Robust control of pneumatic actuators based on a simplified model with delayed input

E Edjekouane, Samer Riachy, M Ghanes, Jean-Pierre Barbot

Robust Control of Pneumatic Actuators Based on a Simplified Model With Delayed Input

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***Abstract*— Due to frictions and air compressibility, the dynamics of pneumatic actuators is often described by a complex fourth-order non linear model. Therefore, simplifying the model of pneumatic actuators is of prime interest to design a controller. In this paper a simple second order model is proposed by modeling the pressure dynamics with a pure time delay on the control input. The Artstein transformation is applied to this model to get a delay-free second order system. Then the delay-free system is stabilized using a robust nonlinear controller. The relevance of the proposed approach is demonstrated through experimental tests.**

I. INTRODUCTION

The control of pneumatic actuators remains a challenge till today in order to obtain better results in positioning, tracking and in terms of robustness (disturbance rejection). This challenge is raised by both industrial and academic partners, across several applications and publications. The pneumatic actuators have a wide field of use ranging from simple process to the complex ones, like in production line, aeronautic and automotive industry. This large place they hold is due to their ease of maintenance, rigidity and safety. However, their dynamic modeling is quite difficult since, the description of the air dynamic is often based on empirical considerations. Moreover, accurate positioning of such an actuator is a difficult task due to the presence of discontinuous friction and air compressibility.

The model used to design controllers for this kind of actuators is often a fourth order model (see [4], [7], [8], [9], [10]). The major drawback of such a model is the difficulty of controller design and closed loop stability analysis. The first contribution of the present work is the introduction of a simplified model which is based on the following observation. A pure time delay is used to model the pressures dynamics. This delay occurs between the moment of the opening of the servovalve and the moment when the force acts on the piston. Consequently, a double integrator with delayed input is obtained. This idea has been raised in the conclusion of [9].

The observation that the pressure dynamics induces a pure time delay was already invoked in [10]. In order to minimize the influence of the delay, the adopted method consisted in fully opening the servovalve during a very short time at the beginning of a piston displacement in order to force quick pressure establishment in the cylinder compartment.

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The second contribution of this paper is the design of a robust controller based on the proposed second order model. The control synthesis is performed in two steps. First, the Artstein transformation is applied to the second order model in order to obtain a system free of delay. Then, a controller based on the concepts of homogeneity and finite-time stability [2] is used to stabilize the transformed system. The proposed controller can be seen as a continuous approximation of a standard second order sliding mode. The stability of the closed loop is analyzed with and without perturbations.

It is clear from [10], that the delay takes different values depending on the opening rate of the servovalve. As a matter of fact, the bigger the opening rate is, the faster the pressure builds up in the cylinder chamber. The delay decreases then as the opening rate of the servovalve increases. However, a constant delay is assumed. The validity of this assumption is supported by the experimental system positioning accuracy.

The paper is organized as follows. Section II recalls the fourth order model, introduces the second order model with input delay and experimentally estimates the delay value. Section III is dedicated to the control design while section IV deals with the stability analysis of the closed loop system. The discretization of the controller is detailed in section V. Section VI presents the experimental tests.

II. DYNAMIC MODEL

A. The fourth order model

The derivation of the dynamic equations of the pneumatic actuator can be found in [9], [10] and [11]. It comes to a model where the state vector is of dimension 4. With y to denote the position of the piston, v its velocity, P_1 and P_2 the pressures in both compartments of the cylinder, the dynamic equations writes:

$$
\begin{aligned}
\dot{y} &= v \\
\dot{v} &= M^{-1}(P_1A_1 - P_2A_2 + \Delta) \\
\dot{P}_1 &= -\frac{kP_1v}{L+y} + \frac{\Omega}{A_1}\left\{\frac{1+\sigma(u)}{2}\frac{\gamma_{1b}P_s}{L+y} - \frac{1-\sigma(u)}{2}\frac{\gamma_{1e}P_1}{L+y}\right\}|u| \\
\dot{P}_2 &= \frac{kP_2v}{L-y} + \frac{\Omega}{A_2}\left\{\frac{1-\sigma(u)}{2}\frac{\gamma_{2b}P_s}{L-y} - \frac{1+\sigma(u)}{2}\frac{\gamma_{2e}P_2}{L-y}\right\}|u|,
\end{aligned}
$$

where $\Omega = k\sqrt{RkT}A_o/U$, $\sigma(u)$ denotes the signum function[1] and A_o the area of the orifice of the servovalve. The control u represents the input voltage to the servovalve while U denotes the maximal input voltage ($|u| \leq U$). P_s denotes

[1] $\sigma(u) = 1$ if $u > 0$, $\sigma(u) = 0$ if $u = 0$, $\sigma(u) = -1$ if $u < 0$.

the supply pressure. Δ represents an external load and can also represent gravity when the cylinder is mounted in a vertical position. T denotes the air temperature, assumed to be constant. The heat coefficient for air is k, while R is the perfect gas constant. $2L$ denotes the total length of the cylinder while M the mass of the moving part (piston, payload etc.). $\gamma_{1b}, \gamma_{2e}, \gamma_{1e}$ and γ_{2b} are given by:

$$\gamma_{1b} = \begin{cases} \sqrt{\frac{2}{k-1}}\left(\frac{P_1}{P_s}\right)^{\frac{k+1}{2k}}\sqrt{\left(\frac{P_1}{P_s}\right)^{\frac{1-k}{k}} - 1} & \text{if} \quad \frac{P_1}{P_s} \geq 0.528 \\ 0.58 & \text{if} \quad \frac{P_1}{P_s} < 0.528. \end{cases}$$

$$\gamma_{2e} = \begin{cases} \sqrt{\frac{2}{k-1}}\left(\frac{P_a}{P_2}\right)^{\frac{k+1}{2k}}\sqrt{\left(\frac{P_a}{P_2}\right)^{\frac{1-k}{k}} - 1} & \text{if} \quad \frac{P_a}{P_2} \geq 0.528 \\ 0.58 & \text{if} \quad \frac{P_a}{P_2} < 0.528. \end{cases}$$

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and P_a denotes atmospheric pressure.

Instead of this model, a second order model with an input delay is proposed. The validity of such a model for the control design purpose is supported by the experimental results. In addition, an interpretation via singular perturbation theory can be given [5]. In fact a two-time scale analysis can be conducted with the fast dynamics consisting of the air pressure and a slow one consisting of the mechanical piston.

B. The proposed model with input delay

The control design is based on the following second order model:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= MGu(t-h) + p \end{aligned} \tag{1}$$

where x_1 and x_2 represent the piston position and velocity respectively. The delay is denoted by h while M denotes the piston and payload mass. Friction forces and gravity are represented by p while G represents an open loop gain. As previously discussed in section I, the delay h is not necessarily constant. However, the following assumption is adopted.

Hypothesis 1: The delay $h > 0$ is assumed to have a bounded constant value.

C. Estimation of the delay

We now proceed to an estimation of the delay h. This is done by open loop tests, where variable delay is observed ranging from 0.01 to 0.06 second as shown in figure 1. In fact, the experiment confirm that the delay decreases as the opening rate of the servovalve increases. However, as required for the control design, the assumption of a constant delay is adopted. The validity of this assumption is confirmed by the experiments.

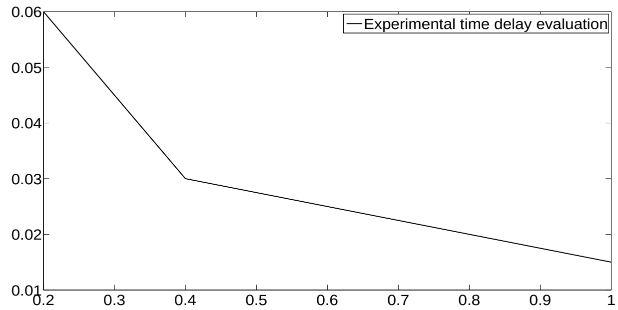

Fig. 1. Time delay (second) versus percentage of servovalve opening

III. CONTROL DESIGN

The Artstein transformation [1] permits to get rid of the delay. For the system (1), it is given by:

$$\begin{aligned} z_1 &= x_1 + hx_2 + MG\int_{t-h}^{t}(t-\tau)u(\tau)d\tau \\ z_2 &= x_2 + MG\int_{t-h}^{t}u(\tau)d\tau. \end{aligned} \tag{2}$$

Differentiating (2) and using (1) lead to the delay-free system

$$\begin{aligned} \dot{z}_1 &= z_2 + hp \\ \dot{z}_2 &= MGu + p. \end{aligned} \tag{3}$$

In the sequel, the following notation is used

$$\lfloor \xi \rceil^{\alpha} = \text{sign}(\xi)|\xi|^{\alpha}, \xi \in \mathbb{R}, \alpha > 0$$

where $|\cdot|$ denotes the Euclidean norm and corresponds to the absolute value for scalar entries. Note that the following rules for derivative are verified except at $\xi = 0$, where the derivative is not defined: $\frac{d\lfloor \xi \rceil^{\alpha}}{d\xi} = \alpha|\xi|^{\alpha-1}$ and $\frac{d|\xi|^{\alpha}}{d\xi} = \alpha\lfloor \xi \rceil^{\alpha-1}$, $\forall \xi \in \mathbb{R}\backslash\{0\}$. The control input

$$u = -k_1\lfloor \dot{z}_1 \rceil^{\alpha} - k_2\left\lfloor z_1 + \frac{k_3}{2-\alpha}\lfloor \dot{z}_1 \rceil^{2-\alpha}\right\rceil^{\frac{\alpha}{2-\alpha}}, \ \alpha \in [0,1], \tag{4}$$

taken from [2] with positive constants k_1, k_2 and k_3, leads to the closed loop system:

$$\begin{aligned} &\dot{z}_1 = z_2 + hp \\ &\dot{z}_2 = -K_1\lfloor z_2 + hp \rceil^{\alpha} - K_2\left\lfloor z_1 + K_3\lfloor z_2 + hp \rceil^{2-\alpha}\right\rceil^{\frac{\alpha}{2-\alpha}} + p, \end{aligned} \tag{5}$$

$K_1 = MGk_1$, $K_2 = MGk_2$, $K_3 = \frac{k_3}{2-\alpha}$. In (4), $\dot{z}_1$ can be computed by using a differentiator. The control (4) with $\alpha \in [0,1)$ ensures, according to [3], a finite time stabilzation of (5) with $p = 0$. A bloc diagram of the closed loop system is given in figure 2.

In particular, with $\alpha = 0$ the system (5) leads to:

$$\begin{aligned} &\dot{z}_1 = z_2 + hp \\ &\dot{z}_2 = -K_1\text{sign}(z_2 + hp) - K_2\text{sign}(z_1 + K_3\lfloor z_2 + hp \rceil^2) + p, \end{aligned} \tag{6}$$

which is a standard second order sliding mode system with unmatched perturbation. Note that due to the perturbation a chattering may appear near the equilibrium $z_1 = z_2 = 0$.

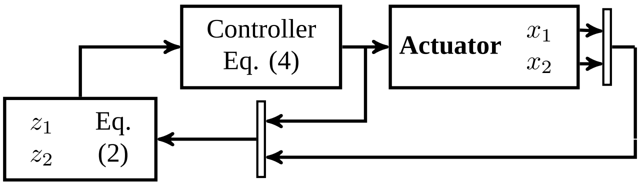

Fig. 2. Bloc diagram of the closed loop system

IV. STABILITY ANALYSIS

The stability analysis is conducted in two steps. In section IV-A, the unperturbed system ($p = 0$) is analyzed while the perturbed one is investigated in section IV-B.

A. Stability of the unperturbed system

The system (5) with $p = 0$ leads to,

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -K_1\lfloor z_2\rceil^{\alpha} - K_2\left\lfloor z_1 + K_3\lfloor z_2\rceil^{2-\alpha}\right\rceil^{\frac{\alpha}{2-\alpha}}. \end{aligned} \tag{7}$$

Proposition 4.1: The system (7) is asymptotically stable. Consequently, $x_1 = x_2 = 0$ is an asymptotically stable equilibrium.

Proof: Consider the Lyapunov function

$$V = K_1\frac{2-\alpha}{2}\left|z_1 + \frac{k_3}{2-\alpha}\lfloor z_2\rceil^{2-\alpha}\right|^{\frac{2}{2-\alpha}} + \frac{\gamma}{2}z_2^2 \tag{8}$$

with, γ a positive constant, chosen such that $K_1(1-K_1k_3) = \gamma K_2$. The derivative of (8) is given by:

$$\dot{V} = -K_1K_2k_3|z_2|^{1-\alpha}\left|z_1 + \frac{k_3}{2-\alpha}\lfloor z_2\rceil^{2-\alpha}\right|^{\frac{2\alpha}{2-\alpha}} - \gamma K_1|z_2|^{1+\alpha}$$

which is negative semidefinite. Then by LaSalle's theorem, $(z_1, z_2) = (0, 0)$ is an asymptotically stable equilibrium point and consequently is $(x_1, x_2) = (0, 0)$ according to (2). ∎

B. Stability of the perturbed system

Consider now, the perturbed system (5). With, $y_1 = z_1$, $y_2 = \dot{z}_1$ and $y = [y_1\ y_2]^T$, it can be rewritten as follows:

$$\dot{y}_1 = y_2 \tag{9}$$

$$\dot{y}_2 = -K_1\lfloor y_2\rceil^{\alpha} - K_2\left\lfloor y_1 + \frac{k_3}{2-\alpha}\lfloor y_2\rceil^{2-\alpha}\right\rceil^{\frac{\alpha}{2-\alpha}} + p.$$

Hypothesis 2: The perturbation p is nonzero but constant. Moreover, a positive constant $\bar{p}$ exists such that $|p| < \bar{p}$.

Proposition 4.2: Let assumptions 1 and 2 be verified. Set $\theta_1 = \max\left(\frac{2k_3}{2-\alpha}\left[\frac{(K_1k_3+\gamma)\bar{p}}{\gamma K_1}\right]^{\frac{2-\alpha}{1+\alpha}},\ 2\left(\frac{2\bar{p}}{K_2}\right)^{\frac{2-\alpha}{\alpha}},\ 2\left[\frac{(1-K_1k_3)\bar{p}}{4K_1K_2k_3}\right]^{\frac{2-\alpha}{\alpha}}\right)$ and $\theta_2 = \max\left(\frac{k_3}{2-\alpha}\left[\frac{(K_1k_3+\gamma)\bar{p}}{\gamma K_1}\right]^{\frac{2-\alpha}{1+\alpha}},\ 2\left(\frac{2\bar{p}}{K_2}\right)^{\frac{2-\alpha}{\alpha}},\ 2\left[\frac{(1-K_1k_3)\bar{p}}{4K_1K_2k_3}\right]^{\frac{2-\alpha}{\alpha}}\right)$ with $\gamma = \frac{K_1(1-K_1k_3)}{K_2}$ and $\alpha \in (0, 1)$. Assume that $\bar{p}$ is sufficiently small such that $\theta_1 < 1$ and $\theta_2 < 1$. Then the ball

$$\mathbb{B} = \{(y_1, y_2); |y_1| \le \theta_1,\ |y_2| \le \theta_2\}$$

attracts any trajectory of (9) initialized within the compact set

$$\mathbb{B}_0 = \left\{(y_1, y_2); |y_1| \le 1,\ |y_2| \le 1, |y_1| + \frac{k_3}{2-\alpha}|y_2|^{2-\alpha} \le 1\right\}.$$

Proof: Consider the Lyapunov function:

$$V = K_1\frac{2-\alpha}{2}\left|y_1 + \frac{k_3}{2-\alpha}\lfloor y_2\rceil^{2-\alpha}\right|^{\frac{2}{2-\alpha}} + \frac{\gamma}{2}y_2^2 \tag{10}$$

and take γ in order to satisfy $K_1(1 - K_1k_3) = \gamma K_2$.

The derivative of V is rewritten as:

$$\begin{aligned} \dot{V} \le{}& -K_1\left|y_1 + \frac{k_3}{2-\alpha}\lfloor y_2\rceil^{2-\alpha}\right|^{\frac{\alpha}{2-\alpha}} k_3|y_2|^{1-\alpha} \quad \times \\ &\left\{K_2\left|y_1 + \frac{k_3}{2-\alpha}\lfloor y_2\rceil^{2-\alpha}\right|^{\frac{\alpha}{2-\alpha}} - \bar{p}\right\} \\ &- \gamma|y_2|[K_1|y_2|^{\alpha} - \bar{p}]. \end{aligned}$$

Set $a = \left|y_1 + \frac{k_3}{2-\alpha}\lfloor y_2\rceil^{2-\alpha}\right|$ and consider the following cases.

- Pick a positive constant θ_1 and assume that $|y_1| > \theta_1$, two cases occur:
  - if $a < \frac{\theta_1}{2}$: Notice first that one has $\theta_1 - \frac{k_3}{2-\alpha}|y_2|^{2-\alpha} < \frac{\theta_1}{2}$ which ensures that $|y_2| > \left(\frac{2-\alpha}{k_3}\frac{\theta_1}{2}\right)^{\frac{1}{2-\alpha}}$. Then the following inequalities

$$\begin{aligned} \dot{V} &\le K_1\left(\frac{\theta_1}{2}\right)^{\frac{\alpha}{2-\alpha}} k_3|y_2|^{1-\alpha}\bar{p} - \gamma K_1|y_2|^{1+\alpha} + \gamma|y_2|\bar{p} \\ &\le -\gamma K_1|y_2|^{1+\alpha} + \left[K_1\left(\frac{\theta_1}{2}\right)^{\frac{\alpha}{2-\alpha}} k_3 + \gamma\right]\bar{p} \\ &\le -\gamma K_1\left(\frac{2-\alpha}{k_3}\frac{\theta_1}{2}\right)^{\frac{1+\alpha}{2-\alpha}} + [K_1k_3 + \gamma]\,\bar{p} \end{aligned}$$

    are verified whenever

$$\boxed{\theta_1 > \frac{2k_3}{2-\alpha}\left[\frac{(K_1k_3+\gamma)\bar{p}}{\gamma K_1}\right]^{\frac{2-\alpha}{1+\alpha}}}$$

    which ensures that $\dot{V} < 0$.
  - if $a > \frac{\theta_1}{2}$: then one has

$$\begin{aligned} \dot{V} &\le -K_1\left(\frac{\theta_1}{2}\right)^{\frac{\alpha}{2-\alpha}}\left[k_3|y_2|^{1-\alpha}\left\{K_2\left(\frac{\theta_1}{2}\right)^{\frac{\alpha}{2-\alpha}} - \bar{p}\right\}\right] \\ &\quad - \gamma|y_2|[K_1|y_2|^{\alpha} - \bar{p}]. \\ &= -|y_2|^{1-\alpha}\left(\gamma K_1\left(|y_2|^{\alpha}\right)^2 - \gamma\bar{p}|y_2|^{\alpha}\right. \\ &\quad \left. + K_1\left(\frac{\theta_1}{2}\right)^{\frac{\alpha}{2-\alpha}} k_3\left\{K_2\left(\frac{\theta_1}{2}\right)^{\frac{\alpha}{2-\alpha}} - \bar{p}\right\}\right) \end{aligned}$$

    which corresponds to a second order polynomial in $|y_2|^{\alpha}$. The polynomial has definite sign (positive) if its parameters satisfies the conditions:

    1) $K_2\left(\frac{\theta_1}{2}\right)^{\frac{\alpha}{2-\alpha}} > 2\bar{p}$ and
    2) $(\gamma\bar{p})^2 < 4\gamma K_1^2k_3\left(\frac{\theta_1}{2}\right)^{\frac{\alpha}{2-\alpha}}\left\{K_2\left(\frac{\theta_1}{2}\right)^{\frac{\alpha}{2-\alpha}} - \bar{p}\right\}$.

    These conditions lead to the following ones appearing in the proposition statement:

    1) $\boxed{\theta_1 > 2\left(\frac{2\bar{p}}{K_2}\right)^{\frac{2-\alpha}{\alpha}}}$ and

2) $\boxed{\theta_1 > 2\left[\frac{(1-K_1k_3)\bar{p}}{4K_1K_2k_3}\right]^{\frac{2-\alpha}{\alpha}}}$

In addition, $\dot{V}$ can be zero if $y_2 = 0$. This is not possible whenever $|y_1| > \left(\frac{\bar{p}}{K_2}\right)^{\frac{2-\alpha}{\alpha}}$ according to (9).

- Pick a positive constant θ_2 and assume that $|y_2|^{2-\alpha} > \frac{2-\alpha}{k_3}\theta_2$, two cases occur:
  - if $a < \frac{\theta_2}{2}$: Notice first that since $\theta_2 - |y_1| < \frac{\theta_2}{2}$ then $|y_1| > \frac{\theta_2}{2}$. The following inequalities

$$\begin{aligned}\dot{V} &\le K_1\frac{\theta_2}{2}k_3|y_2|^{1-\alpha}\bar{p} - \gamma K_1|y_2|^{1+\alpha} + \gamma|y_2|\bar{p}\\ &\le -\gamma K_1|y_2|^{1+\alpha} + \left[K_1\frac{\theta_2}{2}k_3 + \gamma\right]\bar{p}\\ &\le -\gamma K_1\left(\frac{2-\alpha}{k_3}\theta_2\right)^{\frac{1+\alpha}{2-\alpha}} + \left[K_1k_3 + \gamma\right]\bar{p}\end{aligned}$$

are verified whenever

$$\boxed{\theta_2 > \frac{k_3}{2-\alpha}\left[\frac{(K_1k_3+\gamma)\bar{p}}{\gamma K_1}\right]^{\frac{2-\alpha}{1+\alpha}}}$$

  - if $a > \frac{\theta_2}{2}$: Then one has

$$\begin{aligned}\dot{V} &\le -K_1\left(\frac{\theta_2}{2}\right)^{\frac{\alpha}{2-\alpha}}\left[k_3|y_2|^{1-\alpha}\left\{K_2\left(\frac{\theta_2}{2}\right)^{\frac{\alpha}{2-\alpha}} - \bar{p}\right\}\right]\\ &- \gamma K_1|y_2|^{\alpha+1} + \gamma\bar{p}|y_2|\\ &= -|y_2|^{1-\alpha}\Bigg(\gamma K_1\left(|y_2|^{\alpha}\right)^2 - \gamma\bar{p}|y_2|^{\alpha}\\ &+K_1\left(\frac{\theta_2}{2}\right)^{\frac{\alpha}{2-\alpha}}k_3\left\{K_2\left(\frac{\theta_2}{2}\right)^{\frac{\alpha}{2-\alpha}} - \bar{p}\right\}\Bigg)\end{aligned}$$

which corresponds to a second order polynomial in $|y_2|^{\alpha}$. The polynomial has definite sign (positive) if its parameters satisfies the conditions:

1) $K_2\left(\frac{\theta_2}{2}\right)^{\frac{\alpha}{2-\alpha}} > 2\bar{p}$ and
2) $(\gamma\bar{p})^2 < 4\gamma K_1^2k_3\left(\frac{\theta_2}{2}\right)^{\frac{\alpha}{2-\alpha}}\left\{K_2\left(\frac{\theta_2}{2}\right)^{\frac{\alpha}{2-\alpha}} - \bar{p}\right\}$.

These conditions lead to the following ones appearing in the proposition statement:

1) $\boxed{\theta_2 > 2\left(\frac{2\bar{p}}{K_2}\right)^{\frac{2-\alpha}{\alpha}}}$ and
2) $\boxed{\theta_2 > 2\left[\frac{(1-K_1k_3)\bar{p}}{4K_1K_2k_3}\right]^{\frac{2-\alpha}{\alpha}}}$

■

According to proposition 4.2, the trajectories of (5) converges within the ball

$$\mathbb{B}_1 = \{(z_1, z_2),\ |z_1| \le \theta_1,\ |z_2| \le \theta_2 + hp\}.$$

V. NUMERICAL ISSUES

A. Derivative estimation

The system is equipped with a position sensor. However, its velocity is also needed for the controller implementation. A derivative estimator is given by:

$$\tilde{x}_2(t) = \frac{6}{T^3}\int_0^T (T-2s)x_1(t-s)ds. \tag{11}$$

This estimator, which has been used in [9], is first introduced in [6]. The convolution (11) is numerically approximated by a discrete one. With d to denote the sampling time, (11) is approximated by $\tilde{\tilde{x}}_2((k+1)d) = \sum_{i=0}^{7} h_1(i) \times x_1((k-i)d)$, $k \in \mathbb{N}$ where h_1 is a finite impulse response linear filter. It is given by $h_1 = [13.89,\ 37.04,\ 9.26,\ 0,\ -9.26,\ -37.04,\ -13.89]$ for our application with $d = 0.004$ and $T = d \times 7 = 0.028$ second.

B. Numerical approximation of z_1, z_2 and u

We assume that the delay h is a multiple of the sampling time d, that is $h = nd$ and $n \in \mathbb{N}$. $\tilde{z}_1$, $\tilde{z}_2$ and $\tilde{u}$ are given by the following discrete convolutions:

$$\begin{aligned}\tilde{z}_1((k+1)d) &= x_1(kd) + h\tilde{\tilde{x}}_2(kd)\\ &+ M\left(\frac{hu(kd-h)}{2} + \sum_{j=2}^{n-1}(jd)u((k-j)d)\right)\end{aligned}$$

$$\begin{aligned}\tilde{z}_2((k+1)d) &= \tilde{\tilde{x}}_2(kd)\\ &+ M\left(\frac{u(kd-h)}{2} + \sum_{j=2}^{n-1}u((k-j)d) + \frac{u(kd)}{2}\right)\end{aligned}$$

$$\begin{aligned}\tilde{u}((k+1)d) &= -k_1\lfloor\tilde{z}_2(kd)\rceil^{\alpha}\\ &- k_2\left[\tilde{z}_1(kd) + \frac{k_3}{2-\alpha}\lfloor\tilde{z}_2(kd)\rceil^{2-\alpha}\right]^{\frac{\alpha}{2-\alpha}}\end{aligned}$$

VI. EXPERIMENTATION

A. Platform description

As depicted in figure 3, the platform is composed by the following elements: a servovalve (Festo, MPYE-5-1-1/8-LF-010-B), a compressor and a pneumatic cylinder Festo, DNCI-32-200-P-A-MU). This cylinder can be briefly described as a double effect cylinder with a piston diameter of 32mm and a stroke length of 200mm. It is equipped with a built-in piston position sensor. The control law which is implemented in Matlab/Simulink is performed through a dSPACE 1103 microcontroller. The system monitoring in real time is realized by the ControlDesk software. Several experiments have been conducted in order to test the performances of the control law and to find the best value for the delay. In the sequel, all the parameters are fixed except the delay h. Note that tests are carried out with and without load for each delay value. The following tuning parameters are used $k_1 = 14$, $k_2 = 1$ and $k_3 = 0.1$.

B. Experimental tests

1) Test 1, $h = 0$ *second:* The objective of this test is to demonstrate that a second order delay-free model is not representative of the pneumatic actuator. With $h = 0$, one has $x = z = y$. Through this test with no load added to the cylinder, we can deduce that the value $h = 0$ is not the convenient one, because figure 4 shows oscillations caused by the miss-modelling of the delay.

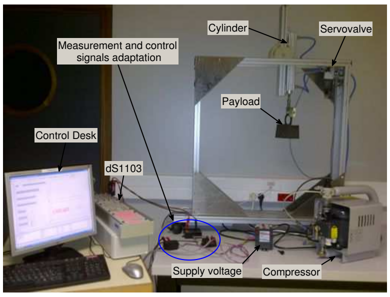

Fig. 3. Experimental setup

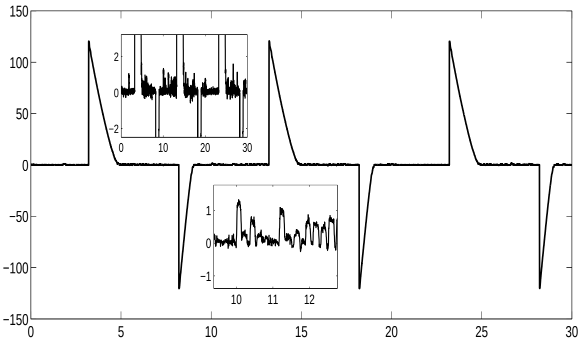

Fig. 4. Error versus time. h = 0, α = 0.1, no load.

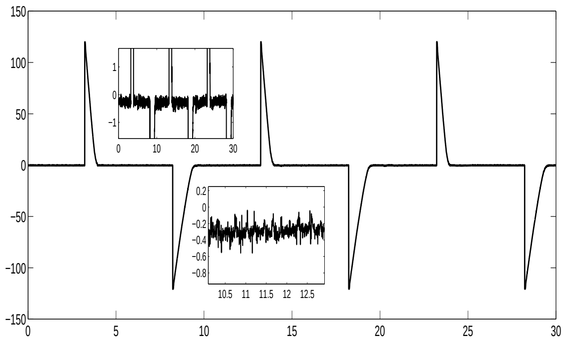

Fig. 5. Error versus time. h = 0.02, α = 0.1, no load.

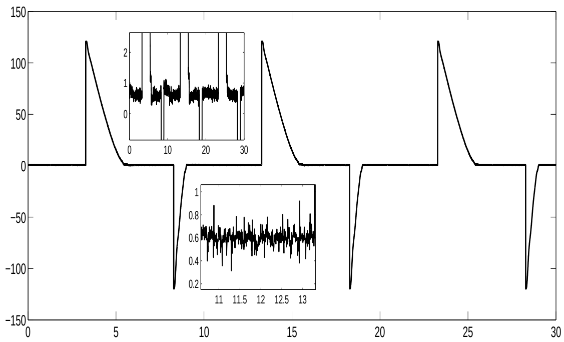

Fig. 6. Error versus time. h = 0.02, α = 0.1, with added load.

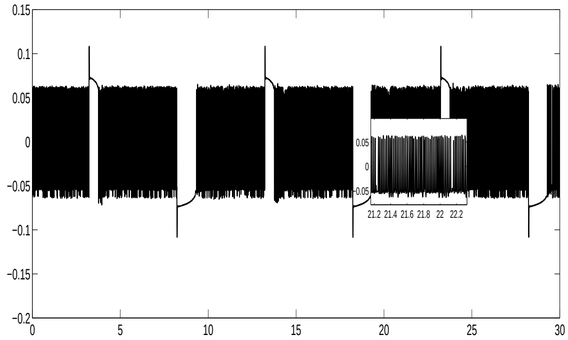

Fig. 7. Control effort versus time . h = 0.02, α = 0.1, no load.

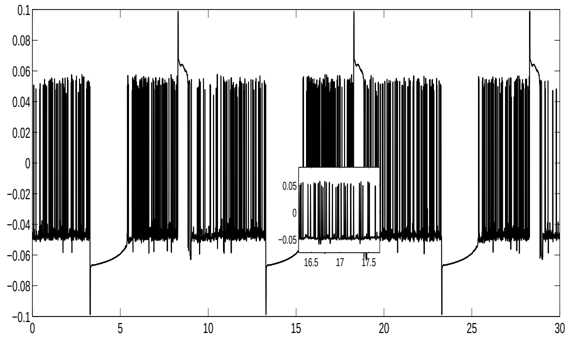

Fig. 8. Control effort versus time. h = 0.02, α = 0.1, with added load.

2) Test 2, h = 0.02 *second:* Better results are observed with a delay value h = 0.02. The robustness of the control law is clearly proved in this case as depicted in figure 5 and 6. Indeed, the static error is less than 0.4mm without load and it is about 0.6mm with an added load. Figure 7 and 8 correspond to the input control signal where a chattering (see equation 6) with frequency approximately about 50Hz can be perceived. It is an acceptable frequency since the natural frequency of oscillation of the servovalve is 100Hz.

3) Test 3, h = 0.06 *second:* A delay of 0.06 second is not adequate since the experiments revealed a much bigger static error (see figures 9, 10, 11 and 12).

VII. Conclusion

A second order model with delayed input is proposed to represent the dynamics of the pneumatic actuator. A homogeneous robust nonlinear controller is synthesized based on the proposed model. The validity of the model as well as the controller were demonstrated experimentally.

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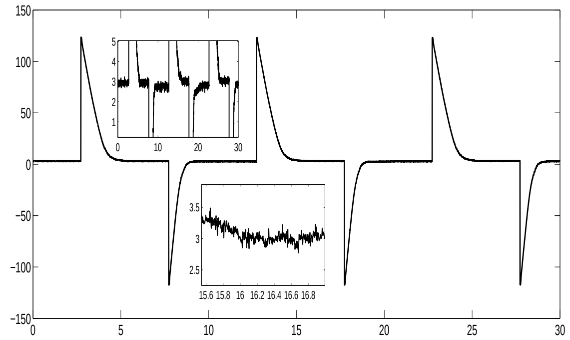

Fig. 9. Error effort versus time. h = 0.06, α = 0.1, no load.

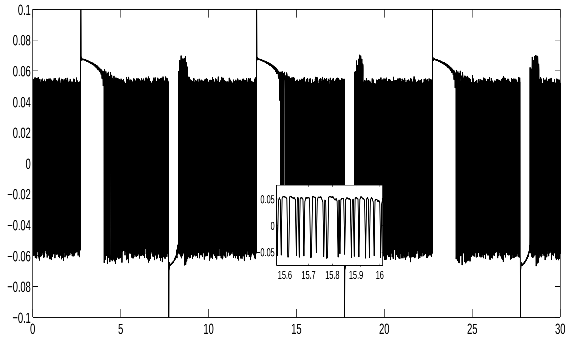

Fig. 10. Control effort versus time. h = 0.06, α = 0.1, no load.

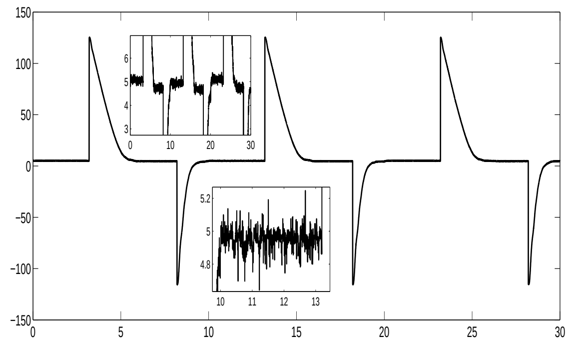

Fig. 11. Error effort versus time. h = 0.06, α = 0.1, with added load.

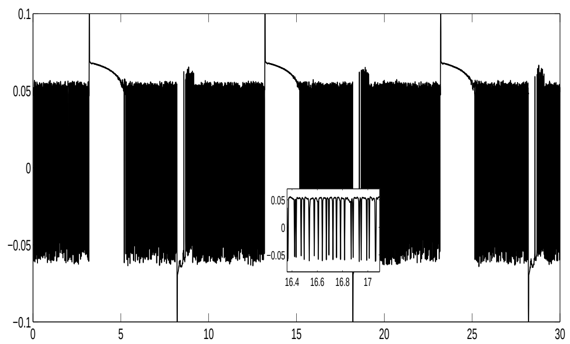

Fig. 12. Control effort versus time. h = 0.06, α = 0.1, with added load.

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