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# History-Preserving Bisimilarity for Higher-Dimensional Automata via Open Maps Extended Abstract

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One of the popular notions of equivalence for non-interleaving concurrent systems is *history-preserving bisimilarity* (*hp-bisimilarity*). *Higher-dimensional automata* (*HDA*) [6], [7] is a non-interleaving formalism for reasoning about behavior of concurrent systems, which provides a generalization (up to hp-bisimilarity) to “the main models of concurrency proposed in the literature” [8].

Using open maps [4], we can show that hp-bisimilarity for HDA has a characterization directly in terms of (higher-dimensional) *transitions* of the HDA, rather than in terms of runs as *e.g.* for Petri nets. Our results imply *decidability* of hp-bisimilarity for finite HDA. They also put hp-bisimilarity firmly into the open-maps framework of [4] and tighten the connections between bisimilarity and weak topological fibrations [1], [5].

A full version of this report is available as [3].

A *precubical set* is a graded set  $X = \{X_n\}_{n \in \mathbb{N}}$  together with mappings  $\delta_k^\nu : X_n \rightarrow X_{n-1}$ ,  $k = 1, \dots, n$ ,  $\nu = 0, 1$ , satisfying the *precubical identity*  $\delta_k^\nu \delta_\ell^\mu = \delta_{\ell-1}^\mu \delta_k^\nu$  for  $k < \ell$ . The mappings  $\delta_k^\nu$  are called *face maps*, and elements of  $X_n$  are called *n-cubes*. Faces  $\delta_k^0 x$  of an element  $x \in X$  are to be thought of as *lower faces*,  $\delta_k^1 x$  as *upper faces*. *Morphisms*  $f : X \rightarrow Y$  of precubical sets are graded mappings  $f = \{f_n : X_n \rightarrow Y_n\}_{n \in \mathbb{N}}$  which commute with the face maps:  $\delta_k^\nu \circ f_n = f_{n-1} \circ \delta_k^\nu$ . This defines a category  $\mathbf{pCub}$  of precubical sets and morphisms.

The category of *higher-dimensional automata* is the comma category  $\mathbf{HDA} = * \downarrow \mathbf{pCub}$  of *pointed precubical sets* and with morphisms which respect the point.

We say that a precubical set  $X$  is a *path object* if there is a (necessarily unique) sequence  $(x_1, \dots, x_m)$  of elements in  $X$  such that  $x_i \neq x_j$  for  $i \neq j$ ,

- for each  $x \in X$  there is  $j \in \{1, \dots, m\}$  for which  $x = \delta_{k_1}^{\nu_1} \dots \delta_{k_p}^{\nu_p} x_j$  for some indices  $\nu_1, \dots, \nu_p$  and a *unique* sequence  $k_1 < \dots < k_p$ , and
- for each  $j = 1, \dots, m-1$ , there is  $k \in \mathbb{N}$  for which  $x_j = \delta_k^0 x_{j+1}$  or  $x_{j+1} = \delta_k^1 x_j$ .

If  $X$  and  $Y$  are path objects with representations  $(x_1, \dots, x_m)$ ,  $(y_1, \dots, y_p)$ , then a morphism  $f : X \rightarrow Y$  is called a *path extension* if  $x_j = y_j$  for all  $j = 1, \dots, m$  (hence  $m \leq p$ ). The category  $\mathbf{HDP}$  of *higher-dimensional paths* (*HDP*) is the subcategory of  $\mathbf{HDA}$  which as objects has pointed path objects, and whose morphisms are generated by isomorphisms and

pointed path extensions.

Following [2], we say that a morphism in  $\mathbf{HDA}$  is *open* if it has the right lifting property with respect to  $\mathbf{HDP}$ , and that  $\mathbf{HDA}$   $X, Y$  are *bisimilar* if there is  $Z \in \mathbf{HDA}$  and a span of open maps  $X \leftarrow Z \rightarrow Y$  in  $\mathbf{HDA}$ . It can be shown [2] that  $X$  and  $Y$  are bisimilar iff  $n$ -cubes with matching lower faces can be matched; this is a straight-forward generalization of ordinary bisimulation for transition systems and appears hence to be rather badly suited for concurrent systems. We can, however, show that this bisimilarity is precisely hp-bisimilarity.

A *cube path* in a precubical set  $X$  is a morphism  $P \rightarrow X$  from a path object  $P$ . Using the notion of adjacency from [7], [8], we can define what it means for two cube paths to be *homotopic*, *i.e.* to represent the same execution up to concurrency. The *unfolding*  $\tilde{X}$  of a  $\mathbf{HDA}$   $X$  is then defined to be the set of homotopy classes of pointed cube paths in  $X$ . With suitable structure maps, this becomes a precubical set, indeed, a *higher-dimensional tree*.

The category of *HDA up to homotopy*  $\mathbf{HDA}_h$  has as objects  $\mathbf{HDA}$  and as morphisms pointed precubical morphisms  $f : \tilde{X} \rightarrow \tilde{Y}$  of unfoldings. Noting that any  $\mathbf{HDP}$  is isomorphic to its own unfolding, we have an embedding  $\mathbf{HDP} \hookrightarrow \mathbf{HDA}_h$ . We can then say that a morphism in  $\mathbf{HDA}_h$  is *homotopy open* if it has the right lifting property with respect to  $\mathbf{HDP}$  and define *homotopy bisimilarity* accordingly.

*Theorem.* Two  $\mathbf{HDA}$  are homotopy bisimilar iff they are hp-bisimilar [8], iff they are bisimilar.

Using an arrow category, we can easily extend the above considerations to the (more interesting) case of *labeled HDA*.

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