$$\begin{aligned}
\text{s.t.} \qquad \dot{q}(\tau^+) &= (\mathbb{I} - \bar{\mathcal{J}}\mathcal{J} + \bar{\mathcal{J}}E\mathcal{J})\dot{q}(\tau^-) \\
\boldsymbol{p}(E) &= (\mathcal{J}M^{-1}\mathcal{J}^\top)^{-1}(E - \mathbb{I})\mathcal{J}\dot{q}(\tau^-) \\
-p_{ni}(E) &\leq 0 \\
\|(p_{ti}(E), p_{t'i}(E))\|_2^2 &\leq (\mu(i,j)p_{ni}(E))^2 \quad \text{(Coulomb-Friction)}
\end{aligned}$$

Quadratic optimization problem with nonlinear constraint

$$d\alpha(x)(\tau) = \dot{q}(\tau^+) - \dot{q}(\tau^-)$$

Example - Extended Bouncing Ball

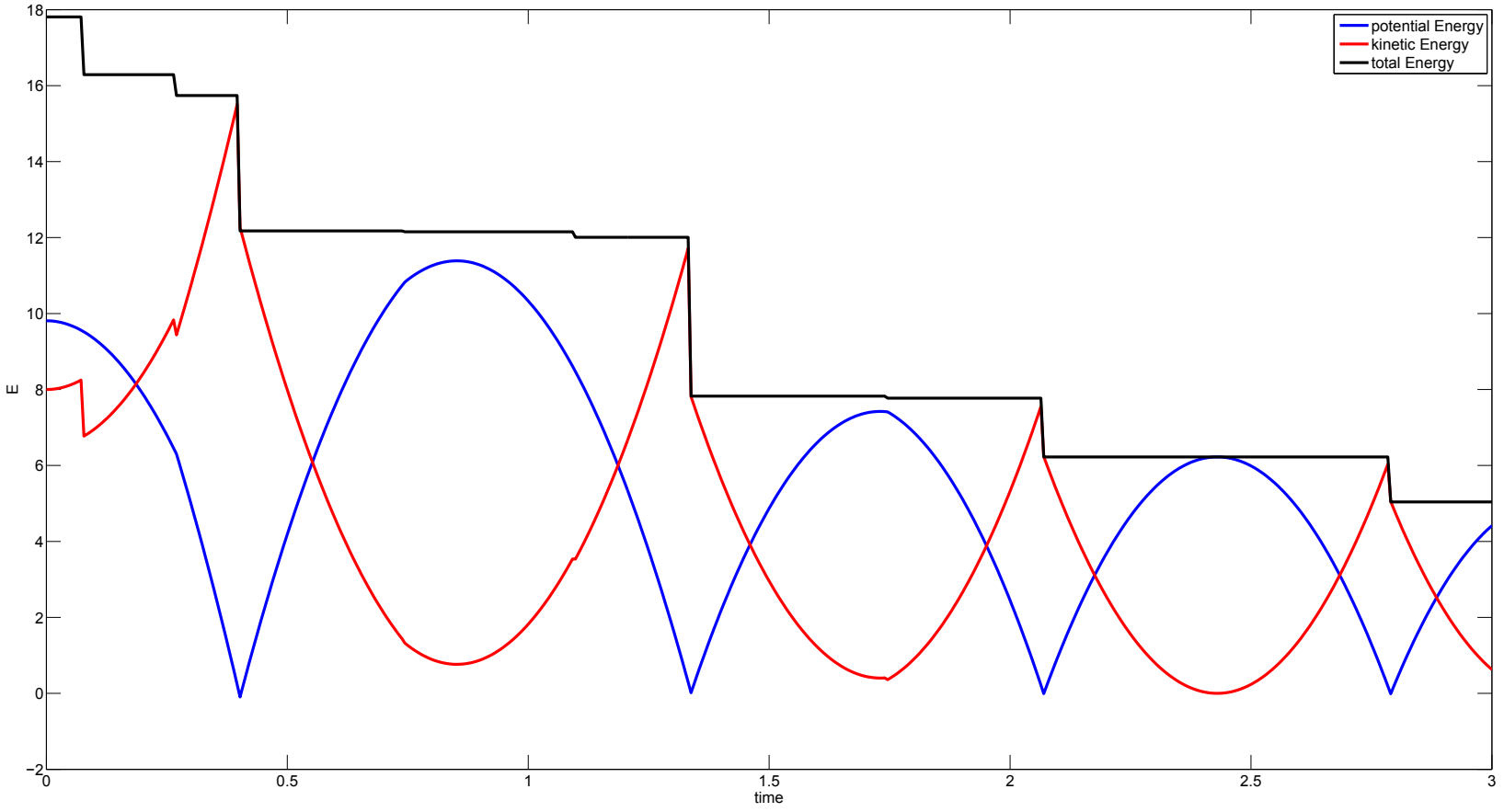


Figure: Energy of the Ball

Extension: Riemann-Stieltjes-Control-Systems

Definition

Let $\alpha(x(u))(t) \in BV([a,b],\mathbb{R}^n)$, $F : ([a,b] \times \mathbb{R}^{2n} \times \mathbb{U}^{n_u}) \to \mathbb{R}^{2n}$ and $G : ([a,b] \times \mathbb{R}^{2n}) \to (\mathbb{R}^{2n} \times \mathbb{R}^n)$. If the function $x : [a,b] \to \mathbb{R}^n$ solves the integral equation

$$x(u)(t) = x_a + \int_a^t F(x(u)(\tau), u)d\tau + \int_a^t G(x(u)(\tau))d\alpha(x(u))(\tau)$$

$$x_a = x(a)$$

for $a \leq t \leq b$ it is called the solution of an impulsive system with control

Remark:

- No impulsive control

Existence of sensitivities

Goal: Calculate Sensitivities w.r.t. control to obtain information for optimization.

Existence of sensitivities

Let an impulsive System with control be given on the Interval $[0, T]$. F and G are supposed to be Lipschitz on $[0, T]$. Let $\alpha(x(u))(\tau), \tau \in (0, T)$, be an impact point given by the switching function $\varsigma(x(u)(t))$. ς has a real zero crossing at $t = \tau$.
It holds

$$\tau(u) = \operatorname*{argmin}_{s \in (0,T)} \left\{ \varsigma(x(u)(s)) = 0 \mid \frac{d}{dt}\varsigma(x(u)(s)) \neq 0 \right\}$$

.
Then there exists an $S(u)$ for that the limit

$$\lim_{\|\varepsilon\| \to 0} \frac{\|x(u+\varepsilon) - x(u) - S(u)\varepsilon\|_\infty}{\|\varepsilon\|_\infty} = 0$$

exists.

Sketch of the proof

- Map the interval $[0, \tau(u)], [\tau(u), T]$ onto $[0, 1], [1, 2]$
- Due to the properties of ς exists $\frac{d\tau(u)}{du}$ in a neighborhood of u
- With the decomposition property of functions of bounded variation it holds

$$\begin{aligned}
&\|x(u+\varepsilon) - x(u) - S(u)\varepsilon\|_\infty \\
&= \|x_c(u+\varepsilon) + x_s(u+\varepsilon) - x_c(u) - x_s(u) - S_c(u)\varepsilon - S_s(u)\varepsilon\|_\infty \\
&\leq \|x_c(u+\varepsilon) - x_c(u) - S_c(u)\varepsilon\|_{\infty,[0,1]} + \|x_c(u+\varepsilon) - x_c(u) - S_c(u)\varepsilon\|_{\infty,[1,2]} \\
&\quad + \|x_s(u+\varepsilon) - x_c s(u) - S_s(u)\varepsilon\|_\infty
\end{aligned}$$

- Estimate the summands with mean value theorem and the lemma of Gronwall.

Optimisation of RCS

- Additional state x_J for continuous costs
- Control-discretization with N equdistant controlgrid $\mathcal{G}_u^N$
- Subdiscretization with M substeps $\mathcal{G}_s^{N \cdot M}$
- Statevector $x(u) \in \mathbb{R}^{2n+1} \times \mathbb{R}^{M+1}$, Controlvector $u = \{u_1, ..., u_M\}$,
- Calculation of the sensitivity matrix using differential-quotient

$$S(u)_{(\cdot,j)} = \frac{x_J(u + \delta u_j) - x_J(u)}{\delta} \quad j = 1, ..., M$$

and the gradient

$$\nabla J(x(u)) = S(u)^\top \dot{x}_J(u)$$

- Update Method using projected gradient-method with Armijo line-search.

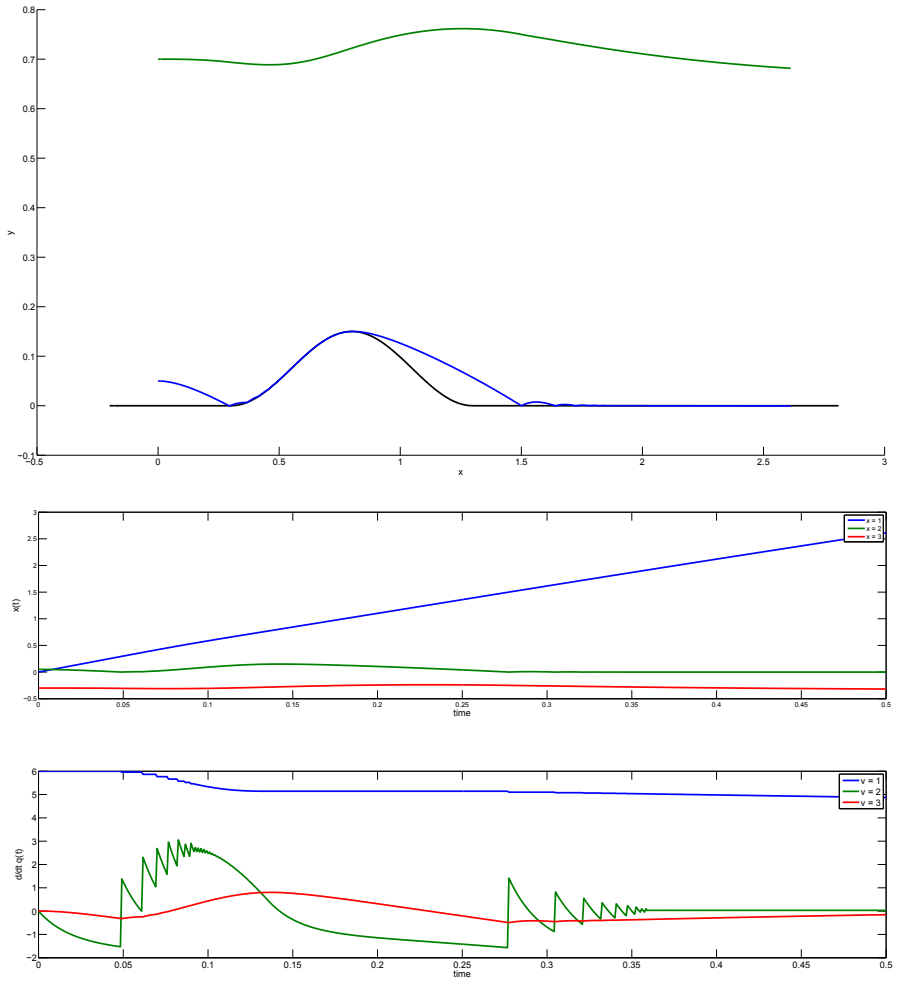
Quartercar - An Example

Quartercar-Model with impact formulation.

$$x(t) = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ x_J \end{pmatrix} \hat{=} \begin{pmatrix} \text{x-position} \\ \text{y-position tire} \\ \text{y-position body} \\ \text{x-velocity} \\ \text{y-velocity tire} \\ \text{y-velocity body} \\ \text{continuous costs} \end{pmatrix}$$

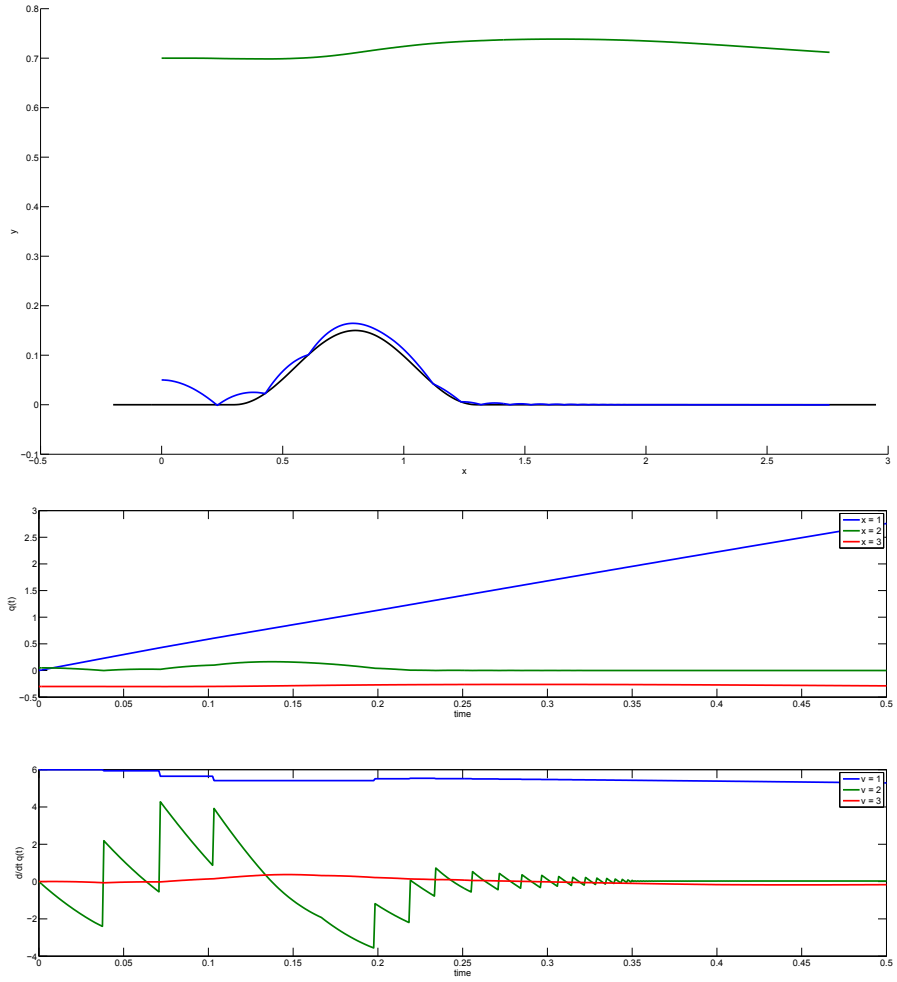
$$x(t, x_0, u) = x_0 + \int_{t_0}^{t} \begin{pmatrix} x_4(\tau) \\ x_5(\tau) \\ x_6(\tau) \\ 0 \\ -gm_w + c_2(x_2 - x_1) + u(x_4 - x_3) \\ -gm_b - c_2(x_2 - x_1) - u(x_4 - x_3) \\ (-gm_b - c_2(x_2 - x_1) - u(x_4 - x_3))^2 \end{pmatrix} d\tau + \int_{t_0}^{t} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} d\alpha(x(\tau)),$$

Initial Trajectory



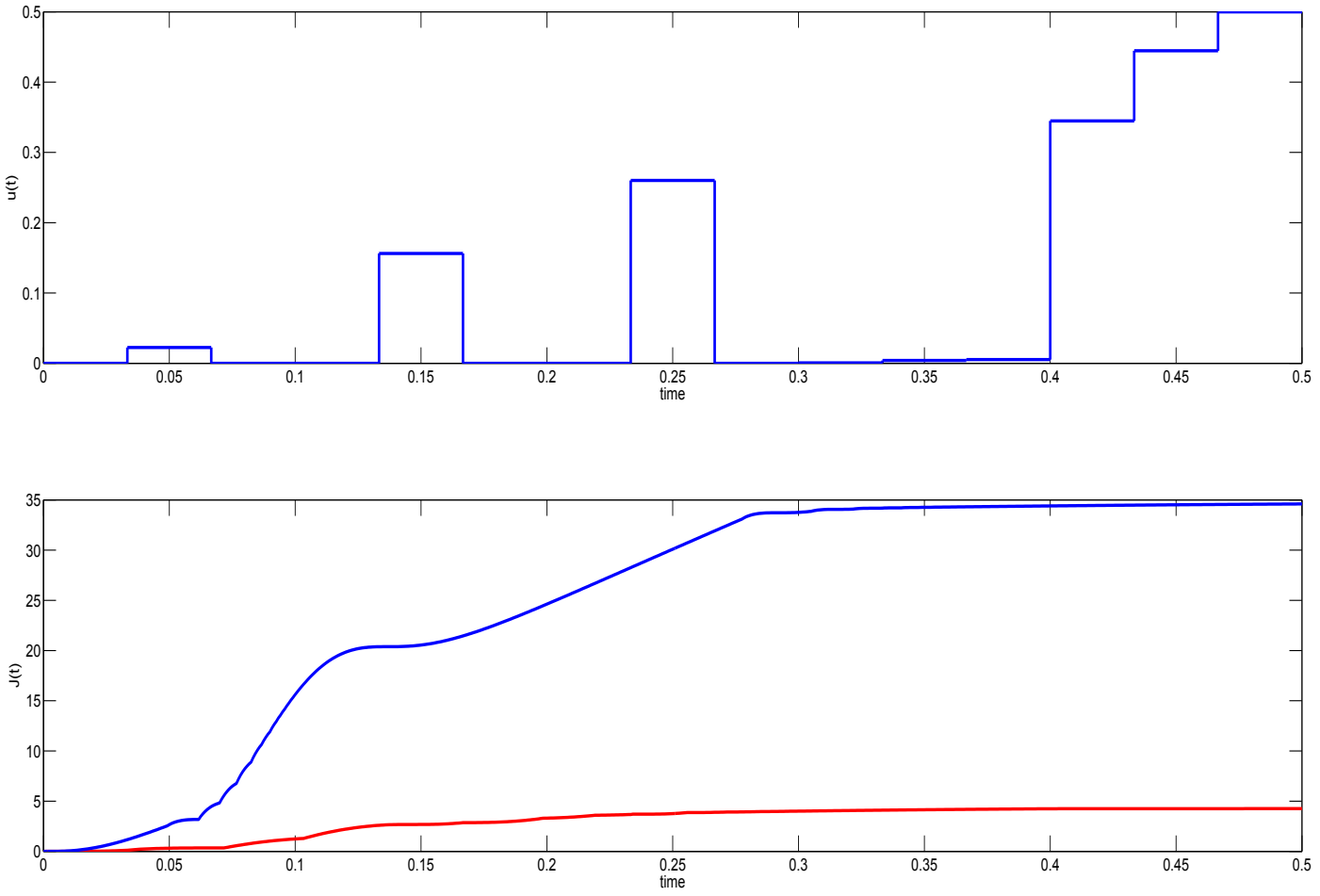
Top: Statespace, bottom: generalized coordinates

Optimized Trajectory



Top: Statespace, bottom: generalized coordinates

Cost-Comparison



Top: optimale control, bottom: cost function for initial and optimized control

Example - Quarter-Car-Animation



Conclusion

We demonstrated

- Modelling of impulsive multi-body-systems using Riemann-Stieltjes-Integrals
- Energy-consistent calculation of impacts using generalized COR
- Optimization with Gradient-Method

Open questions:

- How about points of contact? (only Hölder-continuous)
- Modelling of safety aspets in cost function
- Comparison to full finite element calculations