

Can We Hear the Shape of a Maximum Entropy Potential From Spike Trains?

Rodrigo Cofre, Bruno Cessac

► **To cite this version:**

Rodrigo Cofre, Bruno Cessac. Can We Hear the Shape of a Maximum Entropy Potential From Spike Trains?. Bernstein Conference 2015, Sep 2014, Goettingen, Germany. hal-01095760

HAL Id: hal-01095760

<https://hal.inria.fr/hal-01095760>

Submitted on 16 Dec 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Can We Hear the Shape of a Maximum Entropy Potential From Spike Trains?

Rodrigo Cofre*, Bruno Cessac

NeuroMathComp Team Inria Sophia-Antipolis

*rodrigo.cofre_torres@inria.fr



We consider a spike-generating stationary Markov process whose transition probabilities are known. We show that there is a canonical potential whose Gibbs distribution, obtained from the Maximum Entropy Principle (MaxEnt), is the equilibrium distribution of this process. We provide a method to compute explicitly and exactly this potential as a linear combination of spatio-temporal interactions. The method is based on the Hammersley Clifford decomposition and on periodic orbits sampling. An explicit correspondence between the parameters of MaxEnt model and the parameters of Markovian models like the Generalized-Linear Model can be established. We also provide numerical results.

Spike Trains

Spike state

$$\omega_k(n) \in \{0, 1\}$$

Spike pattern

$$\omega(n) = (\omega_k(n))_{k=1}^N$$

Spike block

$$\omega_m^n = \{\omega(m)\omega(m+1)\dots\omega(n)\}$$

Raster plot

$$\omega \stackrel{\text{def}}{=} \omega_0^T$$

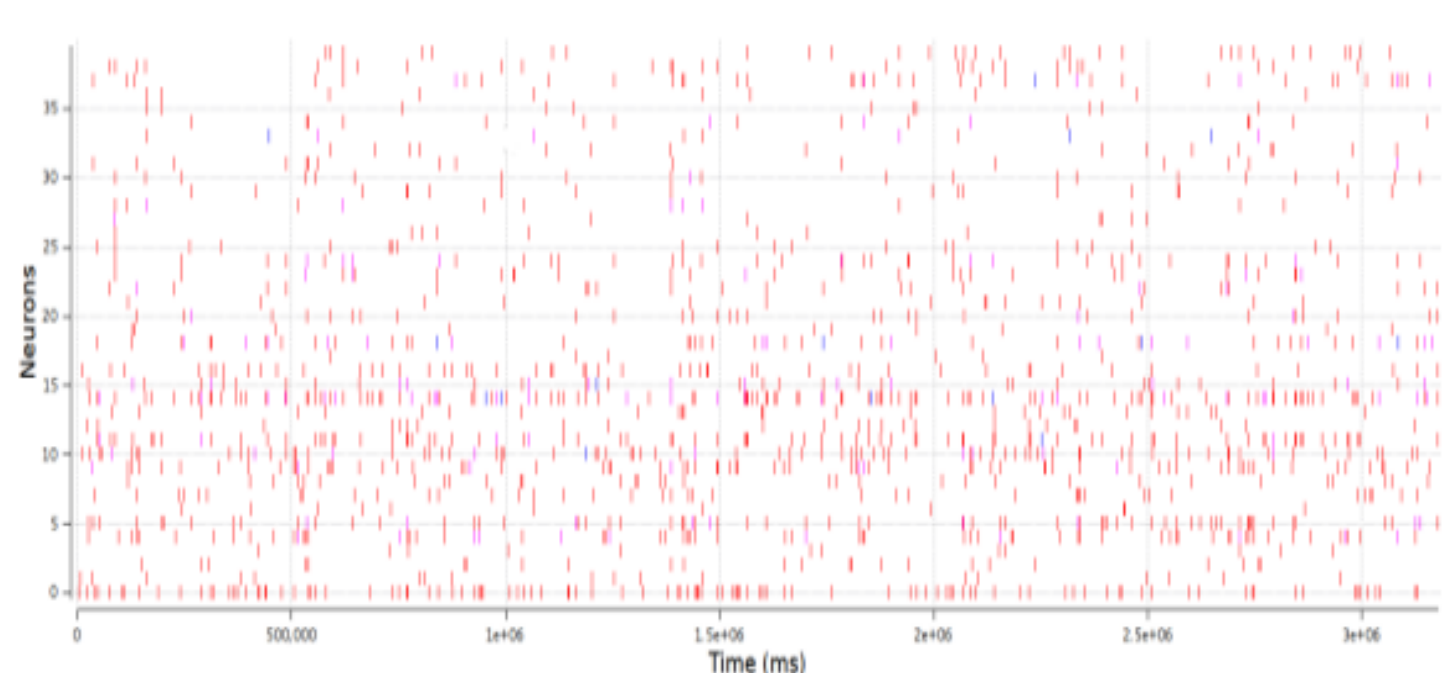


Figure: Raster plot/Spike train.

Maximum Entropy Method

$$h[\mu] = -\sum_{\omega_0^D} \mu[\omega_0^D] \mathbb{P}[\omega(D)|\omega_0^{D-1}] \log \mathbb{P}[\omega(D)|\omega_0^{D-1}]$$

$$\mathcal{P}[\mathcal{H}_\beta] = \sup_{\nu \in \mathcal{M}_{inv}} (h[\nu] + \nu[\mathcal{H}_\beta]) = h[\mu] + \mu[\mathcal{H}_\beta]$$

Maxent is a method for finding the probability distribution that is consistent with known constraints expressed in terms of averages of patterns, but is otherwise as unbiased as possible.

Caveats of Maxent in spike train statistics

- (i) It assumes stationarity in the data;
- (ii) The choice of constraints is ad-hoc;
- (iii) The interpretation of the coefficients shaping the Gibbs distribution (Lagrange multipliers) is controversial.

Markovian Models

In order to relate spike trains to the neural tissue properties and stimulus that generate these spiking activity, dynamical neural networks modeling is necessary.

$$P[\omega(D) | \omega_0^{D-1}] = f(b + K \cdot x + H(\omega_0^D))$$

$$\phi(\omega_0^D) = \log P[\omega(D) | \omega_0^{D-1}]$$

Class of equivalent potentials which originates the same probability

$$\mathcal{H}^{(2)}(\omega_0^D) = \mathcal{H}^{(1)}(\omega_0^D) - f(\omega_0^{D-1}) + f(\omega_1^D) + \Delta$$

$$\Delta = \mathcal{P}[\mathcal{H}^{(2)}] - \mathcal{P}[\mathcal{H}^{(1)}]$$

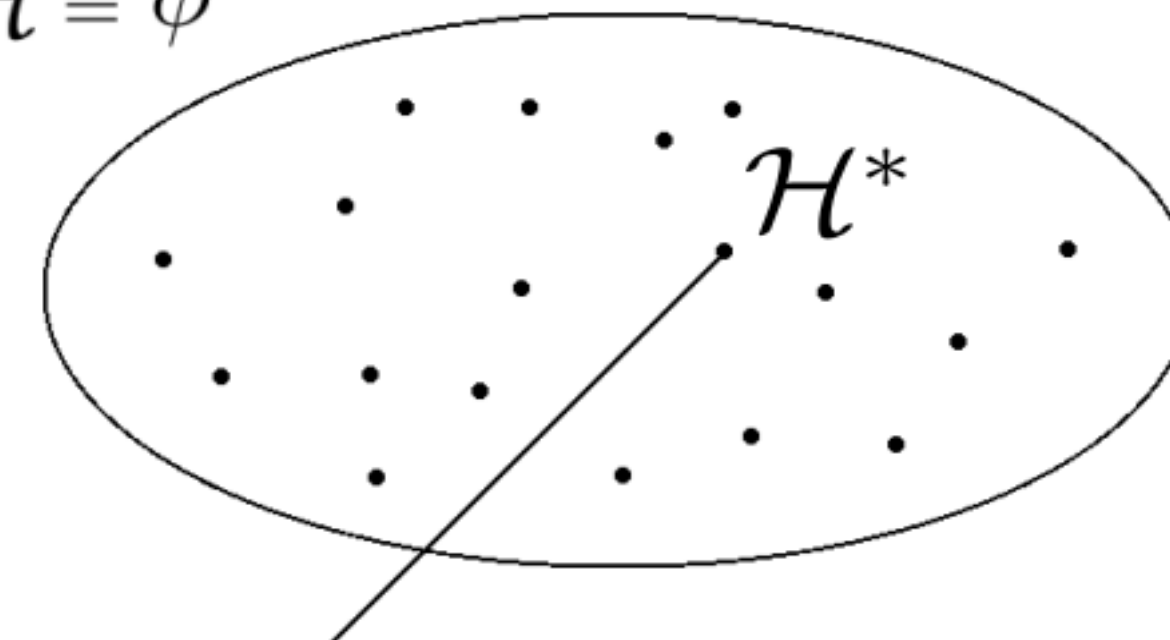
Publications

Estimating Maximum Entropy distributions from Periodic Orbits in Spike Trains, R. Cofre, B. Cessac, Research Report, open access through HAL-INRIA

"Hearing the Maximum Entropy Potential of a spike-generating Markov process", R. Cofre, B. Cessac, submitted, open access through HAL-INRIA

There is an infinite family of equivalent potentials Which one select?

$$\mathcal{H} \cong \phi$$



$$\mathcal{H}^* = \min \|h\|_0$$

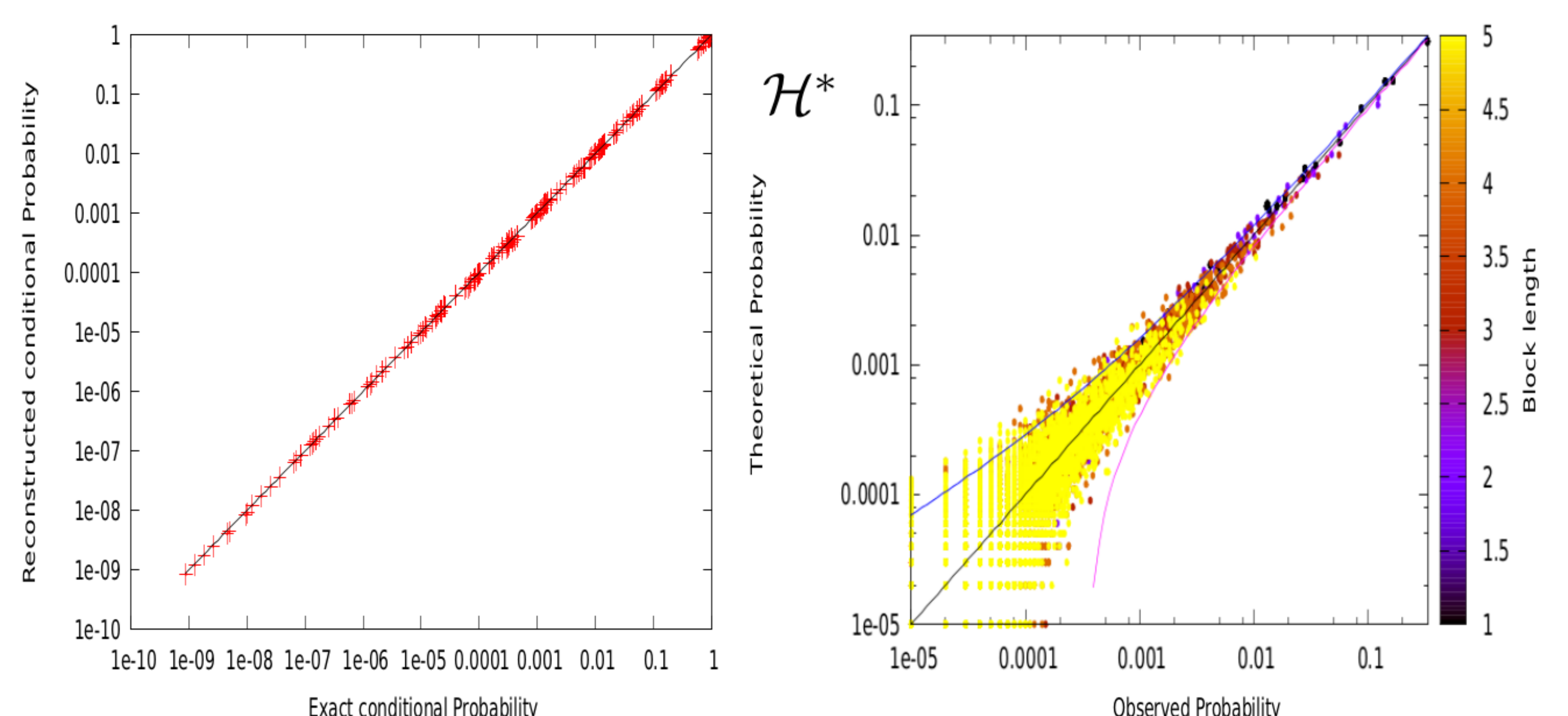
Summing over Periodic Orbits

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} \mapsto \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \mapsto \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

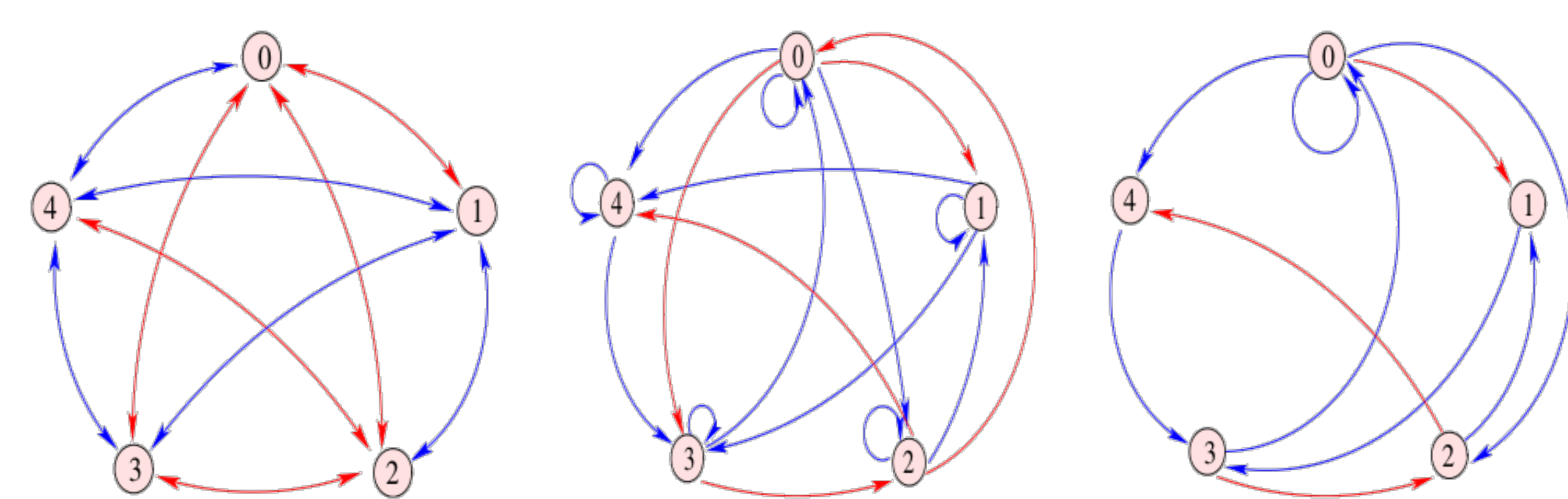
$$\sum_{n=1}^{\tau} \mathcal{H}^{(2)}(\omega^{(l_n)}) = \sum_{n=1}^{\tau} \mathcal{H}^{(1)}(\omega^{(l_n)}) + \tau \Delta$$

$$\sum_{l' \subseteq l} \sum_{n=1}^R h_{\sigma^{n l'}}^{(2)} = \sum_{l' \subseteq l} \sum_{n=1}^R h_{\sigma^{n l'}}^{(1)} + R \Delta$$

Numerical example



Effective v/s real synaptic interactions



Conclusions

Our results allow to establish a link between synaptic connectivity, stimulus and effective connectivity. Our method works as well for finite size rasters, where transition probabilities can be estimated from empirical averages. Our method opens up new possibilities which allow a better understanding of the role of different neural

Acknowledgements

This work was supported by the French ministry of Research and University of Nice (EDSTIC), INRIA, ERC-NERVI number 227747, KEOPS ANR-CONICYT and European Union Project # FP7-269921 (BrainScales), Renvision # 600847 and Mathemacms # FP7-ICT-2011.9