Evolutionary Games for Multiple Access Control: From Egoism to Altruism

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***Abstract*—This paper studies multiple access games with a large population of mobiles decomposed into several groups. Mobiles interfere with each others through many local interactions. We assume that each mobile (or player) cooperates with his group by taking into account the performance of his group. The degree of cooperation not only covers the fully non-cooperative behavior and the fully cooperative behavior, but also the fully altruistic behavior. To do so, we make use of the evolutionary game theory which we extend to cover this kind of behavior. We define and characterize the equilibrium (called Evolutionary Stable Strategy) for these games and establish the optimal level of cooperation that maximizes the probability of successful transmission. We also study the game dynamics both in its classical form and in the presence of delay. Interestingly, we show that, in order to maximize the system performance, the mobiles should be less cooperative.**

I. Introduction

Wireless networks are growing increasingly less structured, adopting many of the characteristics of ad-hoc networks. However, the dynamic interactions arising in these networks make it difficult to analyze and predict system performance, thereby inhibiting the development of wireless technologies. Game theory with several concepts, provides a powerful mathematical framework that can accommodate the preferences and requirements of various stakeholder in a given process. Recently, there has been a surge in research activities that employ game theory to model and analyze the performance of various networks, such as communication networks, neural networks, computer networks, social networks, biologically inspired networks, etc.

Evolutionary game theory has became a central tool for predicting and even designing evolution in many fields. Its origins come from biology where it was first introduced by [1], [2] to model conflicts among animals. It differs from classical game theory by (i) its focusing on the evolution dynamics of a fraction of members of the population that use a given strategy, and (ii) in the notion of Evolutionary Stable Strategy (ESS, [1]) which includes robustness against a deviation of a whole (possibly small) fraction of the population that may wish to deviate (this is in contrast with the standard Nash equilibrium concept that only incorporates robustness against deviation of a single user). Although ESS has been initially defined in the context of biological systems, more and more applications can be found studying multiple access protocols [3], [4], multihoming [5], delay tolerant networks [6], evolution of transport protocols [7] and resources competition in the Internet [8].

In this paper, we consider a large population of sensors or relays that are deployed in a large area to detect the occurrence of a specific event. We assume that the population is decomposed into different groups. We consider an Aloha system in which mobiles interfere with each other through many local interactions (e.g., access points, throwboxes) where the collision can happen if more than one mobile transmit a packet in the same time slot. In particular, we consider that each node seeks to maximize some combination between its own performance and the performance of its group. We study the impact of cooperation in the context of multiple access control in many possible behaviors such as altruist behavior and fully non-cooperative behavior. In many problems, assumption about selfishness or rationality has been often questioned by economists and other sciences. Many research works shown that even in a simple game and controlled environment, individuals do not act selfishly. They are rather either altruistic or malicious. Several explanations have been considered for such behavior of players. Fairness reasons are argued by Fehr [9] to consider the joint utility model, while reciprocity among agents are considered in [10]. Cooperation among users, often referred to as altruism, are discussed in [11], [12], [13], [14]. Some of the models in [15] argues that the partial altruism mimics closely users' behavior often observed in practice.

In this paper, we present a new model for evolutionary games which takes into account both the altruism and selfishness of agents. Firstly, we begin by defining this new concept, driving it in several ways and exploring its major characteristics. The major focus of this paper is to study how the level of cooperation impacts the profile of population as well as the global performance of the system. Our theoretical results unveil some behaviors. More specifically, we show that when all users increase their level of cooperation, then the performance of the system is not necessary improved. In fact, for some scenarios, the performance of groups may lead to an improvement by adopting selfishness instead of altruism. This happened when the density of nodes is high. For low density, the degree of cooperation may indeed improve the performance of all groups.

[‡] This work has been partially supported by the European Commission within the framework of the CONGAS project FP7-ICT-2001-8-317672

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The paper is structured as follows. We first formalize in the next section the system model. We then present, in Section III, the evolutionary game model that includes the cooperation aspect. In Section IV, we compute the expression of the ESS. We study in section V the replicator dynamics in the classical and delayed forms. In Section VI, we proceed to some optimization issues through the analysis of the probability of success. Section VII shows some numerical investigations on the equilibrium, the probability of success and the replicator dynamics. Section VIII concludes the paper.

II. System Model

Consider an Aloha system composed of a large population of mobiles (or sensors) operating in a low traffic condition. Mobiles are randomly deployed over a plane and each mobile may interact with a mobile in its group or with a mobile in another group. The channel is assumed to be ideal for transmission and all errors are only due to collision. A mobile decides to transmit a packet or not to transmit to a receiver when they are within transmission range of each other. Interference occurs as in the Aloha protocol: if more than one neighbor of a receiver transmit a packet at the same time, then there is a collision.

We assume, in particular, that we can ignore cases of interaction in which more than two sensors or relays transmit simultaneously causing interference to each others. An example where we may expect this to hold is when sensors are deployed in a large area to monitor the presence of some events, e.g., in Delay Tolerant Networks (DTNs) where the network is assumed to be sparse and the relay density is low. Under this setting, communication opportunities arise whenever two nodes are within mutual communication range because of the mobility pattern.

The size of each group of mobiles G_i is denoted by α_i with $\sum_{i=1}^N \alpha_i = 1$. Let μ be the probability that a mobile i has its receiver $R(i)$ within its range. When a mobile i transmits to $R(i)$, all mobiles within a circle of radius R centred at $R(i)$ cause interference to the node i for its transmission to $R(i)$. This means that more than one transmission within a distance R of the receiver in the same slot induce a collision and the loss of mobile i packet at $R(i)$. Accordingly, each mobile has two actions: either to transmit T or to stay silent S. A mobile of group G_i may use a mixed strategy $\mathbf{p}_i = (p_i, 1-p_i)$ where p_i is the probability to choose the action T. If a mobile transmits a packet, it incurs a transmission cost of Δ. The packet transmission is successful if the other users do not transmit (stay silent) in that given time slot. If a mobile transmits successfully a packet, it gets a reward of V. We suppose that the payoff V is greater than the cost of transmission, i.e., $\Delta < V$.

III. Utility Functions

As already mentioned, we study a new aspect of evolutionary games for multiple access games where each mobile cooperates with other mobiles of his group in order to improve the performance of his group. Let β be the degree of cooperation. The utility of a tagged mobile choosing action a within group G_i is a convex combination of the utility of his group and his own utility, namely

$$U^i_{\text{user}}(a, \mathbf{p}_{-i}) = \beta U^i_{\text{group}}(a, \mathbf{p}_{-i}) + (1-\beta) U^i_{\text{self}}(a, \mathbf{p}_{-i}) \quad (1)$$

where U_{group} is the utility of the group to which the tagged player belongs and U_{self} is the individual utility of that player.

When the mobile plays T, resp. S, the utility of the group is given resp. by

$$\begin{aligned} U^i_{\text{group}}(T, p_i, p_{-i}) &= \mu\Big[\gamma + (1-\gamma)\sum_{j=1}^N \alpha_j(1-p_j)\Big] \\ U^i_{\text{group}}(S, p_i, p_{-i}) &= \mu(1-\gamma)\alpha_i p_i \end{aligned}$$

with γ being the probability that a mobile is alone in a given local interaction. Analogously, the selfish utility when the mobile chooses strategy T is

$$U^i_{\text{self}}(T, p_i, p_{-i}) = \mu\Big[(1-\triangle) - (1-\gamma)\sum_{j=1}^N \alpha_j p_j\Big]$$

while the selfish utility of user i when playing S is zero, namely

$$U^i_{\text{self}}(S, p_i, p_{-i}) = 0$$

Combining the above results, the utility of a mobile of class i using strategy T is given by

$$U^i_{\text{user}}(T, p_i, p_{-i}) = \mu\Big[1 - (1-\beta)\Delta - (1-\gamma)\sum_{j=1}^N \alpha_j p_j\Big],$$

while the utility of a mobile of class i when he plays S is

$$U^i_{\text{user}}(S, p_i, p_{-i}) = \mu\beta(1-\gamma)\alpha_i p_i$$

IV. Computing the ESS

In evolutionary games, the most important concept of equilibrium is the ESS, which was introduced by [16] as a strategy that, if adopted by most members of a population, it is not invadable by mutant strategies in its suitably small neighbourhood. In our context, the definition of ESS is related to the robustness property inside each group. To be evolutionary stable, the strategy $\mathbf{p}^*$ must be resistant against mutations in each group. There are two possible interpretations of $\epsilon-$ deviations in this context:

1) A small deviation in the strategy by all members of a group. If the group G_i plays according to strategy $\mathbf{p}^*_i$, the $\epsilon-$ deviation, where $\epsilon \in (0,1)$, consist in a shift to the group's strategy $\bar{\mathbf{p}}_i = \epsilon \mathbf{p}_i + (1-\epsilon)\mathbf{p}^*_i$;
2) The second is a deviation (possibly large) of a small number of individuals in a group G_i, that means that a fraction ϵ of individuals in G_i plays a different strategy $\mathbf{p}_i$.

After mutation, the average of a non-mutant will be given $U^i_{\text{user}}(p^*_i, \epsilon p_i + (1-\epsilon)p^*_i, p^*_{-i})$. Analogously, we can construct the average payoff of mutant $U^i_{\text{user}}(p_i, \epsilon p_i + (1-\epsilon)p^*_i, p^*_{-i})$. A strategy $\mathbf{p}^* = (p_1^*, p_2^*, \ldots, p_N^*)$ is an ESS if $\forall\ i$ and $p_i \neq p_i^*$, there exists some $\epsilon_i \in (0,1)$, which may depend on p_i, such that for $\epsilon \in (0, \epsilon_i)$

$$U^i_{\text{user}}(p_i^*, \epsilon p_i + (1-\epsilon)p_i^*, p_{-i}^*) > U^i_{\text{user}}(p_i, \epsilon p_i + (1-\epsilon)p_i^*, p_{-i}^*) \quad (2)$$

Equivalently, p^* is an ESS if and only if it meets best reply conditions:

- Nash equilibrium condition:
$$U^i_{\text{user}}(p_i^*, (p_i^*, p_{-i}^*)) > U^i_{\text{user}}(p_i, (p_i^*, p_{-i}^*))$$
- Stability condition:
$$\text{If } \mathbf{p_i} \neq \mathbf{p}_i^*, \text{ and } U^i_{\text{user}}(p_i^*, (p_i^*, p_{-i}^*)) = U^i_{\text{user}}(p_i, (p_i^*, p_{-i}^*))$$
$$\text{then } U^i_{\text{user}}(p_i, (p_i, p_{-i}^*)) < U^i_{\text{user}}(p_i^*, (p_i, p_{-i}^*))$$

A. Characterization of the equilibria

In this section, we provide the exact characterization of the equilibria induced by the game. We distinguish pure ESS equilibria and mixed ESS. Before studying the existence of ESS, we introduce some definitions needed in the sequel.

Definition 1.

- *A fully mixed strategy $\mathbf{p}$ is the strategy when all actions for each group have to receive a positive probability, i.e., $0 < p_i < 1\ \forall i$.*
- *A mixer (pure) group i is the group that uses a mixed (pure) strategy $0 < p_i < 1$ (resp. $p_i \in \{0,1\}$).*
- *An equilibrium with mixed and non mixed strategies is an equilibrium when there is at least a pure group and a mixer group.*

Proposition 1 characterizes the condition on the existence of a fully mixed ESS.

Proposition 1.

1) *For $\gamma < 1 - \frac{1-(1-\beta)\Delta}{(\beta+N)\min \alpha_i}$ then there exists a unique fully mixed ESS $p^* = (p_i^*)_{i=1,\ldots,N}$, where*
$$p_i^* = \frac{1-(1-\beta)\Delta}{\alpha_i(\beta+N)(1-\gamma)}$$
2) *For $1 - \frac{1-(1-\beta)\Delta}{(\beta+N)\min\alpha_i} \leq \gamma \leq 1 - \frac{1-(1-\beta)\Delta}{1+\beta \max \alpha_i}$, then there exists a unique ESS with mixed and non mixed strategies.*
3) *For $\gamma > 1 - \frac{1-(1-\beta)\Delta}{1+\beta\max\alpha_i}$, then there exists a fully pure ESS where all groups play pure strategy T.*

Proof:

1) *From the definition of ESS in (2), we have $\forall i \in \{1, \ldots, N\}$,*
$$(p_i - p_i^*)\Big[U^i_{user}(T, \epsilon p_i + (1-\epsilon)p_i^*, p_{-i}^*) - U^i_{user}(S, \epsilon p_i + (1-\epsilon)p_i^*, p_{-i}^*)\Big] < 0$$
Thus
$$(p_i - p_i^*)\Big[1 - (1-\beta)\Delta - (1-\gamma)\sum_{j=1}^N \alpha_j p_j^* - (1-\gamma)\beta\alpha_i p_i^*\Big] + \epsilon(p_i - p_i^*)^2\Big[-(1-\gamma)(1+\beta)\alpha_i\Big] < 0$$
The mixed Nash equilibrium is obtained when the first term of the previous inequality is strictly negative. While $(p_i - p_i^*)$ can be positive or negative for $p_i^* \notin \{0,1\}$, the following equation holds
$$1 - (1-\beta)\Delta - (1-\gamma)\sum_{j=1}^N \alpha_j p_j^* - (1-\gamma)\beta\alpha_i p_i^* = 0$$
By summing this equation from 1 to N, we get
$$N(1-(1-\beta)\Delta) - N(1-\gamma)\sum_{j=1}^N \alpha_j p_j^* - (1-\gamma)\beta\sum_{i=1}^N \alpha_i p_i^* = 0$$
and so
$$\sum_{j=1}^N \alpha_j p_j^* = \frac{N(1-(1-\beta)\Delta)}{(1-\gamma)(\beta+N)}$$
Finally, we deduce
$$p_i^* = \frac{1-(1-\beta)\Delta}{\alpha_i(1-\gamma)(\beta+N)}$$
Hence p^* is fully mixed Nash equilibrium if $\gamma < 1 - \frac{1-(1-\beta)\Delta}{(\beta+N)\min\alpha_i}$. Furthermore, since $-(1-\gamma)(1+\beta)\alpha_i) < 0$, the stability condition is always satisfied which implies that p^* is an ESS. This complete the proof of (1).
2) *Without loss of generality, we assume that the sizes of groups are ordered as follows: $\alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_N$. We assume that for a given value of γ we have n_T groups playing strategy T and $N - n_T$ groups play mixed strategy. Hence the profile of population becomes $(T, \ldots, T, p_{n_T+1}, \ldots, p_N)$. For mixed group i ($i \in \{n_T+1, \ldots, N\}$), we have the following relation*
$$1 - (1-\beta)\Delta - (1-\gamma)\sum_{j=1}^{n_T}\alpha_j - (1-\gamma)\cdot \sum_{j=n_T+1}^{N}\alpha_j p_j - (1-\gamma)\beta\alpha_i p_i = 0$$
Thus, a strategy of a mixer group is given by
$$p_i^* = \frac{1-(1-\beta)\Delta - (1-\gamma)\alpha_T}{(1-\gamma)\alpha_i(\beta+N-n_T)}$$
where $\alpha_T = \sum_{j=1}^{n_T}\alpha_j$. For pure groups playing T, the following inequality holds: $\forall i \in \{1, \ldots, n_T\}$
$$1 - (1-\beta)\Delta - (1-\gamma)[\alpha_T + \alpha_i(\beta+N-n_T)] \geq 0$$

which completes the proof of 2).

3) *Assume that $\forall i \in \{1, \dots, N\}$, the group i transmit all the time ($p_i^* = 1$). The Nash equilibrium conditions become : $\forall i \in \{1, \dots, N\}$,*

$$\begin{aligned} 1-(1-\beta)\Delta-(1-\gamma)(1+\beta\alpha_i) > 0 \\ \Rightarrow \gamma > 1-\frac{1-(1-\beta)\Delta}{1+\beta\alpha_i} \end{aligned} \quad (3)$$

and the the proof is complete.

■

The previous proposition claims that an increased network density results in more transmission which leads to more collision. It also states that, in order to avoid collision between mobiles belonging to the same group, the cooperation degree tends to decrease the probability of transmission within the same group.

Proposition 2. *At the Nash equilibrium, there is no group playing pure strategy S.*

Proof: Let p^* be the ESS. By contradiction, suppose that there exists a group k playing the strategy S at ESS, i.e., $p_k^* = 0$. The Nash equilibrium condition for group k becomes

$$p_k\Big[1-(1-\beta)\Delta-(1-\gamma)\sum_{j=1, j\neq k}^{N}\alpha_j p_j^*\Big] < 0$$

$$\Rightarrow 1-(1-\beta)\Delta-(1-\gamma)\sum_{j=1, j\neq k}^{N}\alpha_j p_j^* < 0$$

If all groups use pure strategy S at ESS, the last condition becomes

$$1-(1-\beta)\Delta < 0$$

But this contradicts our assumptions on β and Δ. Hence there exists at least a group l playing strategy p_l^* such that $p_l^* \in]0,1]$. The Nash equilibrium condition is expressed as following:

$$\begin{aligned} (p_l - p_l^*)\Big[1-(1-\beta)\Delta-(1-\gamma)\cdot \\ \sum_{j=1, j\neq k}^{N}\alpha_j p_j^* - (1-\gamma)\beta\alpha_l p_l^*\Big] < 0 \end{aligned}$$

Thus

$$1-(1-\beta)\Delta-(1-\gamma)\sum_{j=1, j\neq k}^{N}\alpha_j p_j^* \geq (1-\gamma)\beta\alpha_l p_l^* \quad (4)$$

Combining conditions (3), (4) and $p_l^* > 0$, we get $(1-\gamma)\beta\alpha_l <$ 0 which is a contradiction. This completes the proof. ■

V. REPLICATOR DYNAMICS

Evolutionary games study not only equilibrium behavior but also the dynamics of competition. We introduce the replicator dynamics which describe the evolution in groups of the various strategies. Replicator dynamic is one of the most studied dynamics in evolutionary game theory. In this dynamic, the frequency of a given strategy in the population grows at a rate equal to the difference between the expected utility of that strategy and the average utility of group i. Hence, successful strategies are more likely to spread over the population.

In this paper, we study the replicator dynamics for the case of two groups. The general case of N groups will be handled in a future work.

A. Replicator dynamics without delay

The proportion of mobiles in the a group i programmed to play strategy T, denoted p_i, evolves according to the replicator dynamic equation given by:

$$\dot{p}_i(t) = p_i(t)[U^i_{\text{user}}(T, p_i(t), p_{-i}(t)) - \bar{U}^i_{\text{user}}(p_i(t), p_{-i}(t))] \quad (5)$$

where

$$\begin{aligned} \bar{U}^i_{\text{user}}(p_i(t), p_{-i}(t)) &= p_i(t)U^i_{\text{user}}(T, p_i(t), p_{-i}(t)) \\ &+ (1-p_i(t))U^i_{\text{user}}(S, p_i(t), p_{-i}(t)) \end{aligned}$$

Thus

$$\begin{aligned} \dot{p}_i(t) = p_i(t)(1-p_i(t))\Big[1-(1-\beta)\Delta-(1-\gamma)\cdot \\ \sum_{j=1}^{2}\alpha_j p_2(t) - (1-\gamma)\beta\alpha_i p_i(t)\Big] \end{aligned} \quad (6)$$

By expressing Equation (6) for $i = 1, 2$, we obtain a system of two non-linear ordinary differential equations (ODEs) in (6). There are several stationary points in which at least one group playing a pure strategy and a unique interior stationary point $p^* = (p_1^*, p_2^*)$ with $0 < p_i^* < 1$. The interior stationary point corresponds to the fully mixed ESS given by Proposition 1 and it is the only stationary point at which all mixed strategies coexist. Assuming that the state space is the unit square and that p^* exists, the dynamic properties of this equilibrium point are brought out in the next theorem.

Theorem 1. *The interior stationary point p^* is globally asymptotically stable in the replicator dynamics.*

Proof: The proof is based on a linearization of the system of non linear ODEs around p^*. We introduce a small perturbation around p^* defined by $x_i(t) = p_i(t) - p_i^*$ for $i = 1, 2$. Keeping only linear terms in x_i, we obtain the following linearized replicator dynamics:

$$\begin{aligned} \dot{x}_i(t) \approx \rho_i\Big[1-(1-\beta)\Delta-(1-\gamma)\sum_{j=1}^{2}\alpha_j(x_j(t)+p_j^*) \\ -(1-\gamma)\beta\alpha_i(x_i(t)+p_i^*)\Big] \end{aligned} \quad (7)$$

with $\rho_i = p_i^*(1-p_i^*)$ and p^* is the interior stationary point of the ODE system. Equation (7) becomes

$$\dot{x}_i(t) \approx -(1-\gamma)\rho_i\Big[\sum_{j=1}^{2}\alpha_j x_j(t) + \beta\alpha_i x_i(t)\Big] \quad (8)$$

This linearized system is of the form $\dot{X}(t) = AX(t)$ where

$$A = -(1-\gamma)\begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}\begin{pmatrix} \alpha(1+\beta) & 1-\alpha \\ \alpha & (1-\alpha)(1+\beta) \end{pmatrix}$$

We note that the previous system is asymptotically stable if the eigenvalues of the matrix A has negative real parts. In order to investigate the eigenvalues of the matrix A, we express the following characteristic polynomial of A:

$$\chi_A = det(\lambda I_2 - A) = \lambda^2 - tr(A)\lambda + det(A)$$

Hence, the determinant and the trace of the matrix B are given resp. by

$$\begin{aligned} det(A) &= (1-\gamma)^2\rho_1\rho_2\alpha(1-\alpha)\Big[(1+\beta)^2 - 1\Big] \\ &= (1-\gamma)^2\rho_1\rho_2\alpha(1-\alpha)\beta(\beta+2) \end{aligned}$$

and

$$tr(A) = -(1-\gamma)(1+\beta)\Big(\rho_1\alpha + \rho_2(1-\alpha)\Big)$$

The discriminant of this polynomial is: $D = tr(A)^2 - 4 \cdot det(A)$. Let λ_1 and λ_2 be two eigenvalues of A. Thus $\lambda_1 + \lambda_2 = tr(A)$ and $\lambda_1\lambda_2 = det(A)$. Since $det(A) \geq 0$ and $tra(A) \leq 0$, it easy to check that the real parts of λ_1 and λ_2 are negative. Hence, the interior fixed point p^* is asymptotically stable in the replicator dynamics. ■

B. Replicator Dynamics with delay

In the classical replicator dynamics, the fitness of strategy a at time t has an instantaneous impact on the rate of growth of the population size that uses it. A more realistic alternative model for replicator dynamic would be to introduce some delay: a mobile belonging to group i perceives the fitness about his group utility after a given delay τ. Hence, the group utility acquired at time t will impact the rate of growth τ time later. Under this assumption, the replicator dynamics equation for the group i is given by:

$$\dot{p}_i(t) = p_i(t)(1-p_i(t))\Big[1-(1-\beta)\Delta - (1-\gamma)\cdot \sum_{j=1}^{N}\alpha_j(\beta p_j(t-\tau) + (1-\beta)p_j(t)) - (1-\gamma)\beta\alpha_i p_i(t-\tau)\Big] \quad (9)$$

Similarly to the non-delayed case, we proceed to the linearization of the replicator dynamics equations by introducing a small perturbation around the interior equilibrium p_i^* defined by $x_i(t) = p_i(t) - p_i^*$. We get the following ODEs system:

$$\dot{x}_i(t) \approx -(1-\gamma)\rho_i\Big[\sum_{j=1}^{N}\alpha_j\Big(\beta x_j(t-\tau) + (1-\beta)x_j(t)\Big) + \beta\alpha_i x_i(t-\tau)\Big] \quad (10)$$

The Laplace transform of the system (10) is given by:

$$[\lambda + (1-\gamma)\rho_i\alpha_i\beta e^{-\tau\lambda}]X_i + (1-\gamma)\rho_i(1-\beta+\beta e^{-\tau\lambda}) \cdot \sum_{j=1}^{N}\alpha_j X_j = 0$$

For the case of two groups, the characteristic equation of the ODEs system is given by:

$$\lambda^2 + \lambda(1-\gamma)(1-\beta+2\beta e^{-2\tau\lambda})(\alpha\rho_1 + (1-\alpha)\rho_2) + (1-\gamma)^2\rho_1\rho_2\alpha(1-\alpha)\Big[2\beta(1-\beta)e^{-\tau\lambda} + 3\beta^2 e^{-2\tau\lambda}\Big] = 0 \quad (11)$$

The zero solution of the linearized system above is asymptotically stable if and only if all solutions of the corresponding characteristic equation (11) have negative real parts. The form of this equation was studied in [17]. The mixed intermediate ESS is an asymptotically stable state in the time-delayed replicator dynamics if and only if
$\tau < \tau_0 = \min(\frac{\pi}{2|\lambda_+|}, \frac{\pi}{2|\lambda_-|})$, with λ_+ and λ_- the roots of the non-delayed characteristic equation ($\tau = 0$). Remember that, according to the proof of Theorem 1, the eigenvalues of the differential system have negatives real parts.

VI. Optimization Issues

According to the structure of the ESS, we try to evaluate the performance of the global system in order to derive the optimal degree of cooperation β. The performance of the system can be presented by the measure of the probability of success in a given local interaction for a mobile randomly selected from all mobiles of all groups. This probability of success is expressed as follows

$$P_{succ} = \gamma\sum_{j=i}^{N}\alpha_i p_i^* + (1-\gamma)\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j p_i^*(1-p_j^*) + p_j^*(1-p_i^*)$$

Let us now study the expression of the probability of success depending on the structure of the game model considered.

A. Fully Mixed ESS

In the fully mixed ESS, the equilibrium is given $\forall i \in \{1,\ldots,N\}$ by

$$p_i^* = \frac{1-(1-\beta)\Delta}{\alpha_i(\beta+N)(1-\gamma)}$$

This gives a probability of success

$$P_{succ} = \frac{N(1-(1-\beta)\Delta)}{(1-\gamma)(\beta+N)^2} \cdot \Big[2(\beta + N(1-\beta)\Delta) - \gamma(\beta+N)\Big]$$

Having this expression, we calculate the level of cooperation β that maximizes the P_{succ}. We find that P_{succ} is maximized for

$$\beta^* = \frac{(4\Delta - \gamma - 2)N}{4N\Delta + \gamma - 2}$$

B. Pure-Mixed ESS

We note that in this structure, there are n_T groups usi pure strategy T at the equilibrium and the other $N - r$ groups using mixed strategies. Then, the probability of succe is expressed by

$$P_{succ} = (\alpha_T + \sum_{j=n_T+1}^{N} \alpha_i p_i^*)$$
$$\cdot \Big(2 - \gamma - 2(1-\gamma)(\alpha_T + \sum_{j=n_T+1}^{N} \alpha_i p_i^*)\Big)$$

with

$$p_i^* = \frac{1 - (1-\beta)\Delta - (1-\gamma)\alpha_T}{(1-\gamma)\alpha_i(\beta + N - n_T)}$$

Finally

$$P_{succ} = \frac{\beta\alpha_T(1-\gamma) + (N-n_T)(1-(1-\beta)\Delta)}{(\beta + N - n_T)^2}$$
$$\cdot \Big[\beta(1-2\alpha_T) + N - n_T$$
$$+ \frac{2(N-n_T)(1-\beta)\Delta + \beta - (N-n_T)}{(1-\gamma)}\Big]$$

We notice here, that $P_{succ}(\beta)$ depends on both n_T and α These two variables are step functions of β. The optim value β^* which maximizes P_{succ} for each value of γ will computed through an iterative algorithm.

C. Fully Pure ESS

When all groups play pure strategy T at the equilibrium, i.e. $p_i^* = 1 \ \forall i \in \{1, \ldots, N\}$

$$\begin{aligned} P_{succ} &= \sum_{i=1}^{N} \alpha_i p_i^* \Big(2 - \gamma - 2(1-\gamma)\sum_{i=1}^{N}\alpha_i p_i^*\Big) \\ &= \gamma \end{aligned}$$

VII. Numerical Results

A. Impact of the transmission cost on the equilibrium

We first investigate the case where two groups compete to access to the medium with $\alpha_1 = \alpha = 0.2$ and $\alpha_2 = 0.8$. In a sparse environment (corresponding to a high value of γ), an anonymous mobile of group G_i is more likely to be alone when transmitting to a destination. This suggests that he will play the strategy T all the time ($p^* = 1$). However, in a dense environment (low values of γ), he is more likely to be in competition with another mobile while transmitting to the destination. In this situation, the strategy played by the mobile i will depend on the cost of transmission Δ. In Figure 1, we consider a low transmission cost ($\Delta = 0.2$). We found that the mobile gives less interest to the effect of collision as the cost of transmitting is very low. In fact, loosing a packet does not affect the mobile's utility compared to what he would earn if the transmission is successful. This fact justifies the aggressive behavior of the mobile. However, when the transmission cost Δ is high, the equilibrium structure differs. In Figure 2, we consider a higher cost ($\Delta = 0.9$). In this case, the mobile becomes more cautious and take into account the effect of collision since it degrades his utility. Thereby, he lowers his level of transmission.

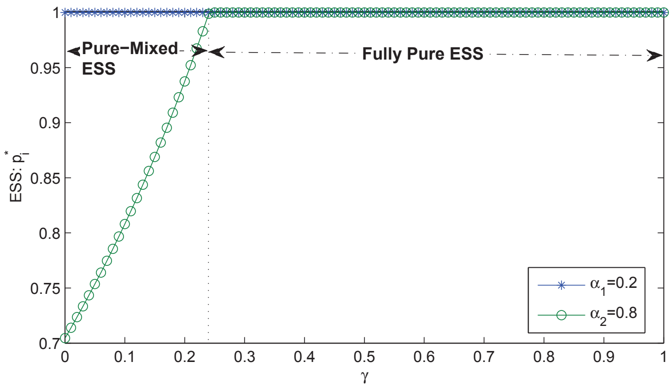

Fig. 1. Evolution of the ESS as a function of γ for $\Delta = 0.2$ and $\beta = 0.1$.

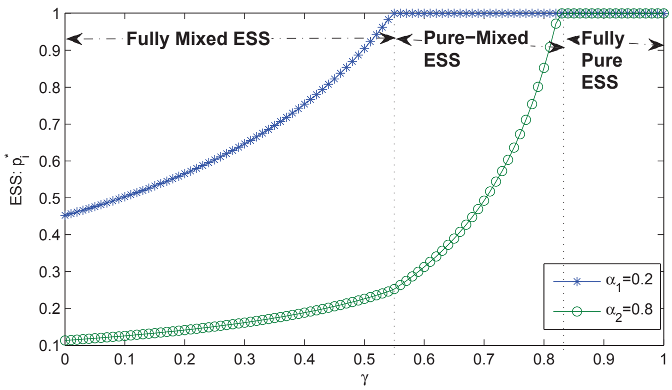

Fig. 2. Evolution of ESS as a function of γ for $\Delta = 0.9$ and $\beta = 0.1$.

B. Impact of the cooperation on the equilibrium

In Figure 3, we keep a high level of transmission cost ($\Delta = 0.9$) and we change the behaviour of the mobiles. We change the degree of cooperation to pass from a nearly egoistic behavior with $\beta = 0.1$ to a nearly altruistic behaviour with $\beta = 0.9$. We notice that by increasing the degree of cooperation, users have more incentive to use strategy T. This suggests that increasing the degree of cooperation among users induces a coordination pattern in which users have more incentive to use strategy T.

C. Impact of the cooperation and the transmission cost on the probability of success

In this section, we investigate the evolution of the P_{succ} according to γ. Intuitively, we can expect that when the mobiles are fully cooperative inside groups, this leads to a better system performance. However, we will show that we obtain a different result. We consider the same system as previously: two interacting groups, the smaller with proportion

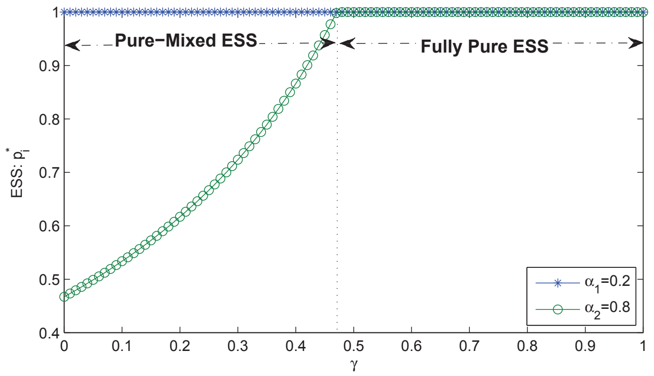

Fig. 3. Evolution of ESS as a function of γ for $\Delta = 0.9$ and $\beta = 0.9$.

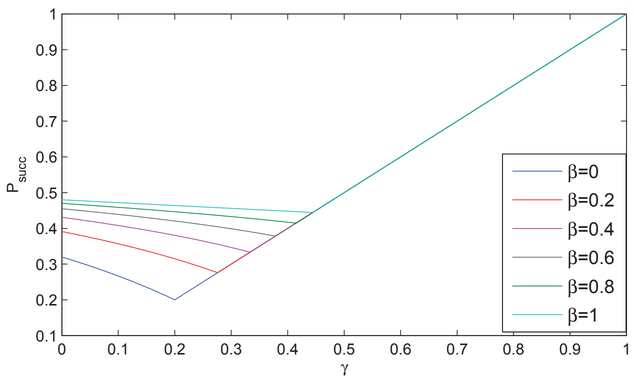

Fig. 4. Variation of P_{succ} with different levels of β for $\Delta = 0.2$.

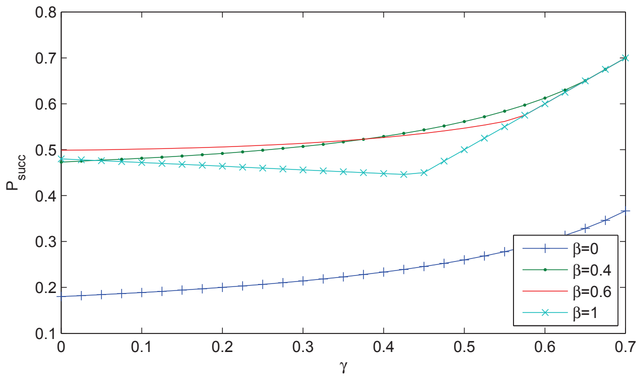

Fig. 5. Variation of P_{succ} with different levels of β for $\Delta = 0.9$.

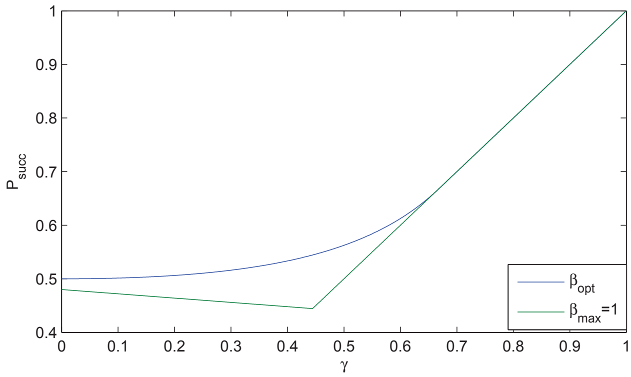

Fig. 6. Comparison of P_{succ} with maximal and optimal β for $\Delta = 0.9$.

$\alpha = 0.2$ and the bigger with proportion $1 - \alpha = 0.8$.
We start with low cost of transmission ($\Delta = 0.2$). In this situation, as shown in Figure 4, the more the mobiles cooperate the more the probability of success increases. We found that the full altruistic behaviour is the unique optimal solution up to a value of $\gamma \simeq 0.45$. Beyond this value of γ, $P_{succ} = \gamma$ and becomes, thus, independent of β. Hence, all levels of cooperation give the same performance of the system .
However, when the cost of transmission becomes high ($\Delta = 0.9$), we notice, through Figure 5, that the P_{succ} takes different values according to the level of cooperation β and we remark that the fully altruistic behavior is no more the optimal solution. In fact, for low values of γ, the level of cooperation that optimizes the performance of the system is unique. The level of cooperation β_{opt} is a decreasing function of γ, which confirms the analytical result. The uniqueness of the level of β_{opt} remains until a value of $\gamma \simeq 0.68$ beyond which several levels of cooperation give the same system performance. Hence, we deduce a counter-intuitive result. We would expect that the fully altruistic behavior is always the best decision that should be adopted to maximize the system performance. However, we found that the mobiles have to be, often, less cooperative. In Figure 6, we represent the margin between the performance of the system when adopting a fully altruistic behavior and this performance when behaving somewhat selfishly but optimally.

D. Impact of the delay on the stability of the replicator dynamics

The presence of delay in the replicator dynamic equations does not influence its convergence to the ESS. However, it has an impact on its stability. We investigate this fact through the following numerical example. We consider $N = 2$, $\alpha = 0.4$, $\Delta = 0.7$, $\gamma = 0.2$ and $\beta = 0.75$. This example corresponds to a fully mixed ESS (see Proposition 1). In Figure 7, we observe that the replicator dynamics converge to the ESS and remain stable, which confirm the Theorem 1. However, for $\tau = 4$ in Figure 8, we obtain the stability but the convergence is much slower. The boundary of stability of the replicator dynamics is $\tau_0 \approx 7.5$. In Figure 9, this boundary increases and we observe, that the replicator dynamics oscillate and become no longer stable.

VIII. Conclusion

In this paper, we have presented a new model of Medium Access Control problem through evolutionary game theory which takes in to account pairwise interactions. Our contribution was to include and investigate the aspect of cooperation between agents of the same group. We have studied the

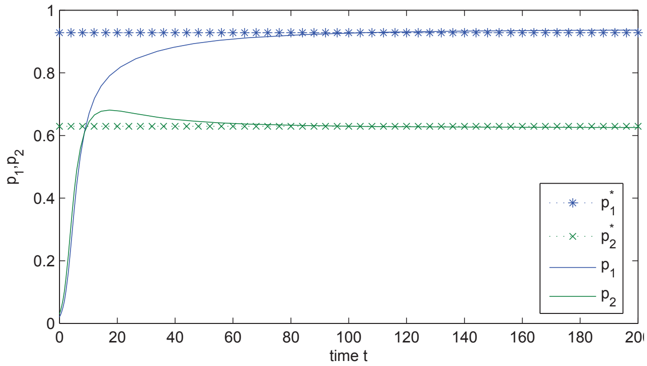

Fig. 7. Stability of the replicator dynamics for $\tau = 0$.

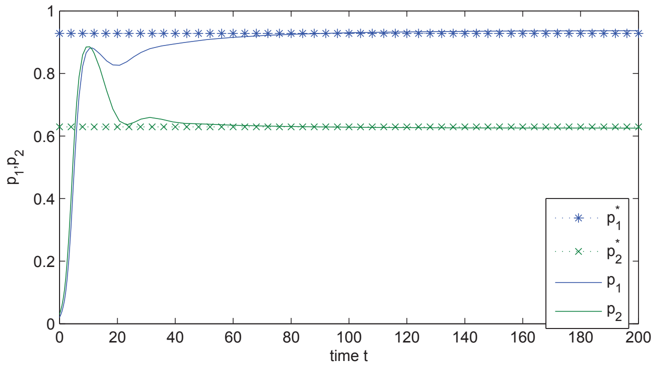

Fig. 8. Stability of the replicator dynamics for $\tau = 4$.

equilibrium of the system through the notion of Evolutionary Stable Strategies and study the effect of transmission cost and cooperation level. We have found that the mobiles tend to transmit less when the energy cost is high, whereas they may profit by the cooperation aspect to rise their transmission levels. Thereafter, we have evaluated the performance of the system in terms of the probability of success. We have studied the stability of replicator dynamics in the classical and delayed form. In a future work, we plan to study the general model with specific distributions.

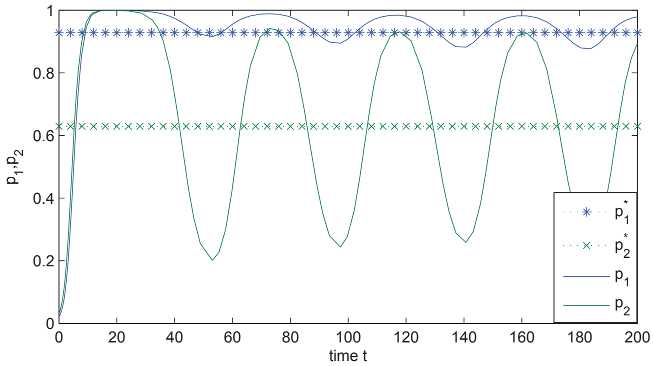

Fig. 9. Instability of the replicator dynamics for $\tau = 12$.

REFERENCES

[1] M. Smith, "Game theory and the evolution of fighting," *In John Maynard Smith, On Evolution (Edinburgh University Press), pp.8-28, 1972.*

[2] J. M. Smith, *Evolution and the theory of Games*. Cambridge University Press, UK, 1982.

[3] A. K. Srinivas Shakkottai, Eitan Altman, "Evolutionary power control games in wireless networks"," *14 Journal on Selected Areas in Communications*, pp. 1207–1215.

[4] H. Tembine, J.-Y. Le Boudec, R. El-Azouzi, and E. Altman, "Mean field asymptotics of markov decision evolutionary games and teams," in *Game Theory for Networks, 2009. GameNets' 09. International Conference on*. IEEE, 2009, pp. 140–150.

[5] S. Shakkottai, E. Altman, and A. Kumar, "The Case for Non-Cooperative Multihoming of Users to Access Points in IEEE 802.11 WLANs," in *IEEE Infocom*, Barcelona, Spain, 2006.

[6] R. El-Azouzi, F. De Pellegrini, H. B. Sidi, and V. Kamble, "Evolutionary forwarding games in delay tolerant networks: Equilibria, mechanism design and stochastic approximation," *Computer Networks*, no. 0, pp. –, 2012. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1389128612003982

[7] E. Altman, R. El-Azouzi, Y. Hayel, and H. Tembine, "The evolution of transport protocols: An evolutionary game perspective," *Computer Networks*, vol. 53, pp. 1751–1759., July 2009.

[8] Y. Zheng and Z. Feng, "Evolutionary game and resources competition in the internet," in *Modern Communication Technologies, 2001. SIBCOM-2001. The IEEE-Siberian Workshop of Students and Young Researchers*, 2001, pp. 51–54.

[9] E. Fehr and K. M. Schmidt, "A theory of fairness, competition and cooperation," *The Quarterly Journal of Economics, pp. 114: 817868, 1999.*

[10] R. B. H. Gintis, S. Bowles and E. Fehr, "Moral sentiments and material interests." *MIT Press, 2005.*

[11] D. K. Levine, "Modeling altruism and spitefulness in experiment," *Review of Economic Dynamics, vol. 1, no. 3, pp. 593622, July.*

[12] J. O. Ledyard, "Public goods: A survey of experimental research. princeton university press," *Princeton University Press, 1997, pp. 111194.*

[13] P. A. Chen and D. Kempe, "Altruism, selfishness, and spite in traffic routing," *in 9th ACM conference on Electronic commerce, 2008, pp. 140149.*

[14] Y. Sharma and D. P. Williamson, "Stackelberg thresholds in network routing games or the value of altruism, games and economic behavior," *Games and Economic Behavior, vol. 67, no. 1, pp. 174 190, 2009.*

[15] J. Sobel, "Interdependent preferences and reciprocity," *Journal of Economic Literature, vol. 43, no. 2, pp. 392436, 2005.*

[16] j. Maynard Smith and G. R. Price, "The logic of animal conflict," *Nature*, vol. 246, pp. 15 – 18, 02 November 1973.

[17] T. Hara and J. Sugie, "Stability region for systems of differential-difference equations," *Funkcialaj Ekvacioj*, vol. 39, no. 1, pp. 69–86, 1996.