Fourth order energy-preserving localy implicit discretization for linear wave equations

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A family of fourth order energy-preserving locally implicit schemes is introduced for linear wave equations. The domain of interest is decomposed into several regions where different fourth order time discretization are used, chosen among a family of implicit or explicit fourth order schemes derived in [1]. The coupling is based on a Lagrangian formulation on the boundaries between the several non conforming meshes of the regions. A global discrete energy is shown to be preserved and leads to global fourth order consistency in time. Numerical results illustrate the good behavior of the schemes and their potential for realistic highly heterogeneous cases or strongly refined geometries (as the Marmousi 2D test case of figure 1), for which using everywhere an explicit scheme can be extremely penalizing because the time step must respect the stability condition adapted to the smallest element or the highest velocities. Accuracy up to fourth order reduces the numerical dispersion inherent to implicit methods used with a large time step, and makes this family of schemes attractive compared to [3] which is only second order accurate in time. The presented technique could be an alternative to local time stepping [2] provided that some limitations are overcame in the future : treatment of dissipative terms, non trivial boundary conditions, coupling with a PML region, fluid structure coupling . . .

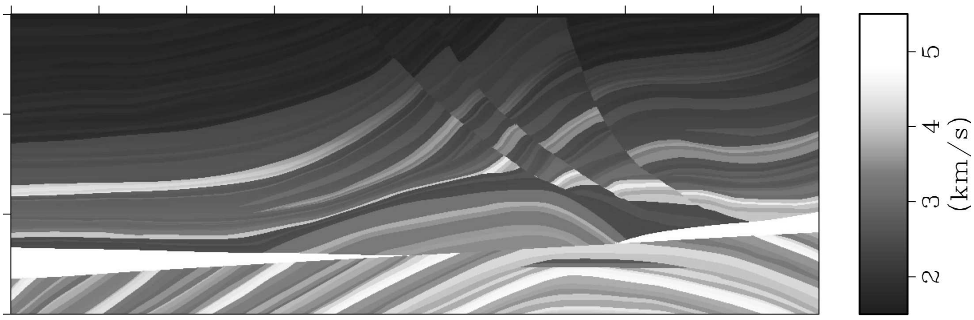

Figure 1: Marmousi 2D benchmark domain presenting high velocity contrasts and refined geometry.

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