# Trigonometrical form of zeta-function for negative (n) with proof for Riemann hypothesis.

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# Trigonometrical form of zeta-function for negative (n) with proof for Riemann hypothesis.

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The Riemann hypothesis is not proved by more, than 150 years. At this paper, I presented new solution for this problem. I found new trigonometrical form of Riemann's zeta function for negative numbers (n). This new form of zeta gives opportunity to prove the Riemann hypothesis. Presented proof isn't complicated for trigonometrical form of zeta function.



## 1. Introduction

Riemann's zeta hypothesis from 1859 [1] is expressed as follows:

CONJECTURE *The non trivial zeros of the Riemann zeta function ζ(s) all have real part Re(s) =1/2.*

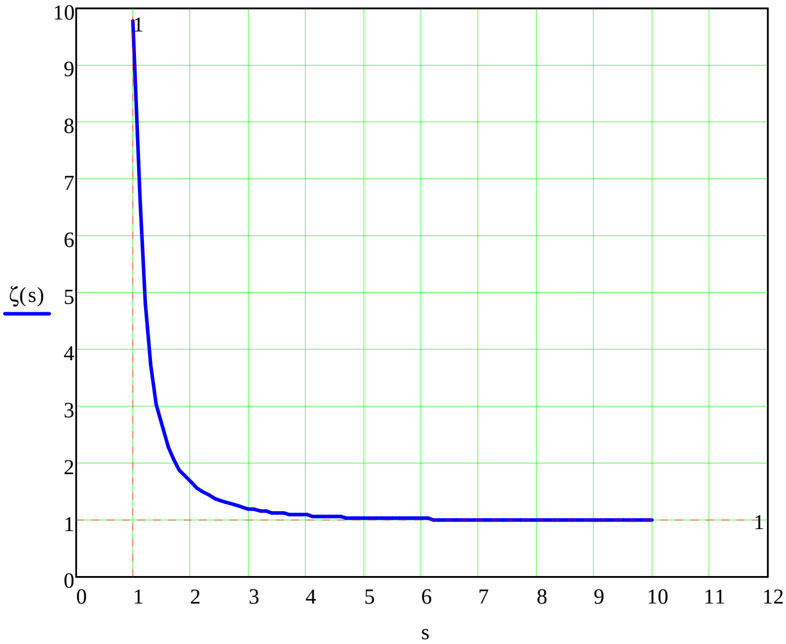
$$z = \frac{1}{2} + i \cdot t$$

The Riemann zeta hypothesis is the most famous of the few still unsolved problems on Hilbert's list of twenty-three mathematical challenges, which he presented in 1900 at the dawn of the new century [2]. It is also one of the seven Millennium Problems [3] named in 2000 by the Clay Mathematics Institute.
From the pure mathematical point of view, to prove this hypothesis is a very challenging task.
Here, I try to prove the Riemann hypothesis by applying a new trigonometrical form of zeta functions.

## 2. Definition

The Riemann zeta function *ζ (s)* is defined for *Re(s)>1* as:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \qquad \mathrm{Re}(s) > 1$$



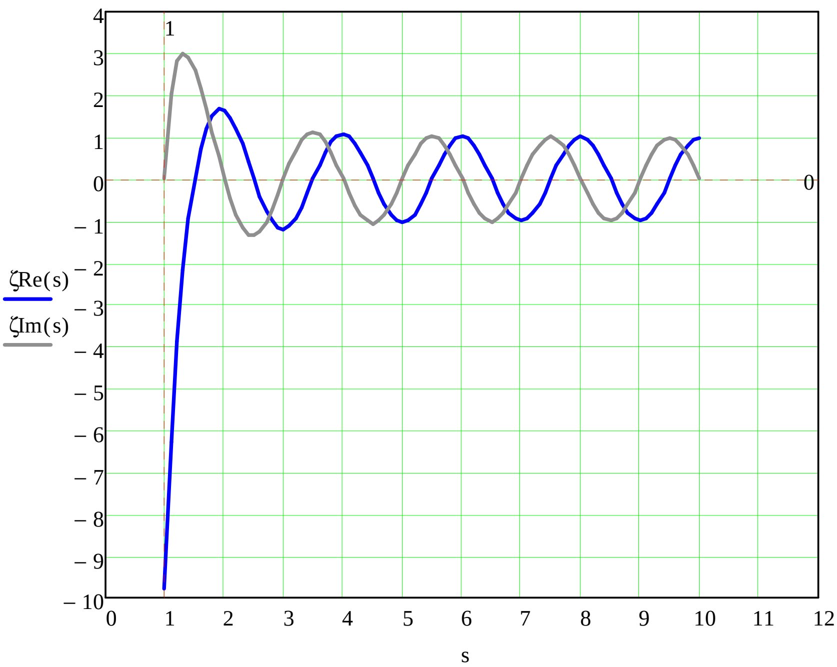

| ζ(s) = | s = |
|---|---|
| 9.788 | 1 |
| 6.603 | 1.1 |
| 4.799 | 1.2 |
| 3.722 | 1.3 |
| 3.043 | 1.4 |
| 2.592 | 1.5 |
| 2.279 | 1.6 |
| 2.052 | 1.7 |
| 1.881 | 1.8 |
| 1.749 | 1.9 |
| 1.645 | 2 |
| 1.56 | 2.1 |
| 1.491 | 2.2 |
| 1.432 | 2.3 |
| 1.383 | 2.4 |
| ... | ... |

**Figure 1. Riemann zeta function *ζ (s)*.**

## 3. Zeta function for negative numbers (n)

For negative numbers (n), Riemann zeta function *ζ (s)* is a complex function. We can select real part and imaginary part of zeta function.

$$\zeta(s) = \sum_{n=-1}^{-\infty} \frac{1}{n^s} \qquad \mathrm{Re}(s) > 1 \qquad \zeta Re(s)=Re(\zeta(s)) \qquad \zeta Im(s)=Im(\zeta(s))$$




ζ(s) =

| | 0 |
|---|---|
| 0 | -9.788 |
| 1 | -6.28+2.041i |
| 2 | -3.883+2.821i |
| 3 | -2.188+3.011i |
| 4 | -0.94+2.894i |
| 5 | 2.592i |
| 6 | 0.704+2.168i |
| 7 | 1.206+1.66i |
| 8 | 1.522+1.106i |
| 9 | 1.664+0.541i |
| 10 | 1.645 |
| 11 | 1.484-0.482i |
| 12 | 1.206-0.876i |
| 13 | 0.842-1.159i |
| 14 | 0.427-1.316i |
| 15 | ... |

**Figure 2. Riemann zeta function for negative (n). Real part and imaginary part.**

## 4. Equivalent form of zeta function for negative numbers (n)

We can express the Riemann zeta function $\zeta(s)$ for negative numbers (n) as equivalent formula. This new equation of zeta consists of two parts. First part is a "normal" zeta function $\zeta(s)$. Second part of equation is a module of negative values. This module is represented by complex numbers.

$$\zeta(s)=\sum_{n=-1}^{-\infty}\frac{1}{n^s}$$

$$\zeta(s)=\sum_{n=-1}^{-\infty}\frac{1}{n^s}=\sum_{n=1}^{\infty}\frac{1}{(-n)^s}=\sum_{n=1}^{\infty}\frac{1}{(-1\cdot n)^s}$$

---

Example: $(a\cdot b)^s=a^s\cdot b^s \qquad a\to -1 \qquad$ *and* $\qquad b\to n$

$(-1\cdot n)^s=(-1)^s\cdot(n)^s$

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$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{(-1\cdot n)^s}=\sum_{n=1}^{\infty}\frac{1}{n^s}\cdot\frac{1}{(-1)^s}=\sum_{n=1}^{\infty}\frac{1}{n^s}\cdot\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}$$

$$\zeta(s)=\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\ \overset{equivalent}{\Leftrightarrow}\ \zeta(s)=\sum_{n=-1}^{-\infty}\frac{1}{n^s}$$

The standard zeta function

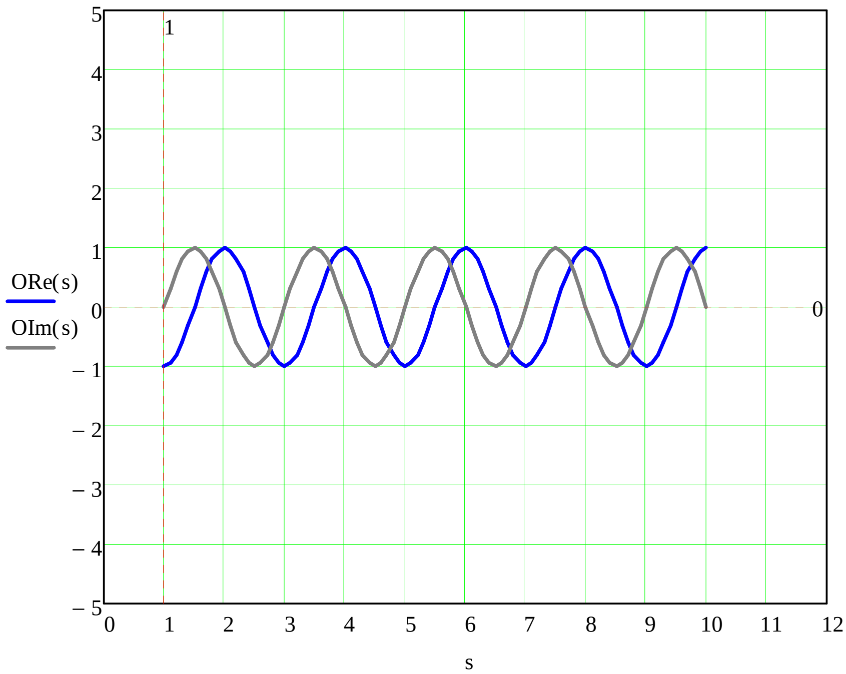
module of negative values (complex numbers)

Below are presented a few numerical values. Zeta function and equivalent form of zeta function for negative numbers :

| s = | $\sum_{n=1}^{\infty}\frac{1}{n^s}$ ζ(s) = | | $\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}$ ζ(s) = 0 | $\mathrm{Re}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right)$ ORe(s) = | $\mathrm{Im}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right)$ OIm(s) = |
|---|---|---|---|---|---|
| 1 | 9.788 | 0 | -9.788 | -1 | 0 |
| 1.1 | 6.603 | 1 | -6.28+2.041i | -0.951 | 0.309 |
| 1.2 | 4.799 | 2 | -3.883+2.821i | -0.809 | 0.588 |
| 1.3 | 3.722 | 3 | -2.188+3.011i | -0.588 | 0.809 |
| 1.4 | 3.043 | 4 | -0.94+2.894i | -0.309 | 0.951 |
| 1.5 | 2.592 | 5 | 2.592i | 0 | 1 |
| 1.6 | 2.279 | 6 | 0.704+2.168i | 0.309 | 0.951 |
| 1.7 | 2.052 | 7 | 1.206+1.66i | 0.588 | 0.809 |
| 1.8 | 1.881 | 8 | 1.522+1.106i | 0.809 | 0.588 |
| 1.9 | 1.749 | 9 | 1.664+0.541i | 0.951 | 0.309 |
| 2 | 1.645 | 10 | 1.645 | 1 | 0 |
| 2.1 | 1.56 | 11 | 1.484-0.482i | 0.951 | -0.309 |
| 2.2 | 1.491 | 12 | 1.206-0.876i | 0.809 | -0.588 |
| 2.3 | 1.432 | 13 | 0.842-1.159i | 0.588 | -0.809 |
| 2.4 | 1.383 | 14 | 0.427-1.316i | 0.309 | -0.951 |
| ... | ... | 15 | ... | ... | ... |

**Figure 3. Numerical comparison of various components.**

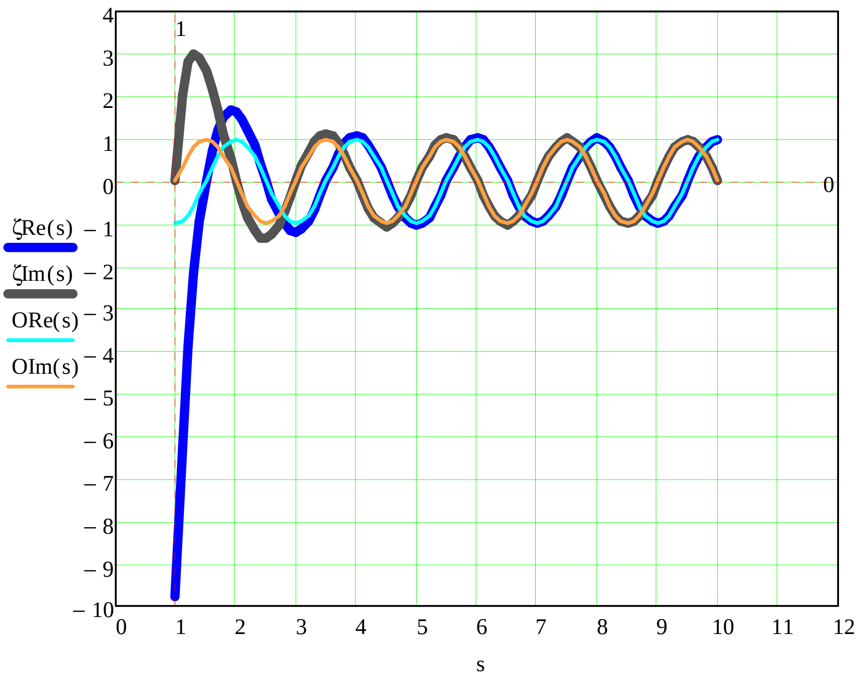
$$One(s) = \frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}} = \frac{1}{i^{2\cdot s}} \qquad i = \sqrt{-1} \qquad ORe(s)=Re(One(s)) \qquad OIm(s)=Im(One(s))$$




$$\mathrm{Re}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right)$$

$$\mathrm{Im}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right)$$

**Figure 4. Characteristic of *One(s).* Variable *One(s)* is a module of negative values (complex numbers). Real part and imaginary part.**




$$\mathrm{Re}\left[\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right]$$

$$\mathrm{Im}\left[\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right]$$

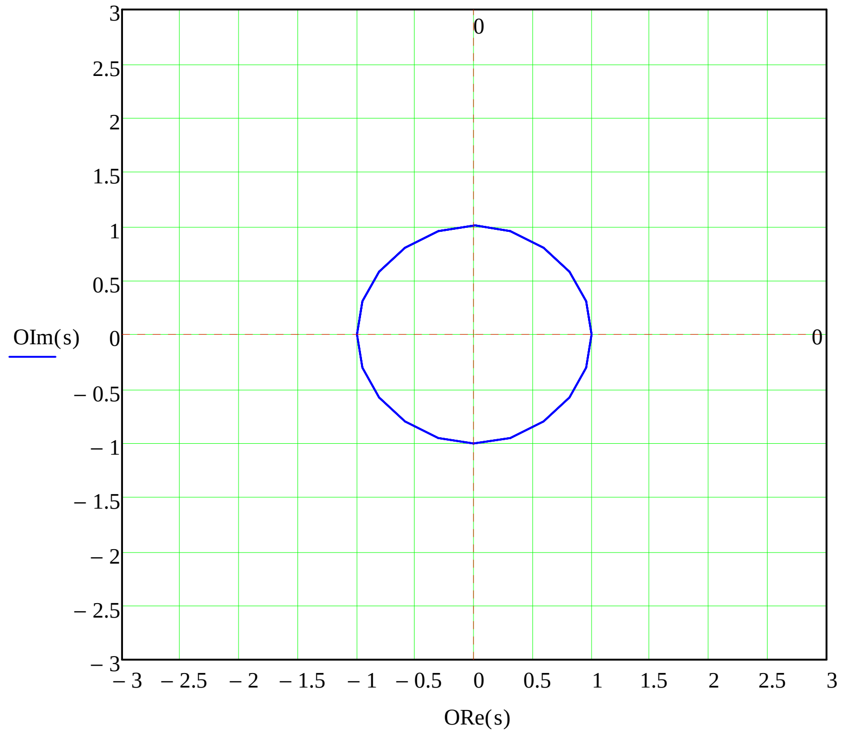
*and*

$$\mathrm{Re}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right)$$

$$\mathrm{Im}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right)$$

**Figure 5. Riemann zeta function for negative (n). Comparision of zeta *ζ (s)* with *One(s).* Real parts and imaginary parts.**

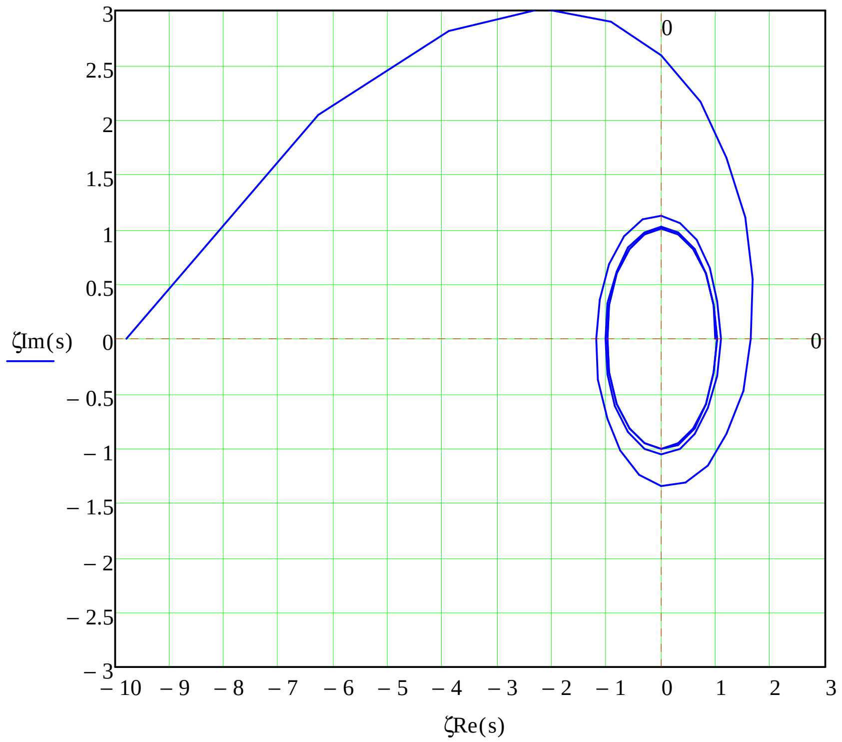
For high value of s (4,5,6,….), ***ζRe(s)*** and ***ORe(s)*** seems to be only one line. Similarly is for imaginary values ( ***ζIm(s)*** and ***OIm(s)***).




$$\mathrm{Re}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right)$$

$$\mathrm{Im}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right)$$

**Figure 6. Characteristic of *One(s)* on comlex plane. Real axis with imaginary axis.**




$$\mathrm{Re}\left[\left(\sum_{n=1}^{\infty}\frac{1}{n^{s}}\right)\cdot\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right]$$

$$\mathrm{Im}\left[\left(\sum_{n=1}^{\infty}\frac{1}{n^{s}}\right)\cdot\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right]$$

**Figure 7. Characteristic of zeta *ζ (s)* on comlex plane. Real axis with imaginary axis.**

## 5. Transformation of zeta function to trigonometrical form (negative n)

Real part and imaginary part of module (***One(s)***), seems to be a some kind of sinusoidal function. That assumption was confirmed by numerical solution and comparison of sinusoidal functions. That was not a big challenge for a computer software like Mathcad. I checked several of sinusoidal functions and choose only two, which possess the same shape as (***ORe(s)*** and ***OIm(s))***. One sinusoidal equivalent for real part ***ORe(s)*** and one sinusoidal equivalent for imaginary part ***OIm(s)***. Discovering of new equation by numerical comparison, probably is not a professional method, but it works. Correct formula of trigonometrical zeta function was presented below.

$$\mathrm{Re}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right) \overset{equivalent}{\Leftrightarrow} \sin\left(-\pi\cdot s+\frac{\pi}{2}\right) \qquad \mathrm{Im}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right) \overset{equivalent}{\Leftrightarrow} \sin(-\pi\cdot s)$$

| s = | ORe(s) = | $\sin\left(-\pi\ s+\frac{\pi}{2}\right)$ = | OIm(s) = | $\sin(-\pi\ s)$ = |
|---|---|---|---|---|
| 1 | -1 | -1 | 0 | 0 |
| 1.1 | -0.951 | -0.951 | 0.309 | 0.309 |
| 1.2 | -0.809 | -0.809 | 0.588 | 0.588 |
| 1.3 | -0.588 | -0.588 | 0.809 | 0.809 |
| 1.4 | -0.309 | -0.309 | 0.951 | 0.951 |
| 1.5 | 0 | 0 | 1 | 1 |
| 1.6 | 0.309 | 0.309 | 0.951 | 0.951 |
| 1.7 | 0.588 | 0.588 | 0.809 | 0.809 |
| 1.8 | 0.809 | 0.809 | 0.588 | 0.588 |
| 1.9 | 0.951 | 0.951 | 0.309 | 0.309 |
| 2 | 1 | 1 | 0 | 0 |
| 2.1 | 0.951 | 0.951 | -0.309 | -0.309 |
| 2.2 | 0.809 | 0.809 | -0.588 | -0.588 |
| 2.3 | 0.588 | 0.588 | -0.809 | -0.809 |
| 2.4 | 0.309 | 0.309 | -0.951 | -0.951 |
| ... | ... | ... | ... | ... |

**Figure 8. Numerical comparison of various components. Comparison of real part *ORe(s)* with a proper sinusoidal function. Similarly for imaginary part *OIm(s)*.**

$$One(s)=\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}$$

$$\mathrm{Re}(One(s))=\mathrm{Re}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right)=\sin\left(-\pi\cdot s+\frac{\pi}{2}\right)$$

$$\mathrm{Im}(One(s))=\mathrm{Im}\left(\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}\right)=\sin(-\pi\cdot s)$$

$$One(s)=\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}=(a+bi)=\mathrm{Re}(One(s))+\mathrm{Im}(One(s))=\sin\left(-\pi\cdot s+\frac{\pi}{2}\right)+\sin(-\pi\cdot s)i$$

$$One(s)=\sin\left(-\pi\cdot s+\frac{\pi}{2}\right)+\sin(-\pi\cdot s)i$$

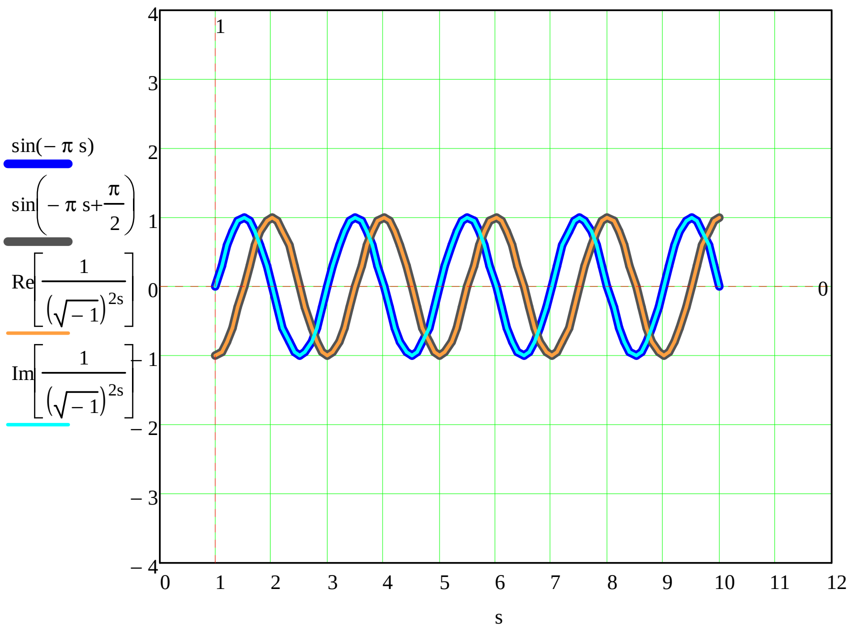



**Figure 9. Visual comparison of real part of module *One(s)* with a trigonometrical equivalent. Similarly for imaginary part of module *One(s)*.**

$$\zeta(s)=\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\frac{1}{\left(\sqrt{-1}\right)^{2\cdot s}}=\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot One(s)$$

$$\zeta(s)=\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\left(\sin\left(-\pi\cdot s+\frac{\pi}{2}\right)+\sin(-\pi\cdot s)i\right)$$

Example: (multiplication of complex numbers)

$$Z_1\cdot Z_2=(a_1+b_1i)\cdot(a_2+b_2i)=(a_1\cdot a_2-b_1\cdot b_2)+(a_1\cdot b_2+a_2\cdot b_1)i$$

$$\zeta(s)=\left(\sum_{n=1}^{\infty}\frac{1}{n^s}+0i\right)\cdot\left(\sin\left(-\pi\cdot s+\frac{\pi}{2}\right)+\sin(-\pi\cdot s)i\right)$$

$$\zeta(s)=\left[\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\left(\sin\left(-\pi\cdot s+\frac{\pi}{2}\right)\right)-0\right]+\left[\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\left(\sin(-\pi\cdot s)\right)+0\right]\cdot i$$

**Trigonometrical version of Riemann zeta function:**

$$\zeta(s)=\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\left(\sin\left(-\pi\cdot s+\frac{\pi}{2}\right)\right)+\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\left(\sin(-\pi\cdot s)\right)i$$

$$\mathrm{Re}(\zeta(s))=\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\left(\sin\left(-\pi\cdot s+\frac{\pi}{2}\right)\right)$$ **real part of zeta function**

$$\mathrm{Im}(\zeta(s))=\left(\sum_{n=1}^{\infty}\frac{1}{n^s}\right)\cdot\left(\sin(-\pi\cdot s)\right)$$ **imaginary part of zeta function**

## 6. Trigonometrical form of zeta function & Riemann hypothesis

Riemann's zeta hypothesis is expressed as follows:

CONJECTURE *The non trivial zeros of the Riemann zeta function ζ(s) all have real part Re(s) =1/2.*

$$z = \frac{1}{2} + i \cdot t$$

We can try testing this hypothesis for trigonometrical version of zeta function. Why not? It's quite simple operation. At first, we should note that the complex function is a zero, if real part is a zero and imaginary part is also a zero.

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Example: (zero & complex numbers)

$z = a + bi$ $\quad$ $i = \sqrt{-1}$

$z = 0$ if $a = 0$ *and* $b = 0$

$z = 0 \longrightarrow z = 0 + 0i$

$\zeta(s) = 0 \xrightarrow{\text{if}} \mathrm{Re}(\zeta(s)) = 0$ *and* $\mathrm{Im}(\zeta(s)) = 0$

---

$$\zeta(s) = 0 = \left(\sum_{n=1}^{\infty} \frac{1}{n^s}\right) \cdot \left(\sin\left(-\pi \cdot s + \frac{\pi}{2}\right)\right) + \left(\sum_{n=1}^{\infty} \frac{1}{n^s}\right) \cdot (\sin(-\pi \cdot s))i$$

(real part of zeta should be a zero) & (imaginary part of zeta should be a zero)

---

### 6.1 Consideration of trigonometrical form of zeta function for *(S є N)*

$$S \in N \quad \Leftrightarrow \quad S \in (1,2,3,4,\ldots,\infty)$$

*Imaginary part of zeta is always a zero* $\mathrm{Im}(\zeta(s)) = 0$ *, for* $S \in (1,2,3,4,\ldots,\infty)$

$$\mathrm{Im}(\zeta(s)) = 0 = \left(\sum_{n=1}^{\infty} \frac{1}{n^S}\right) \cdot (\sin(-\pi \cdot S)) \qquad \textit{because} \qquad \sin(-\pi \cdot S) = 0$$

*Real part of zeta* $\mathrm{Re}(\zeta(s))$ *is not a zero for* $S \in (1,2,3,4,\ldots,\infty)$

$$\mathrm{Re}(\zeta(s)) = \left(\sum_{n=1}^{\infty} \frac{1}{n^S}\right) \cdot \left(\sin\left(-\pi \cdot S + \frac{\pi}{2}\right)\right) = \left(\sum_{n=1}^{\infty} \frac{1}{n^S}\right) \cdot sign \qquad sign \in \{-1,1\}$$

$$\mathrm{Re}(\zeta(s)) = \left(\sum_{n=1}^{\infty} \frac{1}{n^S}\right) \cdot sign \qquad sign \in \{-1,1\}$$

$$sign = \sin\left(-\pi \cdot S + \frac{\pi}{2}\right) = \sin\left(-\pi \cdot \left(S - \frac{1}{2}\right)\right)$$ *, where:*

$$\sin\left(-\pi \cdot S + \frac{\pi}{2}\right) = 1 \qquad \textit{for} \qquad S \in (2,4,6,8,\ldots,\infty)$$

$$\sin\left(-\pi \cdot S + \frac{\pi}{2}\right) = -1 \qquad \textit{for} \qquad S \in (1,3,5,7,\ldots,\infty)$$

| S = | $\sin\left(-\pi S + \frac{\pi}{2}\right)$ | $\left(\sum_{n=1}^{m} \frac{1}{n^S}\right) \cdot \sin\left(-\pi S + \frac{\pi}{2}\right)$ |
|---|---|---|
| 1 | -1 | -9.788 |
| 2 | 1 | 1.645 |
| 3 | -1 | -1.202 |
| 4 | 1 | 1.082 |
| 5 | -1 | -1.037 |
| 6 | 1 | 1.017 |
| 7 | -1 | -1.008 |
| 8 | 1 | 1.004 |
| 9 | -1 | -1.002 |
| 10 | 1 | 1.001 |

**Figure 10. Numerical solution for trigonometrical form of zeta function *(S ϵ N).***

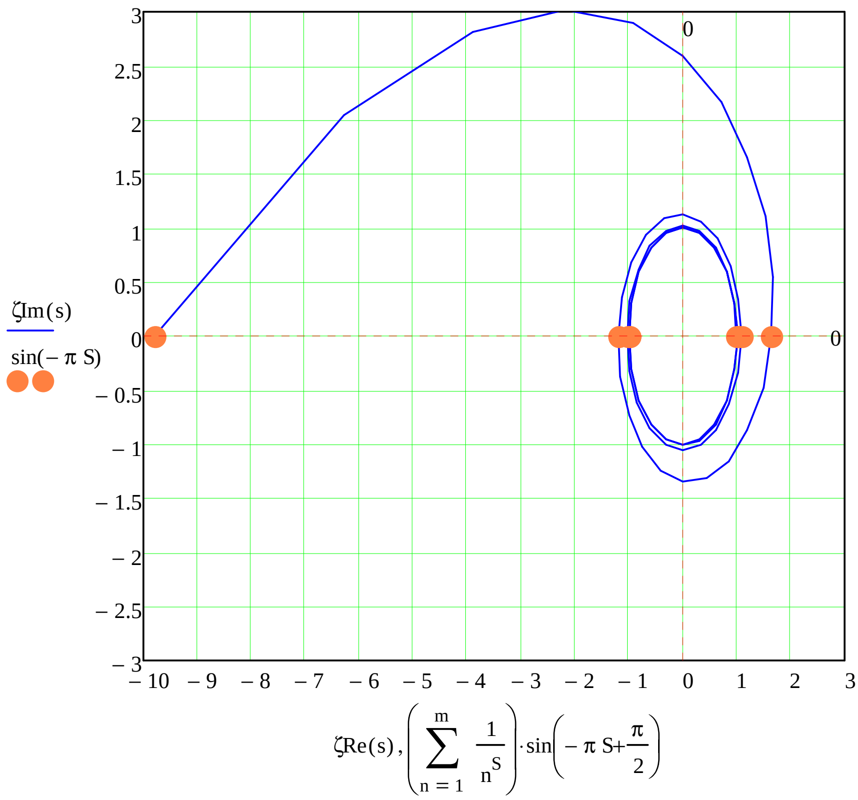


**Figure 11. Visual presentation for solved points. Real part of zeta is not a zero (*ζRe(s)≠0*). Imaginary part of zeta is always zero (*ζIm(s)=0*).**

### 6.2 Consideration of trigonometrical form of zeta function for *(S + 1/2)*

$$\left(S+\frac{1}{2}\right) \quad \Leftrightarrow \quad S \in (1,2,3,4,\ldots,\infty)$$

*Real part of zeta* $\mathrm{Re}(\zeta(s))$ *:*

$$\mathrm{Re}(\zeta(s)) = \left(\sum_{n=1}^{\infty}\frac{1}{n^{S}}\right)\cdot\left(\sin\left(-\pi\cdot S+\frac{\pi}{2}\right)\right) = \left(\sum_{n=1}^{\infty}\frac{1}{n^{S}}\right)\cdot\left(\sin\left(-\pi\left(\cdot S-\frac{1}{2}+\frac{1}{2}\right)\right)\right) = \left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot(\sin(-\pi\cdot S))$$

adding constant $x=\frac{1}{2}$ $\qquad x=\frac{1}{2}$

$$\mathrm{Re}(\zeta(s)) = \left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot(\sin(-\pi\cdot S)) = 0 \quad \textit{for} \quad S \in (1,2,3,4,\ldots,\infty) \quad \textit{and} \quad x=\frac{1}{2}$$

$$\mathrm{Re}(S) = \frac{1}{2} \quad \textit{for} \quad S \in (1,2,3,4,\ldots,\infty)$$

*Real part of zeta is a zero* $\mathrm{Re}(\zeta(s)) = 0$ *, but always for a real constant* $\mathrm{Re}(S) = \frac{1}{2}$

*Imaginary part of zeta* $\mathrm{Im}(\zeta(s))$ *:*

$$\mathrm{Im}(\zeta(s)) = \left(\sum_{n=1}^{\infty}\frac{1}{n^{S}}\right)\cdot(\sin(-\pi\cdot S)) = \left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot\left(\sin\left(-\pi\cdot\left(S+\frac{1}{2}\right)\right)\right) = \left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot \textit{sign}$$

adding constant $x=\frac{1}{2}$ $\qquad x=\frac{1}{2}$

$$\mathrm{Im}(\zeta(s)) = \left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot \textit{sign} \qquad \textit{sign} \in \{-1,1\}$$

$$\textit{sign} = \sin\left(-\pi\cdot\left(S+\frac{1}{2}\right)\right)$$

$$\sin\left(-\pi\cdot\left(S+\frac{1}{2}\right)\right) = 1 \quad \textit{for} \quad S \in (1,3,5,7,\ldots,\infty)$$

$$\sin\left(-\pi\cdot\left(S+\frac{1}{2}\right)\right) = -1 \quad \textit{for} \quad S \in (2,4,6,8,\ldots,\infty)$$

$$\mathrm{Im}(\zeta(s)) = \left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot \textit{sign} \quad \textit{for} \quad \left(\frac{1}{2}\right) \textit{ and } S \in (1,2,3,4,\ldots,\infty) \quad \textit{sign} \in \{-1,1\}$$

*Imaginary part of zeta* $\mathrm{Im}(\zeta(s))$ *is not a zero for* real constant $\mathrm{Re}(S) = \frac{1}{2}$ *and* $S \in (1,2,3,4,\ldots,\infty)$

| $S =$ | $S + \frac{1}{2} =$ | $\sin\left[-\pi\left(S + \frac{1}{2}\right)\right]$ | $\left(\sum_{n=1}^{m} \frac{1}{n^{S+0.5}}\right) \cdot \sin\left[-\pi\left(S + \frac{1}{2}\right)\right]$ |
|---|---|---|---|
| 1 | 1.5 | 1 | 2.592 |
| 2 | 2.5 | -1 | -1.341 |
| 3 | 3.5 | 1 | 1.127 |
| 4 | 4.5 | -1 | -1.055 |
| 5 | 5.5 | 1 | 1.025 |
| 6 | 6.5 | -1 | -1.012 |
| 7 | 7.5 | 1 | 1.006 |
| 8 | 8.5 | -1 | -1.003 |
| 9 | 9.5 | 1 | 1.001 |
| 10 | 10.5 | -1 | -1.001 |

**Figure 12. Numerical solution for trigonometrical form of zeta function *(S+1/2).***

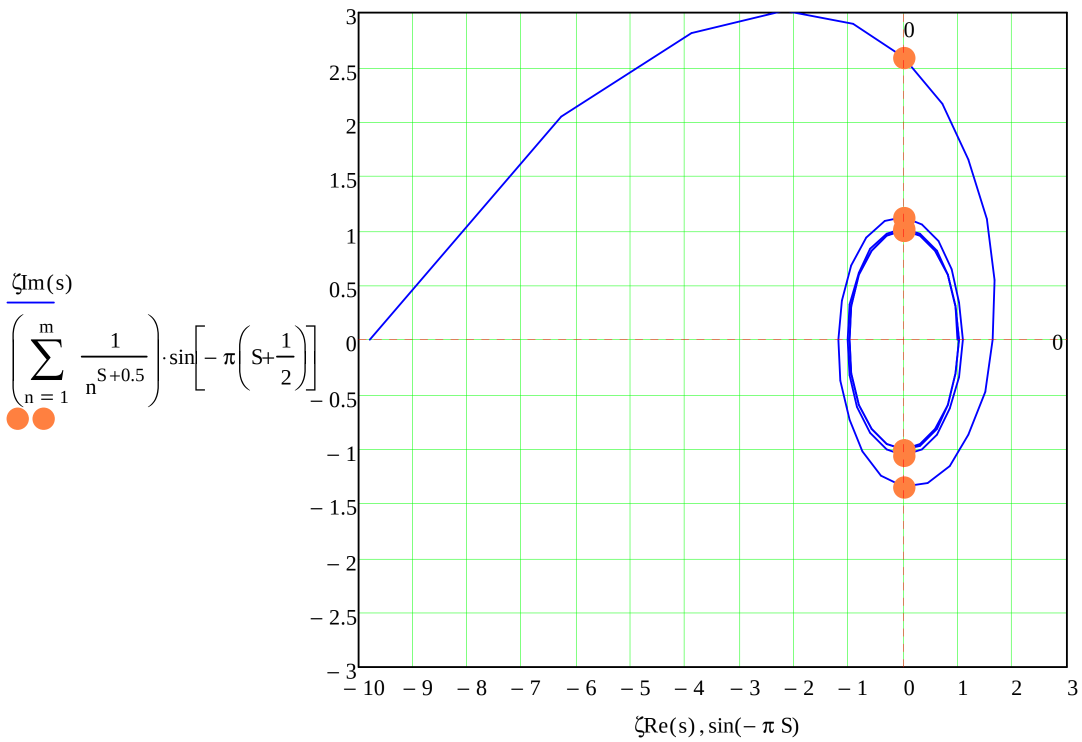


**Figure 13. Visual presentation for solved points. Imaginary part of zeta is not a zero (*ζIm(s) ≠0*). Real part of zeta is a zero (*ζRe(s)=0*) , but always for a real constant *(Re(S)=1/2).***

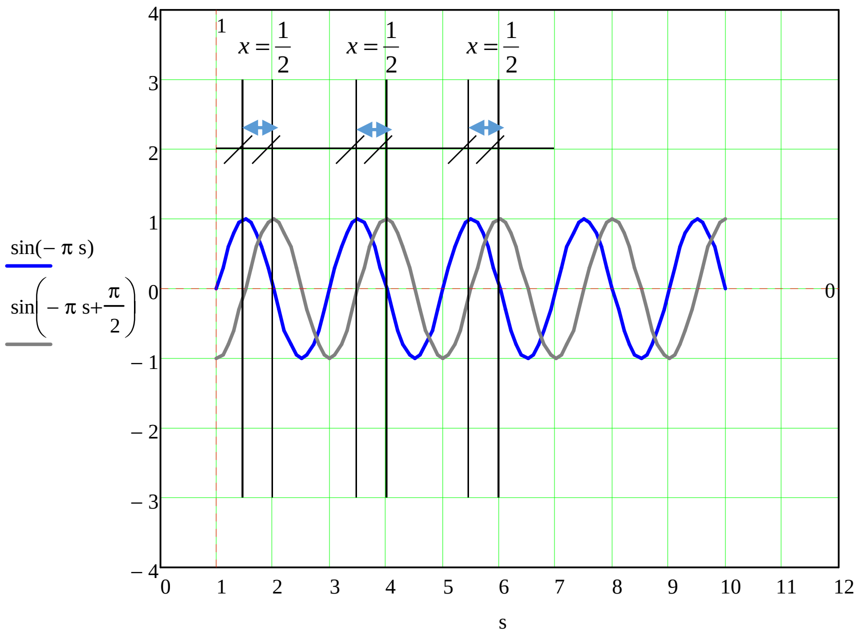


**Figure 14. Visual comparison of trigonometrical functions. Distance among the functions is always constant *(x=1/2)*.**

$$\sin\left(-\pi\cdot S+\frac{\pi}{2}\right)=\sin\left(-\pi\cdot\left(S-\frac{1}{2}\right)\right)=\sin\left(-\pi\cdot\left(S-\frac{1}{2}+\frac{1}{2}\right)\right)=\sin(-\pi\cdot S)$$

$$x=\frac{1}{2}$$

## 7. Proof for Riemann hypothesis

The proof was already composed at previous point (6.2). Now, I demonstrate only a short summary.

Hypothesis: $z=\frac{1}{2}+i\cdot t$ *, where* $\mathrm{Re}(S)=\frac{1}{2}$ *and* $\mathrm{Im}(S)=i\cdot t$

Trigonometrical form of zeta function is expressed as follows:

$$\zeta(s)=0=\left(\sum_{n=1}^{\infty}\frac{1}{n^{s}}\right)\cdot\left(\sin\left(-\pi\cdot s+\frac{\pi}{2}\right)\right)+\left(\sum_{n=1}^{\infty}\frac{1}{n^{s}}\right)\cdot(\sin(-\pi\cdot s))i$$

(real part of zeta should be a zero) & (imaginary part of zeta should be a zero)

*Real part of zeta is a zero , but always for a real constant* $\mathrm{Re}(S)=\frac{1}{2}$ *and* $S\in(1,2,3,4,\ldots,\infty)$

$$\mathrm{Re}(S)=\frac{1}{2}\Longrightarrow\mathrm{Re}(\zeta(s))=0$$

$$\mathrm{Re}(\zeta(s))=\left(\sum_{n=1}^{\infty}\frac{1}{n^{S+1/2}}\right)\cdot(\sin(-\pi\cdot S))=0\qquad\textit{for}\qquad S\in(1,2,3,4,\ldots,\infty)$$

*Imaginary part of zeta is not a zero for* real *constant* $\mathrm{Re}(S)=\frac{1}{2}$ *and* $S\in(1,2,3,4,\ldots,\infty)$

It was shown on (Figure 12) and (Figure 13).
At Riemann hypothesis, imaginary part was defined as: $i\cdot t$

$$\mathrm{Re}(S)=\frac{1}{2} \Longrightarrow \mathrm{Im}(\zeta(s))=\left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot\left(\sin\left(-\pi\cdot\left(S+\frac{1}{2}\right)\right)\right)$$

$$\mathrm{Im}(S)=\left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot\left(\sin\left(-\pi\cdot\left(S+\frac{1}{2}\right)\right)\right)\cdot i=i\cdot t$$ *, where*

$$t=\left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot\left(\sin\left(-\pi\cdot\left(S+\frac{1}{2}\right)\right)\right)$$ *or*

$$t=\left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot sign \quad \textit{and} \quad sign=\sin\left(-\pi\cdot\left(S+\frac{1}{2}\right)\right)$$

Hypothesis: $z=\frac{1}{2}+i\cdot t$ *, where* $\mathrm{Re}(S)=\frac{1}{2}$ *and* $\mathrm{Im}(S)=i\cdot t$

$$z=\frac{1}{2}+i\cdot t=\mathrm{Re}(S)+\mathrm{Im}(S)=\frac{1}{2}+\left(\sum_{n=1}^{\infty}\frac{1}{n^{S+\frac{1}{2}}}\right)\cdot\left(\sin\left(-\pi\cdot\left(S+\frac{1}{2}\right)\right)\right)\cdot i$$

*for* $S\in(1,2,3,4,\ldots,\infty)$

## 8. References


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