Can We Hear the Shape of a Maximum Entropy Potential From Spike Trains?

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We consider a spike-generating stationary Markov process whose transition probabilities are known. We show that there is a canonical potential whose Gibbs distribution, obtained from the Maximum Entropy Principle (MaxEnt), is the equilibrium distribution of this process. We provide a method to compute explicitly and exactly this potential as a linear combination of spatio-temporal interactions. The method is based on the Hammersley Clifford decomposition and on periodic orbits sampling. An explicit correspondence between the parameters of MaxEnt model and the parameters of Markovian models like the Generalized-Linear Model can be established. We also provide numerical results.

Spike Trains

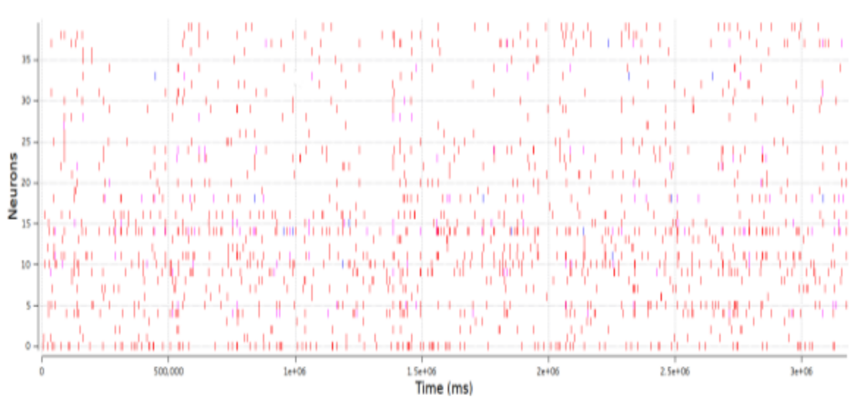


Figure: Raster plot/Spike train.

Spike state

$$\omega_k(n) \in \{0, 1\}$$

Spike pattern

$$\omega(n) = \left(\omega_k(n)\right)_{k=1}^{N}$$

Spike block

$$\omega_m^n = \{\omega(m)\,\omega(m+1)\ldots\omega(n)\}$$

Raster plot

$$\omega \stackrel{\text{def}}{=} \omega_0^T$$

Maximum Entropy Method

$$h[\mu] = -\sum_{\omega_0^D} \mu[\omega_0^D]\ \mathbb{P}[\omega(D)|\omega_0^{D-1}]\ \log \mathbb{P}[\omega(D)|\omega_0^{D-1}].$$

$$\mathcal{P}[\mathcal{H}_\beta] = \sup_{\nu \in \mathcal{M}_{inv}} \left(h[\nu] + \nu[\mathcal{H}_\beta]\right) = h[\mu] + \mu[\mathcal{H}_\beta]$$

Maxent is a method for finding the probability distribution that is consistent with known constraints expressed in terms of averages of patterns, but is otherwise as unbiased as possible.

Caveats of Maxent in spike train statistics

(i) It assumes stationarity in the data;

(ii) The choice of constraints is ad-hoc;

(iii) The interpretation of the coefficients shaping the Gibbs distribution (Lagrange multipliers) is controversial.

Markovian Models

In order to relate spike trains to the neural tissue properties and stimulus that generate these spiking activity, dynamical neural networks modeling is neccesary.

$$P\left[\omega(D)\,|\,\omega_0^{D-1}\right] = f\left(b + K\cdot x + H(\omega_0^D)\right)$$

$$\phi\left(\omega_0^D\right) = \log P\left[\omega(D)\,|\,\omega_0^{D-1}\right]$$

Class of equivalent potentials which originates the same probability

$$\mathcal{H}^{(2)}\left(\omega_0^D\right) = \mathcal{H}^{(1)}\left(\omega_0^D\right) - f\left(\omega_0^{D-1}\right) + f\left(\omega_1^D\right) + \Delta$$

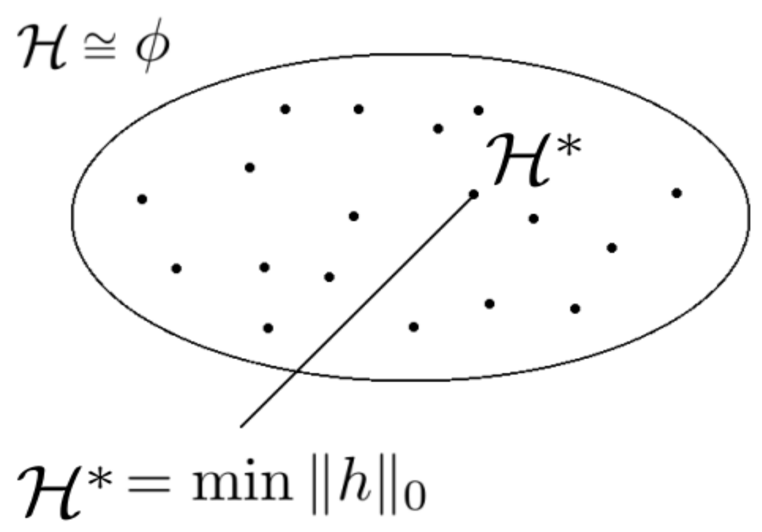
$$\Delta = \mathcal{P}\left[\mathcal{H}^{(2)}\right] - \mathcal{P}\left[\mathcal{H}^{(1)}\right]$$

Publications

Estimating Maximum Entropy distributions from Periodic Orbits in Spike Trains, R. Cofre, B. Cessac, Research Report, open access through HAL-INRIA

"Hearing the Maximum Entropy Potential of a spike-generating Markov process", R. Cofre, B. Cessac, submitted, open access through HAL-INRIA

There is an infinite family of equivalent potentials Which one select?



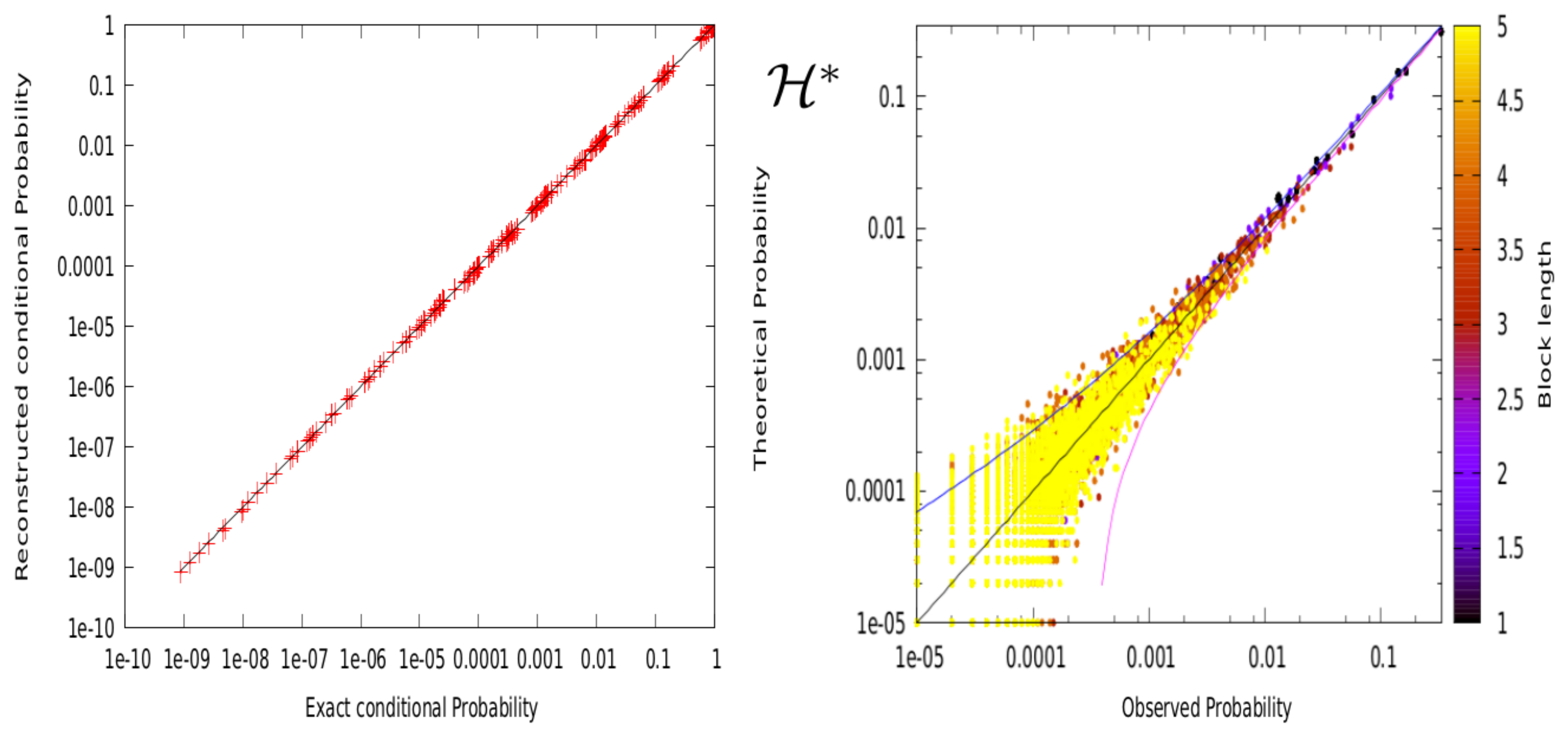
Summing over Periodic Orbits

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} \mapsto \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \mapsto \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

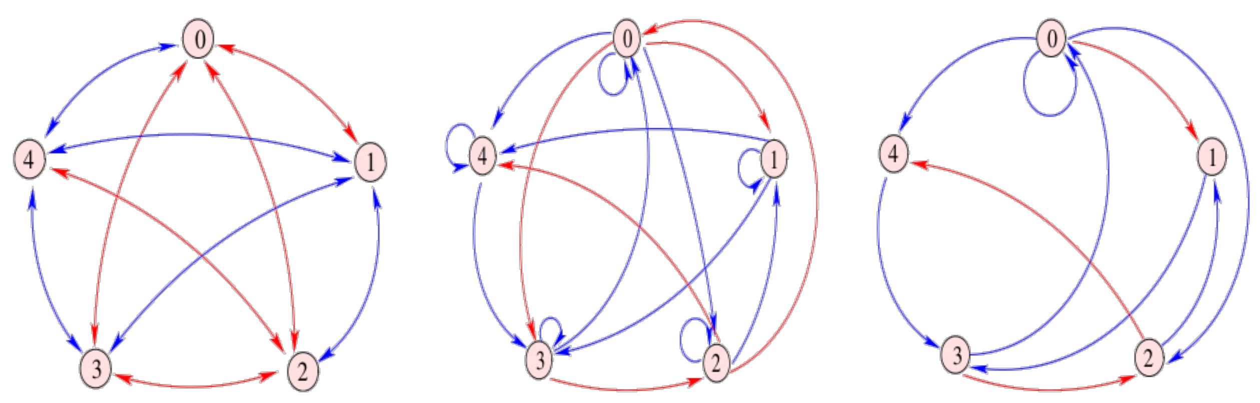
$$\sum_{n=1}^{\tau} \mathcal{H}^{(2)}\left(\omega^{(l_n)}\right) = \sum_{n=1}^{\tau} \mathcal{H}^{(1)}\left(\omega^{(l_n)}\right) + \tau\Delta,$$

$$\sum_{l' \sqsubseteq l}\sum_{n=1}^{R} h^{(2)}_{\sigma^n l'} = \sum_{l' \sqsubseteq l}\sum_{n=1}^{R} h^{(1)}_{\sigma^n l'} + R\Delta$$

Numerical example



Effective v/s real synaptic interactions



Conclusions

Our results allow to establish a link between synaptic connectivity, stimulus and effective connectivity. Our method works as well for finite size rasters, where transition probabilities can be estimated from empirical averages. Our method opens up new possibilities which allow a better understanding of the role of different neural

Acknowledgements

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