Sensitivity analysis of stochastic user equilibrium and its application to delivery services pricing

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1 Introduction

In e-commerce the delivery of products is a crucial part for the success of a shop. An efficient delivery system should offer various services and predict customers' behavior. The latter are influenced by the price of a delivery service, but also by its quality (congestion effect induced by customers' choices). In this study, we introduce a bilevel model to optimize a delivery system. At the upper level, the provider controls services' tariffs. At the lower level, users react by choosing their delivery service according to an utility function which incorporates the provider's tariffs and the congestion effect. We model the customers' reaction using *Stochastic User Equilibrium (SUE)*. We also present a sensitivity analysis for the SUE that gives the explicit expression of the derivatives of customers distribution with respect to services' tariffs. Based on a local search that exploit the derivatives information, a new heuristic algorithm for the bilevel delivery services pricing problem is developed and compared to others existing methods.

2 The delivery system

For the delivery of ordered products, customers choose among a set $\mathcal{J}$ of services. We consider two types of services: delivery at home (DH) and delivery at relay station (RS). The disutility of service j perceived by a customer depends on tariff A_j set by the provider, and on the quality of service or the congestion effect induced by customers' choices. Denoting by λ the customers arrival rate, and by p_j the fraction of customers choosing service $j \in \mathcal{I}$, the general form of the disutility c_i of service i is $c_i(p_i) = A_i + \alpha_i f_i(p_i)$, where f is the congestion function of service i and α_i a monetary conversion coefficient .

- DH service is modeled using an M/D/1 queue with a First-In-First-Out (FIFO) service discipline. The congestion function is given by the service delay

$$f(p_i, \lambda, d) = d + \frac{\lambda p_i d^2}{2(1 - \lambda p_i d)}.$$

- RS service is modeled using a M/M/K/K queue where K is the capacity of the relay. The congestion function is given by the Erlang's formula

$$f(p_i, \lambda, K) = \frac{\frac{(\lambda p_i)^K}{K!}}{\sum_{j=0}^{K} \frac{(\lambda p_i)^j}{j!}}$$

A first way to model customers' behavior is the *Deterministic User Equilibrium (DUE)* also known as the Wardrop equilibrium. Here, customers have full rationality and select the service with the smallest cost. In the real world, the full rationality assumption appears to be very strong (lack of information, individual preference, ...). It is more accurate to consider that users can make errors in their decision. Thus we use *Stochastic User Equilibrium (SUE)* to model user's decision process. The Logit based model (L-SUE) is often used for this issue. In L-SUE, the error term follows a Gumble distribution. In this work we compare LSUE to a Nested-Logit based SUE (N-SUE), more appropriate to the case study. In the latter, correlated services are grouped in nests. For our application, DH services form a nest, and RS ones form another nest. We consider that, at first, a customer selects a nest, and then he selects the delivery service. In the two cases, The SUE is computed using the Method of Successive Averages (Sheffi 1995, Bekhor et al. 2003).

3 The control problem

Knowing that customers follow a stochastic user equilibrium, the provider can control services parameters in a way that optimize some criteria. This yields to a *Mathematical Program with an Equilibrium Constraint (MPEC)*. The provider (leader) controls service tariffs A_i, delivery capacities D_i and relay capacities K_i. Denoting by u the leader's variables, $u = (A_i, D_i, K_i)$, and by p the user equilibrium, $p(u) = (p_i(u))_{i\in\mathcal{I}}$. (MPEC) can be formulated as follows:

$$(MPEC)\begin{cases} max\ F(u,p) & (1) \\ s.t\ L_i \leq u_i \leq U_i\ \forall i \in \mathcal{I} & (2) \\ p = p^\star(u) & (3) \end{cases} \qquad \begin{array}{c} F(u,p) = \sum_{i\in\mathcal{I}} \lambda A_i p_i - a_i D_i - b_i K_i. \\ F(u,p) = \sum_{i\in\mathcal{I}} p_i(u) c_i(u,p). \end{array}$$

Where constraint (3) describes that p is a solution for equilibrium problem parameterized by u. The leader's objectif function can be maximizing profit $F(u,p)$ or minimizing the total cost $F(u,p)$.

The resulting program is in general non convex, and hence it is difficult to find a global optimum (Yang et al. 1994). Sensitivity Analysis based heuristic (SABH), has been used to find efficient solution for such MPEC in the field of traffic control (Yang et al. 1994, Friesz et al. 1990). The key point of SABH lies in how to evaluate the user equilibrium (follower variables) at a given point of services' tariffs (leader variables). Sensitivity analysis gives explicit expression of the derivatives of the formers with respect to the latters. Having these informations, a descent direction (the gradient of $F(u,p)$) is build and used to update leader variables (Friesz et al. 1990). This process is iterated until no improvement of the leader's objective value is provided.

In this work we apply this method for the resolution of a bilevel program with a stochastic user equilibrium (MPSEC), in the case where the provider controls only services' tariffs. We also, introduce a new way to find the descent direction based on a local search that exploits sensitivity informations. Numerical experimentations show better results for the new descent direction regarding the quality of the solution and the execution time.

Further works will treat the case where the provider controls tariffs and capacities of services. This leads to a challenging MPSEC where a part of the upper level variables are integer.

4 Conclusion and perspectives

This work combines *Bilevel Programming* and *Stochastic User Equilibrium (SUE)* for the optimization of a delivery services system. A new heuristic algorithm based on sensitivity analysis is presented. Closely to the delivery system, futur works could consider heterogenous customers in a multi-class model.

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