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Coordination of Epidemic Control Policies: A Game Theoretic Perspective

Lorenzo Maggi⋆, Francesco De Pellegrini⋆, Alexandre Reiffers⋄‡,
P. Jean-Jacques Herings† and Eitan Altman⋄

Abstract—**We consider two neighbouring countries in which a pandemic disease spreads. Countries face a trade-off between the social costs of the epidemic diffusion and the monetary costs in order to avoid the insurgence of pandemics. However, due to migration of people across countries, the treatment efforts by one country generate a positive externality for the neighbouring country. Both countries can negotiate on the healthcare cost that each has to sustain. But, they do so subject to a central authority (CA) who can impose penalties to both countries whenever they cannot reach an agreement. We analyse the outcome of such situation via the Nash bargaining concept. Next we show how the CA should design penalties to *i)* ensure that revealing the true migration flow data is a self-enforcing behaviour, and to *ii)* enforce that the NB solution adheres to certain fairness properties.**

Index Terms—**Epidemic games, Nash bargaining, Mechanism Design, Truth revealing.**

I. Introduction

The dynamics of contagious diseases over a population of individuals under health treatment is a traditional topic in mathematical biology [1].

On the one hand, when a certain disease spreads across its population, a country incurs *healthcare monetary costs* stemming from hospitalization and treatment of infected individuals. On the other hand, countries also need to face societal costs which increase with the number of infected individuals living on their soil. One of them is clearly of ethical nature: countries should guarantee the permanence of a healthy environment within their borders. Moreover, there exist also several indirect costs related to the temporary reduction of workforce. Such costs altogether amount to the *social cost* incurred by every country subject to an epidemic outbreak.

In this paper, we explore an epidemic model where we also account for the effect of mobility across countries. Due to individuals moving across borders, epidemic processes developing in each country are subject to coupling effects. Hence, healthcare treatments performed in one country also have an impact on the pandemic spread in the neighbouring country. In economic terminology, we can say that healthcare efforts in one country generate positive externalities for the other country. Hence, this gives rise to a game situation. We consider the situation where two countries bargain on the healthcare costs that each one has to sustain. If negotiations succeed, then the countries come up with an allocation which is Pareto optimal. If instead the two countries are not able to come up with an agreement, then a Central Authority (CA) has the power to inflict monetary penalties to both countries. We resort to the concept of Nash bargaining solution (NBS) to predict the outcome of such negotiations.

The penalty imposition has a three-fold objective. The CA wants to ensure that the two countries make efficient usage of resources for the control of epidemics, i.e., by ensuring Pareto optimality. Secondly, by choosing appropriate penalties the CA can induce a malicious country possessing private migration flow data to reveal them, so as to guarantee a correct healthcare cost sharing. Finally, the penalties imposed (or threatened) by the CA can enforce the resulting NBS to adhere to some socially desirable fairness properties.

We derive our results under the assumption that the infection process evolves at a time scale which is much slower than the one underlying the migration process. This allows us to exploit a technique that aggregates in a simple manner all the parameters at stake in a single differential equation, i.e., migration rates, healthcare effort costs, and epidemic spread speeds, in the single countries.

The paper is organized as follows. In Section I-A we resume the related works on the optimal control and games models developed in literature for epidemics. Section II introduces our epidemic and game-theoretic model. In Section III we compute the Nash bargaining solution (NBS) to the healthcare cost sharing negotiations, under both convex and linear cost assumptions. Section IV provides insights on the application of our solution concept to the design of penalties inflicted by the Central Authority (CA). Specifically, in Section IV-A we show the expression of self-enforcing truth-revealing penalties, that minimize the probability that a malicious country has an incentive not to disclose private migration flow statistics. Finally, in Section IV-B we show how to enforce a fairness criterion. The reader may find all the proofs in the extended version of this paper at [2].

A. Related works

Epidemic diffusion is a mainstream research topic in mathematical biology [1]. Also, the presence of critical recovery rates is well known [3]: the so called epidemic threshold poses a recovery rate below which a pandemic affects the whole population, and above which it dies

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out. Results on epidemic thresholds have been derived for networked systems using the NIMFA mean-field approach [4]–[6]. The notion of epidemic threshold is core in the model we propose. Optimal treatment campaigns for the susceptible-infected-susceptible (SIS) model was solved in the seminal work [7]: the cost model we adopt resembles the standard cost proposed there. Recent developments appeared in [8] for the SIR model. Similarly, [9] provides optimal pulse control policies for vaccination. Recent studies have investigated the key role of migration in order to describe the epidemic process which may develop across communities [10], [11]. Our research is partially motivated by this observation. The paper [12] provides optimal control strategies for coupled populations under budget constraints. The authors also demonstrate that a strategy giving preferential treatment in the region with the higher prevalence of infected proves the worst possible strategy. Because of immigration flows, healthcare efforts in one country have positive externalities for the other country. Game theory is the standard tool to deal with situations involving externalities. Very few papers address a game-theoretical analysis of epidemic diffusion. The paper [13] discusses malware prevention for computer networks: the key observation is that the health of a player depends on the recovery rate of the other peers. The authors of [14] study a game for the so called *spillover effect* due to the coupling of infection levels and drug resistance among hospitals. [15] studies a two-countries game with no migration. Similar to our case, objective for each country is to minimize the number of infective individuals over the entire time horizon given a finite budget for vaccination. Each country allocates such finite budget so as to minimize the number of infected individuals within their own population.

Main contribution: We provide a framework for studying the orchestration of healthcare efforts among neighbouring countries with cross-migration. Countries face a trade-off between the social costs for the epidemic diffusion and the monetary costs for healthcare efforts. We further provide tools for an optimal and fair design of penalties that a CA should inflict to the countries when an agreement is not reached, in such a way that disclosing the real migration statistics is a self-enforcing behaviour.

II. System and Game Model

We consider hereafter two neighbouring countries, in which a pandemic disease spreads. People migrate at a constant rate Kp_{ij} from country i to country j. Our model is developed under a *fast mixing assumption:* we assume that *the migration process occurs at a much faster scale than the infection process*. The parameter K determines the relative speed of the two processes. We denote $N_i(t)$ as the number of people in country i at time t. Then, we can write

$$\dot{N}_1(t) = K\big[p_{21}N_2(t) - p_{12}N_1(t)\big] \quad (1)$$
$$\dot{N}_2(t) = K\big[p_{12}N_1(t) - p_{21}N_2(t)\big] = -\dot{N}_1(t). \quad (2)$$

Symbol	*Meaning*
X_i	number of infected individuals in country i
N_i	population of country i at the stationary regime
N	total (and constant) number of citizens
λ_i	infection rate in country i
γ_i	recovery rate in country i
γ^*	Pareto optimal aggregated recovery rate
γ_i^{NB}	recovery rate at Nash bargaining solution (NBS) for country i
α_i	migration level from country $-i$ to i
P_i	monetary penalty to country i
$f_i'(\gamma_i)$	healthcare costs for country i to ensure a recovery rate γ_i
J_i	cost function for country i
$\tilde{\alpha}_m$	migration level declared by malicious country m
$\mathcal{P}(\alpha_m)$	set of self-enforcing truth-revealing penalties at α_m

Table I: Main notation used throughout the paper

We first note that, under the flow conservation argument in (2), the total number of people in the system is constant, and we denote it as N:

$$N := N_1(t) + N_2(t), \quad \forall\, t \geq 0.$$

Let us now compute the stationary solutions N_1, N_2 by setting $\dot{N}_1(t) = \dot{N}_2(t) = 0$:

$$N_i = \alpha_i N, \quad i = 1,2 \quad (3)$$
$$\alpha_1 = \frac{p_{21}}{p_{12}+p_{21}} = 1 - \alpha_2.$$

The evolution dynamics of the number $X_1(t), X_2(t)$ of infected individuals at time t in country 1,2, respectively, are coupled via the mobility of individuals between the two countries. In order to make the model more realistic, we should assume that, during the incubation period[1] people migrate with the same rate as the healthy individuals, hence they are potentially able to spread the disease in the neighbouring country. On the other hand, individuals who already show symptoms should have a reduced mobility. Nevertheless, in order to keep the model analytically tractable, we assume that the incubation period is negligible, and *the same mobility pattern holds for both infected and non-infected individuals*. Hence, we are allowed to write the same equations (1) and (2) for $X_1(t), X_2(t)$, i.e.,

$$\dot{X}_1(t) = K\big[p_{21}X_2(t) - p_{12}X_1(t)\big] \quad (4)$$
$$\dot{X}_2(t) = K\big[p_{12}X_1(t) - p_{21}X_2(t)\big] = -\dot{X}_1(t), \quad (5)$$

We model the epidemic spread evolution in each country as a classic SIS compartmental model [1]. We call λ_i the rate of infection in country i and we denote γ_i the control variable representing the recovery rate of infected individuals in country i. It is induced by the treatment efforts operated over the population. When K is big enough to have $Kp_{ij} \gg \lambda_k, \gamma_k$ for all i, j, k, then our *fast mixing assumption* holds, and we can exploit the aggregation technique utilized in [16] to approximate *i)* the number of people in country i as constant in t and equal to its asymptotic value N_i, $i = 1, 2$. Moreover, from (4,5) we can consider *ii)* the number of infected individuals in the two countries as proportional to their total population,

[1]During the incubation period, an individual already got exposed to a pathogenic organism but does not show any symptoms yet

i.e., $X_1(t)/X_2(t) = N_1/N_2$, for all $t \geq 0$. Therefore, we can write

$$\dot{X}_1(t) = \lambda_1 X_1(t)\big(N_1 - X_1(t)\big) - \gamma_1 X_1(t) \tag{6}$$
$$\dot{X}_2(t) = \lambda_2 X_2(t)\big(N_2 - X_2(t)\big) - \gamma_2 X_2(t) \tag{7}$$
$$\frac{X_1(t)}{X_2(t)} = \frac{N_1}{N_2} = \frac{p_{21}}{p_{12}}. \tag{8}$$

We remark that the recovery rate is static, i.e., γ_i does not depend on t. The disease spread and its treatment are considered over the finite time interval $[0, T]$.

Finally, if we define $X(t) := X_1(t) + X_2(t)$, and sum equations (6) (7), we obtain

$$\begin{cases} \dot{X}(t) = \lambda X(t)\big(N - X(t)\big) - \gamma X(t) \\ X(0) = X_1(0) + X_2(0) := x_0 \end{cases} \tag{9}$$

where

$$\lambda = \lambda_1 \alpha_1^2 + \lambda_2 \alpha_2^2 \tag{10}$$
$$\gamma = \alpha_1 \gamma_1 + \alpha_2 \gamma_2. \tag{11}$$

Hence, the aggregation technique in [16] allows us to express with the single equation in (9) the epidemic spread dynamics in both countries, since clearly $X_i(t) = \alpha_i X(t)$ for all $t \in [0; T]$. The solution of the aggregated ODE dynamics (9) is given by the following standard logistic dynamics:

$$X(t) = \frac{V}{1 + \left(\frac{V}{x_0} - 1\right) e^{-\lambda V t}}$$

where $V := V(\gamma_1, \gamma_2) = N - \frac{\gamma}{\lambda}$.
The threshold effect on the epidemic control of the SIS system is resumed in the following.

Lemma 1 (Epidemic Threshold)**.** *$X(t)$ is monotone and converges asymptotically to $\max(V, 0)$, i.e.,*

$$\lim_{t\to\infty} X(t) = \max(V, 0) \tag{12}$$
$$\dot{X}(t) > 0 \quad \forall\, t \geq 0, \quad \text{if } X_0 < \max(V, 0)$$
$$\dot{X}(t) < 0 \quad \forall\, t \geq 0, \quad \text{if } X_0 > \max(V, 0).$$

A. Game model

In our framework, we assume that the healthcare systems of each country perform treatments against a certain pandemic disease. As mentioned before, each country incurs a social cost as well as a healthcare cost. The cost function that country i seeks to minimize can be expressed as a linear combination of the total number of infected individuals over time and of the healthcare costs [7], i.e.,

$$J_i(\gamma_i, \gamma_{-i}) = \int_0^T X_i(v)dv + f_i(\gamma_i) \tag{13}$$

where f_i is a convex and strictly increasing continuous function defined as

$$f_i = f_i'' \circ f_i'$$

where f_i'' and f_i' are also strictly increasing and continuous, with $f_i'(0) = f_i''(0) = 0$. The function $f_i' : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ maps the recovery rate γ_i into the total healthcare expenditure $f_i'(\gamma_i)$ for country i over the whole time window $[0; T]$. On the other hand, $f_i'' : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ accounts for the sensitivity of country i to the healthcare expenditure versus the social cost of epidemics. For instance, we can expect that a country with efficient healthcare system is characterized by a slowly increasing function f_i', while an advanced social welfare program is associated with a slowly decreasing function f_i''. We notice that the cost function J_i depends on the level of treatment effort of both countries through X_i.
We point out that f_i' can be thought as an increasing function of the time window length T; nevertheless, for simplicity of notation, we prefer keep this dependence implicit.
We also remark that, given the monotonicity of (f_1', f_2'), *we can identify the recovery rate γ_i with the cost $f_i'(\gamma_i)$ of healthcare efforts for country $i = 1, 2$.* Hence, we will make use of the two terms interchangeably.

After some calculations, it is possible to derive the closed form of (13) as follows:

$$J_i(\gamma_i, \gamma_{-i}) = \alpha_i \Omega\big(V(\gamma_i, \gamma_{-i})\big) + f_i(\gamma_i) \tag{14}$$

where

$$\Omega(V) = \frac{1}{\lambda} \log\Big(1 - \frac{x_0}{V}\big(1 - e^{\lambda V T}\big)\Big). \tag{15}$$

It is apparent from the expression of the cost function in (14) that the healthcare investments γ_{-i} of country $-i$ generates a positive externality for country i via the total number of infected individuals $\alpha_i \Omega(V(\gamma_i, \gamma_{-i}))$.

Cost function approximation: We now provide an approximation for the cost function J_i that will prove itself useful later in the paper. Let γ_{-i} be fixed, and consider J_i as a function of γ_i only. Let $\bar{\gamma}_i$ the minimum recovery rate that guarantees the epidemics to die out, i.e., $V(\bar{\gamma}_i, \gamma_{-i}) = 0$. Then J_i can be approximated as

$$J_i(\gamma_i, \gamma_{-i}) \approx \begin{cases} \alpha_i \Omega + f_i(\gamma_i) & \forall\, \gamma_i > \bar{\gamma}_i \ (V < 0), \\ \alpha_i \Omega\big(V(\gamma_i, \gamma_{-i})\big) + f_i(\bar{\gamma}_i) & \forall\, \gamma_i \leq \bar{\gamma}_i \ (V \geq 0). \end{cases} \tag{16}$$

where $\Omega := \Omega(0)$. This approximation, that preserves the continuity of J_i in $V = 0$, follows the following observations:

- $\gamma_i \geq \bar{\gamma}_i$ (healthcare cost dominates): this is the socially desirable condition $V \leq 0$ where $\Omega(V)$ is bounded by $\Omega(0)$ and varies very slowly in V. Hence $\Omega(V)$ can be approximated as $\Omega(0)$.
- $\gamma_i < \bar{\gamma}_i$ (social cost dominates): this case corresponds to the outbreak of the epidemic disease, i.e., $V > 0$. The number of infected shows a fast increase with V, since $\Omega(V) \approx VT$ for $V > 0$, and this term dominates the healthcare cost.

Hence, under the approximation (16), $J(., \gamma_{-i})$ reaches its minimum at $\bar{\gamma}_i$, i.e., $V = 0$.

B. Pareto optimality

In the previous section we have derived the expression of the per country cost function for a given healthcare cost pair

(γ_1, γ_2). Now, we shall characterize the set of Pareto optimal cost allocations. We will then study the case when the two countries come up with one cost allocation among the Pareto ones via bargaining.

Definition 1. *The set of Pareto optimal cost allocations is the set of costs (γ_1, γ_2) such that there does not exist (γ_1', γ_2') with $J_i(\gamma_1', \gamma_2') \leq J_i(\gamma_1, \gamma_2)$ for $i = 1, 2$, where the strict inequality holds for at least one country.*

In order to compute set of Pareto optimal allocations we first need to introduce further concepts and notation. Let γ^* be the aggregated healthcare cost associated to $V = 0$, i.e., from (11):

$$\gamma^* = \lambda N = (\lambda_1 \alpha_1^2 + \lambda_2 \alpha_2^2) N. \tag{17}$$

According to Lemma 1, the healthcare cost γ^* can be also interpreted as the minimum aggregated cost that ensures the total number of infected individuals across countries $X(t)$ to asymptotically vanish, i.e.,

$$\gamma^* = \inf \left\{ \gamma : \lim_{t \to \infty} X(t) = 0 \right\}.$$

We notice that the recovery rate γ^* is actually a linear combination between the recovery rates in the single countries, as it is clear from (11). Therefore, there are infinite ways for sharing γ^* between the two countries. We call Γ the set of all feasible pairs of recovery rates, and hence of all cost feasible healthcare cost divisions, giving rise to γ^*:

$$\Gamma = \left\{ (\gamma_1, \gamma_2) \geq 0 \;\middle|\; \alpha_1 \gamma_1 + \alpha_2 \gamma_2 = \gamma^* \right\}.$$

We observe that the cost for both countries within Γ simplifies, and writes

$$J_i(\gamma_1, \gamma_2) = \alpha_i \Omega + f_i(\gamma_i) \quad \forall (\gamma_1, \gamma_2) \in \Gamma, \tag{18}$$

where the integral cost Ω is given by the expression (15) evaluated at $V = 0$, i.e.,

$$\Omega := \Omega(0) = \int_0^T X(v) dv \Big|_{V=0} = \frac{1}{\lambda} \log \Big(1 + x_0 \lambda T \Big). \tag{19}$$

Finally, let $\mathcal{J}_\Gamma$ be the set of cost allocations associated to Γ, i.e.,

$$\mathcal{J}_\Gamma := \left\{ (J_1, J_2) : \; J_i = J_i(\gamma_1, \gamma_2), i = 1, 2, \;\middle|\; (\gamma_1, \gamma_2) \in \Gamma \right\}.$$

Fact 1. *The set of cost allocations $\mathcal{J}_\Gamma$ associated to Γ can be expressed as*

$$\mathcal{J}_\Gamma = \Big\{ (J_1, J_2) : J_2 = \alpha_2 \Omega + f_2 \left(\tfrac{\gamma^*}{\alpha_2} - \tfrac{\alpha_1}{\alpha_2} f_1^{-1}(J_1 - \alpha_1 \Omega) \right), \quad \alpha_1 \Omega \leq J_1 \leq \alpha_1 \Omega + f_1 \left(\tfrac{\gamma^*}{\alpha_1} \right) \Big\}. \tag{20}$$

Now we are ready to characterize the set of Pareto healthcare costs (or, equivalently, recovery rates).

Theorem 1. *Under the approximation (16), the set of Pareto optimal recovery rate allocations coincides with Γ.*

Evidently, if the two countries start a negotiation on how to share the healthcare costs, they can never agree on any allocation that does not lie on the Pareto frontier, since by jointly deviating they can both be better off. On the other hand, it is still unclear which allocation inside Γ should be chosen by the two countries. Predicting the outcome of such negotiation is not a trivial task. In fact, each country contributes to the overall disease spread through different migration rates Kp_{ij}. Also, each country perceives differently the trade-off between social and healthcare costs, in our framework, via the shape of functions f_i''. We suggest to study the outcome of the bargaining problem by resorting to the well known concept of *Nash bargaining solution* (NBS) [17].

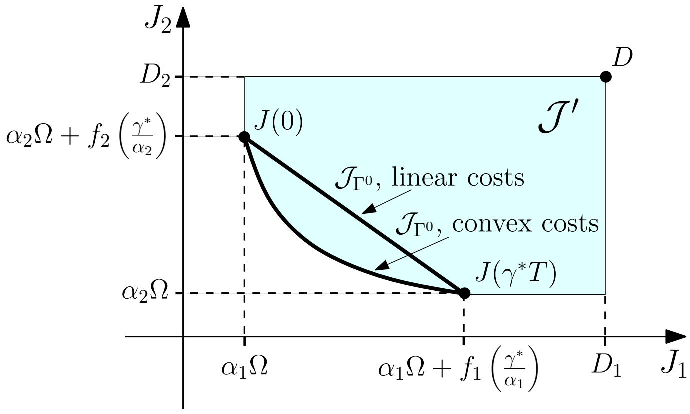

Figure 1: The feasibility region with penalties $P_1, P_2 > 0$. The Nash bargaining solution lies on the Pareto frontier.

III. Nash Bargaining

The Nash bargaining solution (NBS) [17] is arguably one of the prominent bargaining solutions in the game theoretical literature. It is the unique cost allocation that satisfies the four axioms of Pareto efficiency, symmetry, invariance to equivalent pay-off representation, and independence of irrelevant alternatives. Moreover, the NBS is the outcome of a non-cooperative bargaining protocol where the players take turns in making proposals and decide at each bargaining round whether to accept or to reject the proposal currently on the table [18], [19].

The NBS heavily relies on the situation that arises when an agreement is not reached by the two bargaining parties, that is commonly called *disagreement point*. Hence, before delving into the computation of NBS we first need to describe the scenario at the disagreement.

A. Disagreement scenario

When no agreement is reached by the two countries on how to share the recovery rate (or equivalently the healthcare costs) within Γ, the CA is assumed to have the power to inflict monetary sanctions $P_i \geq 0$ to each countries $i = 1, 2$. Here below we provide three motivations for this model choice.

i) By threatening to impose penalties, the CA wishes to enforce a socially desirable healthcare cost allocation. In fact, any allocation which is not Pareto optimal can be improved upon by both countries by jointly deviating and sharing the costs differently.

ii) By tilting the value of the penalties P_1, P_2, it turns out that the CA can ensure that

ii.a) disclosing private migration data is a self-enforcing behaviour for a malicious country, that otherwise may be enticed not to reveal them to its own advantage (see Section IV-A);

ii.b) the outcome of the bargaining adheres to some fairness principles (see Section IV-B).

We call $D = (D_1, D_2)$ the pair of values of the cost function at the *disagreement*, i.e., when negotiations between the two countries on the healthcare cost sharing do not succeed. At the disagreement, we assume that each country $i = \{1, 2\}$ has no visibility on the healthcare strategy of the neighbour. Therefore, each country $i = 1, 2$ needs to face the possible insurgence of the worst case scenario, in which the neighbour country chooses not to treat its citizens at all, i.e., $\gamma_{-i} = 0$. Therefore, the country i needs to implement the minimax strategy γ_i^d that, under the approximation in (16), can be computed as follows:

$$\gamma_i^d = \frac{\gamma^*}{\alpha_i} \approx \underset{\gamma_i \geq 0}{\operatorname{argmin}} \max_{\gamma_{-i} \geq 0} J_i(\gamma_i, \gamma_{-i}) = \underset{\gamma_i \geq 0}{\operatorname{argmin}} J_i(\gamma_i, \gamma_{-i} = 0). \quad (21)$$

Thus, it turns out that the conservative minimax healthcare plan at the disagreement point induces each country to actually invest on the healthcare system for the two countries together. Finally, the cost D_i for country i at the disagreement point writes

$$D_i = \int_0^T X^{(\gamma_i^d, \gamma_{-i}=0)}(v)dv + f_i''\left(f_i'(\gamma_i^d) + P_i\right) = \alpha_i\Omega + f_i''\left(f_i'\left(\frac{\gamma^*}{\alpha_i}\right) + P_i\right).$$

We are now in the position of characterizing the set of achievable cost allocations. Via side-payments, both countries can always achieve any cost J which is higher than the one obtained at the disagreement (see Eq. 18). We call $\mathcal{J}$ the set of achievable cost allocations:

$$\mathcal{J} = \left\{ J = (J_1, J_2) : \; J \geq J' \;\middle|\; J' \in \mathcal{J}_\Gamma \right\}.$$

On the other hand, each country cannot possibly accept as an outcome of the bargain a cost function value which is higher than the one it can achieve in the absence of an agreement. Therefore, we can characterize the feasible cost set $\mathcal{J}$ as

$$\mathcal{J}' = \mathcal{J} \cap \left\{ J = (J_1, J_2) : \; J \leq D \right\}$$

By combining equations in (18), we come up with an explicit expression for $\mathcal{J}'$, which is the set of utilities (J_1, J_2) fulfilling the following system of nonlinear equations:

$$\mathcal{J}' = (J_1, J_2) : \begin{cases} \alpha_1\Omega \leq J_1 \leq D_1 \\ \alpha_2\Omega \leq J_2 \leq D_2 \\ J_2 \geq \alpha_2\Omega + f_2\left(\frac{\gamma^*}{\alpha_2} - \frac{\alpha_1}{\alpha_2} f_1^{-1}(J_1 - \alpha_1\Omega)\right) \end{cases} \quad (22)$$

B. Nash bargaining computation

We can now formulate the NBS as the cost pair (J_1^{NB}, J_2^{NB}) that solves the following optimization problem [17]:

$$(J_1^{NB}, J_2^{NB}) = \underset{(J_1,J_2)\in\mathcal{J}'}{\operatorname{argmax}} (D_1 - J_1)(D_2 - J_2). \quad (23)$$

In practice, the NBS maximizes the product of the individual gains with respect to the disagreement point.

We remark that the feasibility region $\mathcal{J}'$ is always nonempty, since $D \in \mathcal{J}$, hence the NBS exists. We also observe that the third inequality in (22) generate a convex set, since both f_1 and f_2 are convex and strictly increasing. The following result easily follows.

Proposition 1. *The feasibility region $\mathcal{J}'$ is convex.*

Let us now compute the expression of the NBS. Since the region of feasible allocations $\mathcal{J}'$ is convex then the NBS is well defined and lies on the Pareto boundary of the feasibility region (23), that we depicted in Figure 1. Then we first compute the expression of the Pareto boundary $\mathcal{J}_\Gamma$ in parametric form:

$$J_1(\theta) = \alpha_1\Omega + f_1\left(\frac{\theta}{\alpha_1}\right)$$
$$J_2(\theta) = \alpha_2\Omega + f_2\left(\frac{\gamma^* - \theta}{\alpha_2}\right), \quad \theta \in [0; \gamma^*].$$

We notice that, when θ assumes values at its boundaries then just one country takes charge of the whole healthcare costs. In fact, if $\theta = 0$ then $\gamma_1 = 0$ and $\gamma_2 = \gamma^*/\alpha_1$. On the other hand, $\theta = \gamma^*$ corresponds to the case $\gamma_1 = \gamma^*$ and $\gamma_2 = 0$.

Since there exists a bijective map between the Pareto boundary $\mathcal{J}_\Gamma$ and the set of optimal healthcare costs Γ, we are then allowed to define the healthcare costs at the NBS as $\gamma^{NB} = \gamma(J^{NB})$, as described below.

Theorem 2. *The cost allocation at the NBS, γ^{NB}, can be expressed as*

$$\begin{cases} \gamma_1^{NB} = \frac{\theta^*}{\alpha_1} \\ \gamma_2^{NB} = \frac{\gamma^*}{\alpha_2} - \frac{\theta^*}{\alpha_2} \end{cases}$$

where θ^* is the solution of the uni-dimensional optimization problem:

$$\theta^* = \underset{0 \leq \theta \leq \gamma^*}{\operatorname{argmax}} \big(D_1 - J_1(\theta)\big)\big(D_2 - J_2(\theta)\big). \quad (24)$$

C. Linear costs

From this section onwards we will consider the special case in which both countries are characterized by increasing linear cost functions f_i', f_i'':

$$\begin{cases} f_i'(x) = \epsilon_i' x \\ f_i''(x) = \epsilon_i'' x \end{cases} \quad \text{where } x \geq 0, \; \epsilon_i', \epsilon_i'' > 0, \; i = 1, 2. \quad (25)$$

By analogy with the nonlinear case, let us call $\epsilon_i = \epsilon_i' \epsilon_i''$. Under the linear assumption (25), we are able to find the explicit expression of the NBS. We first observe that in the linear case the cost function J_i for both countries can be rewritten as

$$J_i(\gamma_i, \gamma_{-i}) = \alpha_i\Omega + \epsilon_i\gamma_i, \quad \forall\, (\gamma_i, \gamma_{-i}) \in \Gamma, \; i = 1, 2. \quad (26)$$

The parametric form of J, where $\theta \in [0; \gamma^*]$, writes

$$J(\theta) = \begin{bmatrix} J_1(\theta) \\ J_2(\theta) \end{bmatrix} = \begin{bmatrix} \alpha_1 \Omega \\ \alpha_2 \Omega + \frac{\epsilon_2}{\alpha_2} \gamma^* \end{bmatrix} + \begin{bmatrix} \frac{\epsilon_1}{\alpha_1} \\ -\frac{\epsilon_2}{\alpha_2} \end{bmatrix} \theta. \tag{27}$$

Moreover, the disagreement point $D = (D_1, D_2)$ can be expressed as

$$D_i = \alpha_i \Omega + \epsilon_i \left(\frac{\gamma^*}{\alpha_i} + \frac{P_i}{\epsilon'_i} \right), \quad i = 1, 2.$$

Now we are finally ready to compute explicitly the NBS in the form of the following Theorem.

Theorem 3. *Let us define θ_0 as*

$$\theta_0 = \frac{1}{2} \left(\alpha_1 \frac{P_1}{\epsilon'_1} - \alpha_2 \frac{P_2}{\epsilon'_2} + \gamma^* \right).$$

Then, under the linear costs assumption in (25),

i. if $\theta_0 \leq 0$, then $J^{NB} = J(\theta = 0)$, and country 2 covers all the healthcare costs, i.e.,

$$\gamma_1^{NB} = 0, \quad \gamma_2^{NB} = \frac{\gamma^*}{\alpha_2};$$

ii. if $0 < \theta_0 < \gamma^*$, then for $i = 1, 2$

$$\gamma_i^{NB} = \frac{1}{2\alpha_i} \left(\alpha_i \frac{P_i}{\epsilon'_i} - \alpha_{-i} \frac{P_{-i}}{\epsilon'_{-i}} + \gamma^* \right);$$

iii. if $\theta_0 \geq \gamma^*$, then $J^{NB} = J(\theta = \gamma^*)$ and country 1 covers all the healthcare costs, i.e.,

$$\gamma_1^{NB} = \frac{\gamma^*}{\alpha_1}, \quad \gamma_2^{NB} = 0.$$

Remark 1. *The expression of the cost allocation at the NBS in Theorem 3 does not depend on the sensitivity of both countries to the social-monetary cost trade-off, i.e., the value of $\epsilon''_1, \epsilon''_2$, which can reasonably assumed to be* unknown *at the CA side. On the other hand, if we assume that the CA has at its disposal good estimate of λ_1, λ_2, and α_1 (and hence of α_2), then the outcome of the bargaining in the linear case can be well predicted and anticipated by the CA itself.*

IV. PENALTY DESIGN

In this section we explore the role of penalties that are inflicted at the disagreement by the CA to both countries in order to induce a desired behaviour from both countries regarding their healthcare program. We first deal with the scenario in which the CA aims at eliciting the truthful disclosure of migration flow data from a malicious country that may want to utilize the private information to its own advantage. In the following we will still assume that the cost functions f'_i and f''_i are linear, i.e., conditions (25) hold.

A. Self-enforcing private migration data disclosure

So far, in this paper we have assumed that the true values of α_1, α_2 are public and available to the two countries and to the CA. In reality, the statistical information on ingoing and outgoing migration flow available to the countries may differ. For example, country m may be in possession of private, unofficial migration data that it may not be willing to disclose, since they may give it an edge over the neighbour country during the negotiation phase. Hence, in this case, the malicious country m would be able to convince country $-m$ that some $\tilde{\alpha}_m \neq \alpha_m = 1 - \alpha_{-m}$ is really the migration level from country $-m$ to country m.
In our model, *the CA suspects that country m has private migration flow data*, and we compute the penalties that the CA should impose to both countries such that country m, in case it really possesses private information, finds (with highest probability) no benefit in cheating and declaring fake migration figures. In other words, we wish to *find the penalties ensuring that disclosing the private migration data is a self-enforcing behaviour for the malicious country m.* The main goal of the CA is acting so as to guarantee a correct healthcare cost sharing.

In order to restrict our search of optimal penalties from a 2- to a 1-dimensional space while still adhering to some desirable *fairness* property, we consider that the CA penalizes at the disagreement the country with the least efficient healthcare system. More specifically, we force the penalties to each country i to be proportional to ϵ'_i.

$$\textbf{Fairness property 1}: \quad \frac{P_1}{\epsilon'_1} = \frac{P_2}{\epsilon'_2} := P. \tag{28}$$

We can reasonably assume that the CA has at its disposal some *a priori* information on the real value α_m, described by an *a priori* probability distribution $p_{\alpha_m}(.)$, i.e.,

$$\int_0^x p_{\alpha_m}(v) dv = \Pr(\alpha_m \leq x), \quad x \in [0; 1].$$

We notice that this model choice incorporates the extreme cases in which *a)* the CA has no information at all on α_m (i.e., p_{α_m} is uniform) and *b)* the CA knows the exact value of α_m (i.e., p_{α_m} is the Dirac's delta centred in α_m).

For simplicity, we assume that the epidemic disease spreads in both countries with the same speed:

$$\lambda_0 := \lambda_1 = \lambda_2.$$

Let us delve into the details of this new scenario. The negotiation between the two countries on the healthcare cost sharing is carried out on the basis of the officially declared migration values $(\tilde{\alpha}_m, 1 - \tilde{\alpha}_m)$. We then call $\tilde{\gamma}_m^{NB}(\tilde{\alpha}_m)$ the Nash solution of the bargaining problem when $\tilde{\alpha}_m$ is the officially declared migration flow level, i.e.,

$$\tilde{\gamma}_m^{NB}(\tilde{\alpha}_m) = \gamma_m^{NB}(\alpha_m = \tilde{\alpha}_m).$$

Remarkably, the malicious country m also modifies the Pareto global healthcare investment level $\tilde{\gamma}^{*(\alpha_m)}(\tilde{\alpha}_m)$ which is different from the optimal γ^*, i.e.,

$$\tilde{\gamma}^{*(\alpha_m)}(\tilde{\alpha}_m) = \gamma^*(\alpha_m = \tilde{\alpha}_m) = N\lambda_0 \left(2\tilde{\alpha}_m^2 - 2\tilde{\alpha}_m + 1\right).$$

Hence, under the unfortunate case $\tilde{\gamma}^{*(\alpha_m)}(\tilde{\alpha}_m) < \gamma^*$, the resulting recovery rate is not sufficient to extinguish the pandemics. We call $\Omega^{(\alpha_m)}(\tilde{\alpha}_m)$ the total number of infected individuals over the time interval $[0;T]$ when $\tilde{\alpha}_m$ has been declared, i.e.,

$$\tilde{\Omega}^{(\alpha_m)}(\tilde{\alpha}_m) = \Omega(\tilde{V}^{(\alpha_m)}(\tilde{\alpha}_m))$$

where $\max(\tilde{V}^{(\alpha_m)}(\tilde{\alpha}_m), 0)$ is the asymptotic value of $X(t)$, i.e.,

$$\tilde{V}^{(\alpha_m)}(\tilde{\alpha}_m) = N - \frac{\tilde{\gamma}^{*(\alpha_m)}(\tilde{\alpha}_m)}{\lambda} = N\left(1 - \frac{2\tilde{\alpha}_m^2 - 2\tilde{\alpha}_m + 1}{2\alpha_m^2 - 2\alpha_m + 1}\right).$$

We point out that, when the true value α_m is declared, $\tilde{V}^{(\alpha_m)}(\alpha_m) = V = 0$. We also remark that $\Omega^{(\alpha_m)}(\tilde{\alpha}_m)$ depends on the true value α_m through the epidemic spread speed λ, while it is a function of the fictitious value $\tilde{\alpha}_m$ via the Pareto healthcare cost $\tilde{\gamma}^*$.
The cost function that country m perceives when declaring $\tilde{\alpha}_m$ instead of the real value α_m is called $\tilde{J}_m^{(\alpha_m)}(\tilde{\alpha}_m)$, and equals

$$\tilde{J}_m^{(\alpha_m)}(\tilde{\alpha}_m) = \alpha_m \tilde{\Omega}^{(\alpha_m)}(\tilde{\alpha}_m) + \epsilon_m \tilde{\gamma}_m^{NB}(\tilde{\alpha}_m).$$

Depending on the value of the penalties P_m, malicious country m may find profitable not to disclose its private information on migration flows - so that the officially declared value $\tilde{\alpha}_m$ is different from the real one, α_m - in order to incur a smaller healthcare cost share. We hence define below the set of self-enforcing truth-revealing penalties.

Definition 2. *The* self-enforcing truth-revealing *set-valued function* $\mathcal{P}: [0;1] \to \mathbb{R}_0^+$ *defines, for each input value of* α_m*, the set of penalties satisfying (28) such that the best strategy for malicious country* m *is declaring the real value of* α_m*:*

$$\mathcal{P}(\alpha_m) = \left\{ P: \; \alpha_m = \operatorname*{argmin}_{\tilde{\alpha}_m \in [0;1]} \tilde{J}_m^{(\alpha_m)}\left(\tilde{\alpha}_m, \frac{P_1}{\epsilon_1'} = \frac{P_2}{\epsilon_2'} = P\right) \right\}.$$

Our main objective is figuring out how the CA can exploit its *a priori* information on α_m in order to properly design the penalty that minimizes the probability that malicious country m has an incentive to declare a fictitious migration parameter $\tilde{\alpha}_m \neq \alpha_m$, i.e., disclosing private data is not a self-enforcing behaviour for country m. This idea is inspired to the very popular concept of *mechanism design* (see e.g. [20], Chap.7).

Definition 3. *Assume that the CA has an a priori probability distribution* p_{α_m} *on the real value of* α_m*. The penalty* P^* *is a* mechanism *for the malicious country* $m \in \{1,2\}$ *whenever its probability of being truth-revealing is maximal, i.e.,*

$$P^* = \operatorname*{argmax}_{P \in \mathbb{R}} \Pr\Big(P \in \mathcal{P}(\alpha_m)\Big), \tag{29}$$

where the probability is computed w.r.t. p_{α_m}*.*

Let $\underline{\alpha}_m = \min(\alpha_m, 1-\alpha_m)$ and $\overline{\alpha}_m = \max(\alpha_m, 1-\alpha_m)$. In order to find the mechanism P^* we need to rely on the same kind of approximation on the cost function $\tilde{J}_m^{(\alpha_m)}(\tilde{\alpha}_m)$ that we utilized in (16), i.e., $\tilde{J}_m^{(\alpha_m)}(\tilde{\alpha}_m)$ approximately equals

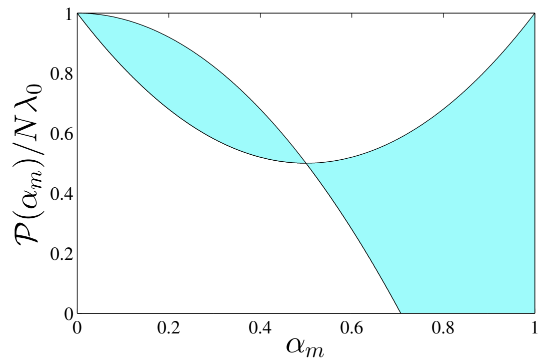

Figure 2: The set-valued function $\mathcal{P}(\alpha_m)$ of self-enforcing truth-revealing penalties is shown in light blue.

$$\begin{cases} \alpha_m \Omega + \epsilon_m \tilde{\gamma}_m^{NB}(\tilde{\alpha}_m), \quad \forall\, \tilde{\alpha}_m \in [0;\underline{\alpha}_m] \cup [\overline{\alpha}_m;1] \\ \alpha_m \Omega\big(\tilde{V}^{(\alpha_m)}(\tilde{\alpha}_m)\big) + \epsilon_m \tilde{\gamma}_m^{NB}(\underline{\alpha}_m), \quad \forall\, \tilde{\alpha}_m \in [\underline{\alpha}_m; \frac{1}{2}] \\ \alpha_m \Omega\big(\tilde{V}^{(\alpha_m)}(\tilde{\alpha}_m)\big) + \epsilon_m \tilde{\gamma}_m^{NB}(\overline{\alpha}_m), \forall\, \tilde{\alpha}_m \in [\frac{1}{2}; \overline{\alpha}_m] \end{cases} \tag{30}$$

In fact, the expression $\tilde{\alpha}_m \in [0;\alpha_m] \cup [1-\alpha_m;1]$ corresponds to $\tilde{V}^{(\alpha_m)}(\tilde{\alpha}_m) \leq 0$, while the condition $\tilde{\alpha}_m \in (\underline{\alpha}_m;\overline{\alpha}_m)$ corresponds to the outbreak of the epidemic disease, since in this case $\tilde{V}^{(\alpha_m)}(\tilde{\alpha}_m) > 0$.

Therefore, we can already claim that malicious country m never has an incentive in declaring $\tilde{\alpha}_m \in (\alpha_m; 1-\alpha_m)$, since the epidemic outbreak would cause its cost function to rapidly increase. This tells us that, even in the case that the CA does not manage to design truth-revealing penalties $(P_1, P_2) \in \mathcal{P}$, the epidemic spread outbreak is forestalled, i.e., for any value of α_m and P, then $\lim_{t\uparrow\infty} X(t) = 0$ even if country m acts maliciously.

Before characterizing the set $\mathcal{P}(\alpha_m)$ of truth-revealing penalties, let us first show the following auxiliary result.

Lemma 2. *Assume that* $0 \leq P \leq N\lambda_0$*. Then, the NBS equals*

$$\tilde{\gamma}_m^{NB}(\tilde{\alpha}_m) = \frac{1}{2\tilde{\alpha}_m}\Big(P(2\tilde{\alpha}_m - 1) + N\lambda_0(2\tilde{\alpha}_m^2 - 2\tilde{\alpha}_m + 1)\Big).$$

Now we are finally ready to characterize the set of truth-revealing penalties.

Theorem 4. *Under the approximation in (30), the set* $\mathcal{P}(\alpha_m)$ *of truth-revealing penalties can be expressed, when* $0 < \alpha_m \leq 1/2$*, as*

$$\mathcal{P}(\alpha_m) = \Big\{P: \; 2\alpha_m^2 - 2\alpha_m + 1 \leq \frac{P}{N\lambda_0} \leq 1 - 2\alpha_m^2\Big\},$$

and, when $1/2 < \alpha_m < 1$*, as*

$$\mathcal{P}(\alpha_m) = \Big\{P: \; \max(1 - 2\alpha_m^2, 0) \leq \frac{P}{N\lambda_0} \leq 2\alpha_m^2 - 2\alpha_m + 1\Big\}.$$

and, when $\alpha_m = \{0,1\}$*, as* $\mathcal{P}(\alpha_m) = \{P \geq 0\}$.

After characterizing in Theorem 4 the set-valued function $\mathcal{P}(\alpha_m)$ of truth-revealing penalties, we can finally compute the expression of the mechanism P^* as the outcome of an optimization problem. The proof is direct consequence of Definition 3 and Theorem 4.

Corollary 1. *Let p_{α_m} be a bounded probability distribution function. The mechanism P^* for malicious country m equals*

$$P^* = \operatorname*{argmax}_{0 \le P \le N\lambda_0} \int_{\mathcal{P}^{-1}(P)} p_{\alpha_m}(v)\, dv,$$

where, if $0 \le x \le \frac{1}{2}$, $\mathcal{P}^{-1}(N\lambda_0 x) = \left[\sqrt{\frac{1-x}{2}}; 1\right]$ and, if $\frac{1}{2} \le x \le 1$,

$$\mathcal{P}^{-1}(N\lambda_0 x) = \left[\frac{2-\sqrt{4-8(1-x)}}{4}; \sqrt{\frac{1-x}{2}}\right] \cup \left[\frac{2+\sqrt{4-8(1-x)}}{4}; 1\right].$$

We remark that if the CA has full information on the migration flow, then the CA knows $\mathcal{P}(\alpha_m)$. Hence, by choosing any $P^* \in \mathcal{P}(\alpha_m)$, P^* is truth-revealing with probability 1, and the malicious country m is never enticed to declare a fictitious value $\tilde{\alpha}_m \neq \alpha_m$. Hence, under the penalty P^*, disclosing all the private migration information is always a self-enforcing behaviour.

Corollary 2. *If $p_{\alpha_m} = \delta_{\alpha_m}$, where δ_{α_m} is the Dirac's delta centred in α_m, then $\Pr(P^* \in \mathcal{P}(\alpha_m)) = 1$.*

Let us now consider the other extreme case, where the CA has no prior information about the true value of α_m, i.e., the *a priori* distribution function p_{α_m} is uniform.

Corollary 3. *If $p_{\alpha_m}(v) = 1$ for all $v \in [0; 1]$, then*

$$P^* = \frac{1}{2} N\lambda_0, \quad \Pr(P^* \in \mathcal{P}(\alpha_m)) = \frac{1}{2}.$$

Remark 2. *We highlight that the information needed at the CA's side to compute the truth-revealing penalty P^* is very limited, and restricted to the total population size N and the epidemic spread rate λ_0 (along with the prior p_{α_m}). Therefore, the CA does not need to estimate the cost function J_i for each country i, which would clearly represent a daunting task.*

B. Fairness

In this final section we show another example of penalty design. Unlike the previous section, we assume that no country possesses private mobility information. We now aim at designing the penalties that ensure the NBS to adhere to a certain fairness property. Specifically, we suggest that the recovery rates ensured by the two countries should be proportioned to the expected number of infected individuals per unit of time in each country in the absence of migration, that is formalized in the following.

$$\textbf{Fairness property 2}: \quad \frac{\gamma_1^{NB}}{\gamma_2^{NB}} = \frac{\lambda_1 N_1(0)}{\lambda_2 N_2(0)} \tag{31}$$

In order to understand the rationale behind the fairness condition (31), consider the case when $\lambda_2 N_2(0) \gg \lambda_1 N_1(0)$, i.e., country 2 contributes to the pandemic spread much more consistently than country 1. Hence, according to property (31), $\gamma_2^{NB} \gg \gamma_1^{NB}$ should hold. But, if there is a large immigration rate from country 2 to 1 ($\alpha_1 \gg \alpha_2$) and both countries have the same sensitivity to healthcare costs ($\epsilon_1' = \epsilon_2'$, $\epsilon_1'' = \epsilon_2''$), for penalties equal and sufficiently large, then the whole cost for the healthcare program is sustained by country 1, i.e. $\gamma_1^{NB} = \gamma^*/\alpha_1$ and $\gamma_2^{NB} = 0$. This solution would clearly be perceived as unfair by the country 1.

Proposition 2. *In order to enforce the fairness condition (31), the penalties (P_1, P_2) need to satisfy the following relation:*

$$\frac{\alpha_2}{\epsilon_2'} P_2 - \frac{\alpha_1}{\epsilon_1'} P_1 = \gamma^* \frac{\alpha_2\lambda_2 N_2(0) - \alpha_1\lambda_1 N_1(0)}{\alpha_2\lambda_2 N_2(0) + \alpha_1\lambda_1 N_1(0)}.$$

If we also impose the fairness condition (28) under which $\frac{P_i}{\epsilon_i'} = P$ for $i = 1, 2$, we conclude that

$$P = \frac{\gamma^*}{\alpha_2 - \alpha_1} \frac{\alpha_2\lambda_2 N_2(0) - \alpha_1\lambda_1 N_1(0)}{\alpha_2\lambda_2 N_2(0) + \alpha_1\lambda_1 N_1(0)}.$$

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