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# Distributed coordination of self-organizing mechanisms in communication networks

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**Abstract**—The fast development of the Self-Organizing Network (SON) technology in mobile networks renders critical the problem of coordinating SON functionalities operating simultaneously. SON functionalities can be viewed as control loops that may need to be coordinated to guarantee conflict free operation, to enforce stability of the network and to achieve performance gain. This paper proposes a distributed solution for coordinating SON functionalities. It uses Rosen’s concave games framework in conjunction with convex optimization. The SON functionalities are modeled as linear Ordinary Differential Equation (ODE)s. The stability of the system is first evaluated using a basic control theory approach. The coordination solution consists in finding a linear map (called coordination matrix) that stabilizes the system of SON functionalities. It is proven that the solution remains valid in a noisy environment using Stochastic Approximation. A practical example involving three different SON functionalities deployed in Base Stations (BSs) of a Long Term Evolution (LTE) network demonstrates the usefulness of the proposed method.

**Keywords**—Self-Organizing Networks, Concave Games, SON Coordination, Stochastic Approximation

## I. INTRODUCTION

The Radio Access Networks (RAN) landscape is becoming increasingly complex and heterogeneous with co-existing and co-operating technologies. SON mechanisms have been introduced as a means to manage complexity, to reduce cost of operation, and to enhance performance and profitability of the network. SONs enable automation of the following management tasks: configuration of newly deployed network nodes (self-configuration), parameter tuning for Key Performance Indicators (KPIs) improvement (self-optimization) and diagnosis and reparation of faulty network nodes (self-healing). In LTE networks, large scale deployment of SON functionalities has started with self-configuration SON functions to simplify the network deployment, and that of self-optimization functions is expected to follow.

Self-optimization mechanisms can be viewed as control loops, that can be deployed in the management or the control plane. The former is often denoted as centralized SON and the latter as distributed SON. In the centralized case, the SON

algorithms are deployed in the Operation and Maintenance Center (OMC) or in the Network Management System (NMS) which are part of the Operation and Support System (OSS). Centralized SON benefits from abundant data (metrics and KPIs) and computational means necessary for processing and running powerful optimization methods [2], [3]. The main drawback of the centralized approach is related to the long time scale that is typically used, in the order of an hour and more, which is related to the periods in which batch of measured data is sent from the RAN to the OMC. Hence the SON algorithms cannot adapt the network to traffic variations that occur at short time scales.

The second approach, namely the distributed SON, is more scalable since the optimization is performed locally involving one or several BSs. The main advantage of the distributed SON is its higher reactivity, namely its ability to track quick variations in the propagation conditions or in the traffic [4], [5] and to adapt system parameters in the time scale of seconds (i.e. flow level duration). The higher reactivity sometimes impacts the type of solution sought, namely a solution which targets local minima instead of global minima. However, distributed optimization can also reach global minima [4].

SON functions are often designed as stand alone functionalities, and when triggered simultaneously, their interactions are not always predictable. The deployment of multiple control loops raises the questions of conflicts, stability and performance. The topics of conflict resolution, coordination, and a framework for managing multiple SON functionalities are receiving a growing interest (see for example [6]–[8]). Most contributions that have addressed the coordination problem between specific SON functionalities, provide a solution implemented in a centralized [9], [10] or distributed [11] fashion. In a centralized solution, the SON coordination can be treated as a multi-objective optimization [12]. From the standardization point of view, the coordination problem has been addressed as a centralized, management-plane problem [13].

Little material has been reported on distributed, control plane solutions for the coordination problem in spite of its higher reactivity, attractiveness from an architecture point of view and potential performance gain. The aim of this paper is to provide a generic coordination mechanism which is practically implementable. The contributions of the paper are the following:

- The problem of SON coordination is analyzed using

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Due to paper size limitations, a longer, more complete version of this article is available [1].

a control theory/stochastic approximation-based framework.

- The case of fully distributed coordination is addressed.
- It is shown that coordination can be formulated as a convex optimization problem.
- The coordination solution is applied to a use case involving 3 SON functions deployed in several BSs of a RAN.

A first version of this paper has already been published in [14]. New results presented here include the formulation of the coordination problem as a convex optimization problem with Linear Matrix Inequality (LMI) constraints: stability constraint and connectivity constraints related to the capability of the self-organizing nodes to exchange information via the transport network. The merit of the proposed solution is its capability to handle a large number of control loops and enforce their stability, as illustrated in the use case of SON deployment in a LTE network. To our knowledge, this is the first generic control plane solution to the problem of SON coordination in a mobile network.

The paper is organized as follows: in Section II we state the proposed model for interaction of SON mechanisms running in parallel and the coordination problem to be solved. In Section III we focus on the case where performance indicators are affine functions of the parameters, and propose a practically implementable coordination mechanism. In Section IV we study the fully distributed coordination with no exchange of information between SON entities, and show the limit of this approach. In Section V the coordination problem is formulated as a convex optimization problem which can be quickly solved with modern computers. In Section VI we illustrate the coordination solution applied to a LTE network with three different SON functions deployed in each BS. Section VII concludes the paper. In appendices A and B we recall the notions of diagonal strict concavity and martingales, respectively.

#### LIST OF NOTATIONS

- $[\bullet]_S^+$  Projection on the set  $S$
- $[A]_{k,k}^{k^{th}}$   $k^{th}$  order leading principal submatrix of  $A$
- $\bar{x}$  Notation for constants often representing target values
- $\mathcal{C}$  Set of matrices having the same particular form (e.g., with zero elements at specific positions)
- $\dot{\alpha}$  Derivative of  $\alpha$  over time
- $\mathbb{E}(\bullet)$  Expectation of a random variable
- $\mathbb{1}_{\{cond\}}(x)$  Indicator function on the set of  $x$  values satisfying the condition  $cond$
- $\nabla_x f$  Gradient of  $f$  with regard to  $x$
- $\text{tr}(\bullet)$  Trace of a matrix
- $\theta$  Vector of parameters
- $\theta^*$  Equilibrium points of system comprising control loops
- $A \prec 0$   $A$  is negative definite
- $A$  Real matrix representing linear system of SON functions
- $A^T$  Transpose of matrix  $A$
- $\det(\bullet)$  Determinant of a matrix
- $\text{eig}(A)$  Eigenvalues of  $A$
- $JF(\bullet)$  Jacobian of  $F(\bullet)$

## II. PROBLEM DESCRIPTION

### A. SON model

A SON mechanism is an entity which monitors a given performance indicator and controls a scalar parameter. The current value of the performance indicator is observed, and the parameter is modified accordingly to attain some objective. We consider  $I > 1$  SON mechanisms operating simultaneously. We define  $\theta_i$  the parameter controlled by the  $i$ -th SON mechanism and  $\theta = (\theta_1, \dots, \theta_I)$  the vector of parameters. The  $i$ -th SON mechanism monitors a performance indicator  $F_i(\theta)$  and updates its parameter  $\theta_i$  proportionally to it.  $F(\theta) = (F_1(\theta), \dots, F_I(\theta))$  is the vector of update of  $\theta$ .

We say that the  $i$ -th SON mechanism operates in *stand-alone* mode if all parameters but  $\theta_i$  are kept constant. The  $i$ -th SON mechanism operating in stand-alone is described by the ODE:

$$\dot{\theta}_i = F_i(\theta), \quad \dot{\theta}_j = 0, j \neq i. \quad (1)$$

We say that the SON mechanisms operate in *parallel* mode if all parameters are modified simultaneously, which is described by the ODE:

$$\dot{\theta} = F(\theta). \quad (2)$$

We say that the  $i$ -th SON mechanism is stable in stand-alone mode if there exists  $\theta_i^{*,i}$  for fixed  $\theta_j, j \neq i$  which is *asymptotically stable* for (1). It is noted that  $\theta_i^{*,i}$  depends on  $\theta_j, j \neq i$ . We say that the SON mechanisms are stable in parallel mode if there exists  $\theta^*$  which is asymptotically stable for (2). Typically, the SON mechanisms are designed and studied in a stand-alone manner, so that stand-alone stability is verified.

However, stand-alone stability does not imply parallel stability. First consider a case where  $F_i(\theta)$  does not depend on  $\theta_j$ , for all  $j \neq i$ . Then (2) is a set of  $I$  parallel independent ODEs, so that stand-alone stability implies parallel stability. On the other hand, if there exists  $i \neq j$  such that  $F_i(\theta)$  depends on  $\theta_j$ , then the situation is not so clear-cut. We say that SON  $i$  and  $j$  *interact*. Namely, interaction potentially introduces *instability*.

In the remainder of this article we will be concerned with conditions for parallel stability, and designing coordination mechanisms to force stability whenever possible.

### B. Stability characterization

Two particular cases of parallel mechanisms will be of interest. The first case is what we will call *zero-finding* algorithms. Each SON mechanism monitors the value of a performance indicator and tries to achieve a fixed target value for this performance indicator. Namely:

$$F_i(\theta) = f_i(\theta) - \bar{f}_i, \quad (3)$$

where  $f_i$  is the performance indicator monitored by SON  $i$  and  $\bar{f}_i$  - a target level for this performance indicator. The goal of the  $i$ -th SON mechanism is to find  $\theta_i^*$  for fixed  $\theta_j, j \neq i$  such that  $f_i(\theta_1, \dots, \theta_i^*, \dots, \theta_I) = \bar{f}_i$ . If  $\theta_i \mapsto f_i(\theta_1, \dots, \theta_i, \dots, \theta_I)$  is strictly decreasing  $1 \leq i \leq I$  then stand-alone stability is assured. Indeed,  $V_i(\theta) = (f_i(\theta) - \bar{f}_i)^2$  would be a Lyapunov function for (3).

Another case of interest is maximization algorithms. Each SON mechanism tries to maximize a given performance indicator. There exists a continuously differentiable function  $g_i$  such that:

$$F_i(\theta) = \nabla_{\theta_i} g_i(\theta). \quad (4)$$

In stand-alone mode, SON  $i$  indeed converges to a local maximum of  $\theta_i \rightarrow g_i(\theta)$ . If we restrict  $\theta$  to a closed, convex and bounded set and if  $\theta_i \mapsto g_i(\theta_1, \dots, \theta_i, \dots, \theta_I)$  is concave  $1 \leq i \leq I$ , we fall within the framework of *concave games* considered in [15]. Note that zero-finding algorithms can be rewritten as maximization algorithms by choosing  $g_i(\theta) = -(f_i(\theta) - \bar{f}_i)^2$ .

An important result of [15] for parallel stability is given in the following theorem. Denote by  $w \in \mathbb{R}_+^I$  a vector of *real positive weights*.

**Theorem 1.** Consider  $g : S \rightarrow \mathbb{R}^I$  with  $S$  a compact convex set. Assume that for all  $1 \leq i \leq I$ , and  $\theta \in S$ ,  $\theta_i \mapsto g_i(\theta_1, \dots, \theta_i, \dots, \theta_I)$  is concave. Then there exists an equilibrium point for the system of ODEs  $\dot{\theta} = w^T \cdot F(\theta)$  that is asymptotically stable in  $S$ .

If in addition  $\sum_{i=1}^I w_i g_i(\theta)$  is diagonally strictly concave then that equilibrium point is unique.

The definition of diagonal strict concavity is given in Appendix A. If we denote by  $J_{F,w}$  the Jacobian of  $w \cdot F(\theta) = [w_1 F_1(\theta), \dots, w_I F_I(\theta)]$ , a sufficient condition for diagonal strict concavity is that  $J_{F,w} + J_{F,w}^T$  is negative definite.

Note that (2) is a special case of the ODEs considered in Theorem 1 with  $w_i = 1, i = 1, \dots, I$ . Without diagonal concavity there is no guarantee that parallel stability occurs, and coordination is needed.

### C. Linear case

In the remainder of this paper, we study the case where  $F$  is affine:

$$F(\theta) = A\theta + b, \quad (5)$$

where  $b$  is a vector of size  $I$  and  $A$  - a matrix of size  $I \times I$ . We assume that  $A$  is invertible and we define  $\theta^* = -A^{-1}b$ . The SON mechanisms running in parallel are described by the linear ODE:

$$\dot{\theta} = A\theta + b = A(\theta - \theta^*). \quad (6)$$

It is noted that in the linear case, we always fall within the scope of zero-finding algorithms described previously, by defining:

$$f_i(\theta) = \sum_{1 \leq j \leq I} A_{i,j} \theta_j, \quad \bar{f}_i = -b_i, \quad (7)$$

$$\dot{\theta}_i = f_i(\theta) - \bar{f}_i. \quad (8)$$

In particular, stand-alone stability occurs if and only if (iff)  $A_{i,i} < 0, 1 \leq i \leq I$ , i.e all the diagonal terms of  $A$  are strictly negative. Parallel stability holds iff all the eigenvalues of  $A$  have a strictly negative real part. We then say that  $A$  is a *Hurwitz matrix*.

For practical systems, performance indicators  $F(\theta)$  need not be linear functions of  $\theta$ . However, as long as they are

smooth, they can be approximated by linear functions using a Taylor expansion in the neighborhood of a stationary point assuming that a Taylor expansion exists. Consider a point  $\theta^*$  with  $F(\theta^*) = 0$ . If the values of  $\theta$  are restricted to a small neighborhood of  $\theta^*$ , we can use the approximation:

$$F(\theta) \approx JF(\theta^*)(\theta - \theta^*), \quad (9)$$

with  $JF(\theta^*)$  being the Jacobian of  $F$  evaluated at  $\theta^*$ . The Hartman-Grossman theorem ([16]) states that on a neighborhood of  $\theta^*$ , stability of the system with linear approximation implies stability of the original, non-linear system. Hence implementing the proposed coordination mechanism where  $A$  is replaced by  $JF$  ensures stability if we constrain  $\theta$  to a small neighborhood of  $\theta^*$ .

The parameters  $A$  and  $b$  might be unknown, and we can only observe noisy values of  $F(\theta)$  for different values of  $\theta$ . The crudest approach is to estimate  $A$  and  $b$  through finite differences:

$$a_{i,j} \approx \frac{f_j(\theta + e_i \delta \theta_i) - f_j(\theta - e_i \delta \theta_i)}{2\delta \theta_i}, \quad (10)$$

$$b_i \approx f_i(0). \quad (11)$$

with  $e_i$  being the  $i$ -th unit vector. The results are averaged over several successive measurements and additive noise is omitted for notation clarity. In general, the measurements of  $F$  are obtained by calculating the time average of some output of the network during a relatively long time, so that a form of the central limit theorem applies and the additive noise is Gaussian. In this case, a better method is to employ *least-squares regression*. Least-squares regression is a well studied topic with very efficient numerical methods ([17]) even for large data sets so that the estimation of  $A$  and  $b$  is not computationally difficult.

From the engineering point of view, the computation of (10) could be performed in the management center of the operator, where data is abundant and a large amount of computing power is available.

Finally, since practical systems do not remain stationary for an infinite amount of time, a database with values of  $A$  and  $b$  for each set of operating conditions must be maintained. In the context of wireless networks, the relationship between parameters and performance indicators changes when the traffic intensity changes because of daily traffic patterns. For instance, during the night traffic is very low, and traffic peaks are observed at the end of the day. A database with estimated values of  $A$  and  $b$  at (for instance) each hour of the day could be constructed.

## III. COORDINATION

### A. Coordination mechanism

If  $A$  has at least one eigenvalue with positive or null real part, convergence to  $\theta^*$  does not occur, and a coordination mechanism is needed. We consider a *linear* coordination mechanism, where  $A$  is replaced by  $CA$  with  $C$  being a  $I \times I$  real matrix. The ODE for the coordinated system is:

$$\dot{\theta} = CA(\theta - \theta^*). \quad (12)$$

The matrix  $A$  derived using (10) is needed to compute the coordination matrix  $C$  (see Section (V)). Once  $C$  is available, the control changes from  $\theta = F(\theta)$  to  $\dot{\theta} = CF(\theta)$  as shown in Figure 1. For SON  $i$ , it changes from  $\dot{\theta}_i = F_i(\theta)$  to  $\dot{\theta}_i = \sum_{k=1}^N C_{i,k} F_k(\theta)$ .

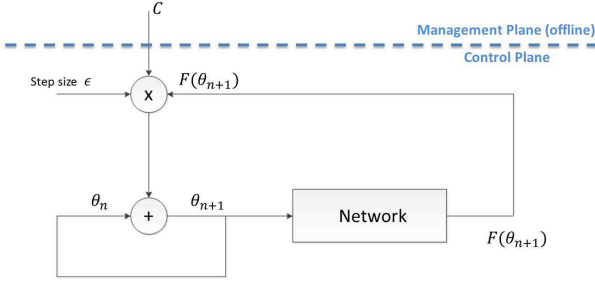


Fig. 1. Coordination system block diagram

The coordination block is located in the management plane. It sends each line of the coordination matrix to the corresponding SON function. This part can be viewed as a feed-forward control since the coordination matrix is generally evaluated once or updated at very large intervals of time. The SON block remains a feedback loop by updating parameters according to measured KPIs, but uses the coordination matrix in these updates as described in Figure 1.

Stability is achieved if there exists a symmetric matrix  $X$  such that:

$$(CA)^T X + XCA \prec 0, \quad X \succ 0, \quad (13)$$

where  $X \succ 0$  denotes that  $X$  is positive definite. In particular,

$$V(\theta) = (\theta - \theta^*)^T X (\theta - \theta^*), \quad (14)$$

acts as a Lyapunov function.

### B. Distributed implementation

The choice for the coordination matrix  $C$  is not unique. For instance  $C = -A^{-1}$  ensures stability. For the coordination mechanism to be scalable with respect to the number of SONs,  $C$  should be chosen to allow *distributed* implementation. We say that SON  $j$  is a neighbor of SON  $i$  if  $\frac{\partial f_j}{\partial \theta_i} \neq 0$ . We define  $\mathcal{I}_i$  the set of neighbors of  $i$ . The coordination mechanism is distributed if each SON needs only to exchange information with its neighbors.

We give an example of a coordination mechanism which can always be distributed. The mechanism is based on a *separable* Lyapunov function. Define the weighted square error:

$$V(\theta) = \sum_{i=1}^I w_i (f_i(\theta) - \bar{f}_i)^2 = (\theta - \theta^*)^T A^T W A (\theta - \theta^*), \quad (15)$$

with  $W = \text{diag}(w)$  i.e the diagonal matrix with diagonal elements  $\{w_i\}_{1 \leq i < I}$ . Coordination is achieved by following the gradient of  $-V$  so that  $V$  is a Lyapunov function:

$$\dot{\theta} = -\nabla_{\theta} V(\theta) = -A^T W A (\theta - \theta^*). \quad (16)$$

Namely, we choose  $C = -A^T W$ . We can verify that the mechanism is distributed:

$$\dot{\theta}_i = \sum_{j=1}^I 2w_j \frac{\partial f_j}{\partial \theta_i} (f_j(\theta) - \bar{f}_j) = \sum_{j \in \mathcal{I}_i} 2w_j \frac{\partial f_j}{\partial \theta_i} (f_j(\theta) - \bar{f}_j). \quad (17)$$

Indeed, the update of  $\theta_i$  only requires knowledge of  $\frac{\partial f_j}{\partial \theta_i}$  and  $f_j(\theta) - \bar{f}_j$ , for  $j \in \mathcal{I}_i$ , and this information is available from the *neighbors* of  $i$ .

It is also noted that such a mechanism can be implemented in an *asynchronous manner*, i.e the components of  $\theta$  are updated in a round-robin fashion, or at random instants, and the average frequency of update is the same for all components. The reader can refer to [18][chapters 6-8] for the round-robin updates and [19][chapter 12] for the random updates. Asynchronous implementation is important in practice because if the SONs are not co-located, maintaining clock synchronization among the SONs would generate a considerable amount of overhead.

### C. Stochastic Control Stabilization

In practical systems, ODEs are replaced by stochastic approximation algorithms. Indeed, the variables are updated at discrete times proportionally to functions values which are noisy.

The noise in the function values is due to the fact that time is slotted and functions are estimated by averaging certain counters during a time slot. For example, the load of a BS in a mobile network is often estimated by evaluating the proportion of time during which the scheduler is busy, and the file transfer time is estimated by averaging the file transfer times of all flows occurring in a certain period of time. The noise is also due to intrinsic stochastic nature of real systems, for example in wireless networks the propagation environment is inherently non-deterministic (because of fading, mobility, etc.) so the Signal to Interference plus Noise Ratio (SINR) will be noisy.

When the noise in the measurements of the function values is of Martingale difference type (see appendix B for basic definition of martingales), the mean behavior of those Stochastic Approximation (SA) algorithms matches with the system of ODEs. Note that we consider Martingale difference type of noise but the SA results hold for much more general noise processes (stationary, ergodic). In [11] for example, SA results are used without the Martingale difference property.

The initial system of control loops is in reality a system of SA algorithms, with one of them written as

$$\theta_i[k+1] = [\theta_i[k] + \epsilon_k (f_i(\theta[k]) + N_k^i)]_{S_i}^+ \quad (18)$$

where  $[\cdot]_{S_i}^+$  is the projection on the interval  $S_i = [a_i, b_i]$ ;  $a_i < b_i \in \mathbb{R}$ ,  $\theta[k] = (\theta_1[k], \dots, \theta_I[k])$  is the vector of parameters after the  $(k-1)$ th update,  $\epsilon_k$  the step of the  $k$ th update and  $N_k^i$  represents the noise in the measurement.

The projection in (18) aims at ensuring that the iterates are bounded. This is also a condition for convergence of the SA algorithm towards the invariant sets of the equivalent ODE.

Most SON algorithms are or can be reduced to the form of (18). For example in [4], a load balancing SON is presented

in this very same form. In [11] relays are self-optimized using also a SA algorithm. In [20], SA algorithms are used for self-optimizing interference management for femtocells. A handover optimization SON which can be rewritten as an SA algorithm is also presented in [21].

We suppose that  $N_k$  is a martingale difference sequence to meet the conditions for stand alone convergence (see [19], [22]). Namely the SA algorithms have the same behavior as their equivalent ODE. Now we want to check if the conditions for the SA equivalence with the limiting ODE are still verified after the coordination mechanism is applied. The coordinated SA for the  $i$ -th mechanism is

$$\theta_i[k+1] = \left[ \theta_i[k] + \epsilon_k \left( \sum_{j=1}^I C_{i,j} (f_j(\theta[k]) + N_k^j) \right) \right]_{S_i}^+ \quad (19)$$

The projection ensures that the iterates are bounded. The question now is to show that  $\sum_{j=1}^I C_{i,j} N_k^j$  is a Martingale difference sequence in order to meet the convergence conditions. Denoting  $\mathcal{F}_k = \left\{ \sum_{j=1}^I C_{i,j} N_l^j, l < k \right\}$ , we have

$$E \left[ \sum_{j=1}^I C_{i,j} N_k^j | \mathcal{F}_k \right] = \sum_{j=1}^I C_{i,j} E \left[ N_k^j | \mathcal{F}_k \right] = 0$$

since  $E[N_k^j | N_l^j, l < k] = 0, j = 1 \dots I$ . So this condition is satisfied ensuring the validity of the coordination method in a stochastic environment.

#### IV. FULLY DISTRIBUTED COORDINATION

In this section we study fully distributed coordination, where the coordination matrix  $C$  is *diagonal*. As said previously, if  $C_{i,j} \neq 0, i \neq j$  then SON  $i$  and  $j$  need to exchange information. In fully distributed coordination, no information is exchanged. We prove two results. For  $I = 2$  fully distributed coordination can always be achieved. For  $I = 3$  it is also possible if  $A^{-1}$  has at least one non-zero diagonal element and impossible otherwise. These results are attractive from a practical point of view: if there are 3 or less SONs to coordinate, it suffices to modify their feedback coefficient, without any exchange of information or interface between them. We say that the system can be coordinated in a *fully distributed* way iff there exists  $c \in \mathbb{R}^I$  such that  $\text{diag}(c)A$  is a Hurwitz matrix.

The following lemma will be useful. It is a consequence of the Routh-Hurwitz theorem ([23]).

**Lemma 1.** *Let  $M$  a  $I \times I$  invertible real matrix. For  $I = 2$ ,  $M$  is a Hurwitz matrix iff*

$$\det(M) > 0, \quad \text{tr}(M) < 0 \quad (20)$$

where  $\text{tr}$  denotes the trace of a matrix.

For  $I = 3$ ,  $M$  is a Hurwitz matrix iff

$$\det(M) < 0, \quad \text{tr}(M) < 0, \quad \text{tr}(M)\text{tr}(M^{-1}) > 1. \quad (21)$$

*Proof:* See [1, Appendix A].  $\blacksquare$

For  $I = 2$  mechanisms, the system can always be coordinated in a fully distributed way as shown by Theorem 2.

**Theorem 2.** *For  $I = 2$ , the system can always be coordinated in a fully distributed way.  $\text{diag}(c)A$  is a Hurwitz matrix iff:*

$$c \in \mathcal{C} = \{c : c_1 A_{1,1} + c_2 A_{2,2} < 0, c_1 c_2 \det(A) > 0\}, \quad (22)$$

and  $\mathcal{C}$  is not empty since:

$$\left( 1, \text{sign} \det(A) \frac{|A_{1,1}|}{2|A_{2,2}|} \right) \in \mathcal{C}. \quad (23)$$

*Proof:*  $\text{tr}(\text{diag}(c)A) = c_1 A_{1,1} + c_2 A_{2,2}$  and  $\det(\text{diag}(c)A) = c_1 c_2 \det(A)$ . Using Lemma 1 proves the first part of the result.  $\mathcal{C}$  is not empty, since one of its elements is given by inspection of the proposed value.  $\blacksquare$

For  $I = 3$  mechanisms, the system can also be coordinated in a fully distributed way providing that the inverse of  $A$  has one non-zero diagonal element as shown by Theorem 3.

**Theorem 3.** *For  $I = 3$ , the system can be coordinated in a fully distributed way if  $B = A^{-1}$  has at least one non-zero diagonal element. Assume that  $B_{2,2} \neq 0$  without loss of generality. Consider  $\epsilon > 0$ , and define  $C(\epsilon) = \text{diag}(1, \epsilon c_2, \epsilon c_3)$  with:*

$$\frac{B_{2,2}}{c_2} + \frac{B_{3,3}}{c_3} < 0, \quad c_2 c_3 \det(A) < 0. \quad (24)$$

A possible choice for  $(c_2, c_3)$  is  $c_2 = -B_{2,2}$  and  $c_3 = -2\text{sign}(\det(A)c_2)|B_{3,3}|$  if  $B_{3,3} \neq 0$  and  $c_3 = -\text{sign}(\det(A)c_2)$  otherwise.

Then there exists  $\epsilon_0$  such that  $C(\epsilon)A$  is a Hurwitz matrix for  $0 < \epsilon < \epsilon_0$ .

If  $B$  has a null diagonal, the system cannot be coordinated.

*Proof:* We have that

$$\begin{aligned} \text{tr}(C(\epsilon)A) &= A_{1,1} + O(\epsilon), \\ \text{tr}(C(\epsilon)A)\text{tr}((C(\epsilon)A)^{-1}) &= \frac{A_{1,1}}{\epsilon} \left( \frac{B_{2,2}}{c_2} + \frac{B_{3,3}}{c_3} \right) + O(1). \end{aligned}$$

Recall that  $A_{1,1} < 0$ . So there exists  $\epsilon_0$  such that  $\text{tr}(C(\epsilon)A) < 0$  and  $\text{tr}(C(\epsilon)A)\text{tr}((C(\epsilon)A)^{-1}) > 1$ , if  $\epsilon > \epsilon_0$ . Using Lemma 1,  $C(\epsilon)A$  is a Hurwitz matrix for  $0 < \epsilon < \epsilon_0$ . The existence of a couple  $(c_2, c_3)$  is given by inspection of the proposed value. If  $B$  has a null diagonal, then  $\text{tr}((\text{diag}(c)A)^{-1}) = 0$  for all  $c$ , so that the conditions of Lemma 1 can never be met.  $\blacksquare$

For  $I > 3$ , the problem becomes more involved. Sufficient conditions for the existence of a diagonal matrix can be found in the literature. In particular Fisher and Fuller (1958) [24] have proven that if there exists a permutation matrix  $P$  such that all leading principal sub-matrices of  $\hat{A} = PAP^{-1}$  are of full rank, then  $A$  can be stabilized by scaling.

A more restrictive version of this condition which gives a simple way to construct the coordination matrix is given in the following theorem.

**Theorem 4.** *If all leading principal sub-matrices of  $A$  are of full rank, then there exists a diagonal matrix  $C = \text{diag}(c_1, c_2, \dots, c_I) \in \mathbb{R}^{I \times I}$  that stabilizes  $A$  (i.e.  $CA$  is Hurwitz).*

*Proof:* Indeed, it then suffices to choose  $c_1, c_2, \dots, c_I$  sequentially such that  $(-1)^i c_1 \dots c_i \det([A]_{i,i}) > 0$  for  $i = 1, \dots, I$  where  $[A]_{i,i}$  is the submatrix of  $A$  comprised of lines 1 through  $i$  and columns 1 through  $i$ . This means that  $\forall k = 1..I$ ,  $(-1)^k \det([CA]_{k,k}) > 0$  which implies by a known result [25, Section 16.7] on negative definite matrices that all eigenvalues of  $CA + (CA)^T$  are strictly negative. ■

Later works have extended the Fisher and Fuller condition to more general cases [26].

## V. COORDINATION AS A CONVEX OPTIMIZATION PROBLEM

This section considers the problem of deriving a coordination matrix  $C$  such that (12) is stable while (6) is not. We begin by recalling a sufficient condition for stability mentioned in Section II-B, applying it for the linear case.

**Theorem 5.** *Suppose there exists a  $I \times I$  matrix  $C$  verifying*

$$(CA)^T + CA \prec 0, \quad (25)$$

*then  $A$  and  $C$  are invertible, and  $\theta^*$  is the only equilibrium point of (12) and it is globally asymptotically stable.*

*Proof:* If  $CA + (CA)^T \prec 0$ , then  $CA$  is invertible, and so the equation  $CF(\theta) = CA(\theta - \theta^*) = 0$  has a unique solution which is  $\theta^*$ .

The global asymptotic stability is obtained from Theorem 1, since condition (25) implies diagonal strict concavity. ■

Note that  $\theta^*$  is also an equilibrium point of (6). In addition to the constraint (25) we need to consider an additional constraint which is related to the capability of the different SON entities to exchange information. For example, if two SONs  $i$  and  $j$  are located in different BSs of a LTE network without a X2 interface between them, then the element  $C_{i,j}$  in matrix  $C$  must be equal to 0. On the other hand, if  $C_{i,j} \neq 0$ , then updating the parameter  $\theta_i$  requires the value of  $F_j(\theta)$ , so we have to be sure that this information can be made available. Typically in a network for example, this relates to interfaces that exist between BSs, so the system constraints will be mapped from the network architecture. We denote by  $\mathcal{C} \in \mathbb{R}^{I \times I}$  the set of feasible matrices which satisfy the system constraints.

Denote the two constraints mentioned above as *stability* and *connectivity* constraints. These two constraints may be verified by a large number of matrices, and the one with the best convergence properties is sought. From convex optimization theory, we know that iterative algorithms converge faster when their condition number is lower [27]. Indeed, the solution of the system of ODEs  $\dot{x} = CAx$  can be written as  $x(t) = e^{tCA}x_0$ . The exponential of a matrix is defined using the power series, so

$$x(t) = \left( \sum_{k=0}^{\infty} \frac{t^k}{k!} (CA)^k \right) x_0.$$

If we choose  $x_0$  as an eigenvector of  $CA$  with the eigenvalue  $\lambda_0$ , we can see that  $x(t) = \left( \sum_{k=0}^{\infty} \frac{t^k}{k!} \lambda_0^k \right) x_0 = e^{\lambda_0 t} x_0$ . The same argument is valid for all the eigenvectors of the matrix  $CA$  so that for a random starting point  $x_0$ , a lower

condition number will ensure a better convergence as the speed of convergence will be homogeneous across the eigenspaces.

Without constraints, the best coordination matrix would be  $-A^{-1}$ , leading to a diagonal matrix  $CA = -I$  with the lowest condition number i.e. 1. When taking the constraints into account, we formulate the convex optimization problem as the minimization of the distance, defined in terms of the Frobenius norm, between the coordination matrix  $C$  and  $-A^{-1}$ :

$$\begin{aligned} & \text{minimize } \|C + A^{-1}\|_F \\ & \text{s.t. } (CA)^T + CA \prec 0; C \in \mathcal{C} \end{aligned} \quad (26)$$

where  $\|\cdot\|_F$  is the Frobenius norm defined for a  $\mathbb{R}^{m \times n}$  matrix  $M$  as

$$\|M\|_F = \sqrt{\sum_{i=1}^m \sum_{j=1}^n |M_{i,j}|^2} = \sqrt{\text{Tr}(M^T M)} = \sqrt{\sum_{i=1}^{\min(m,n)} \sigma_i^2} \quad (27)$$

with  $\sigma_i$  being the singular values of  $M$ . It is noted that the Frobenius norm is often used in the literature for finding a preconditioner that improves the convergence behavior of iterative inversion algorithms [28].

The stability constraints are expressed in the form of LMIs. LMIs are a common tool used in control theory for assessing stability. Solving convex optimization problems with LMI constraints is a tractable problem for which fast solvers are available [29].

From the implementation point of view, the coordination process can be performed in two steps as follows. In the first step, a centralized coordination server gathers and processes data to derive the matrix  $A$ , and performs the optimization problem (26) to obtain the coordination matrix  $C$ , and sends each line of the matrix  $C$  to the corresponding SON entity. This step is performed off-line. The second step is the on-line control process where each SON performs the coordinated control, while satisfying the connectivity constraints, by using the appropriate line of matrix  $C$ .

## VI. SON COORDINATION USE CASE: APPLICATION TO WIRELESS NETWORKS

In this section we illustrate instability and coordination in the context of RANs using a use case involving 3 SONs deployed in several BSs of a LTE network.

### A. System Model

Consider three SON mechanisms deployed in the BSs of a LTE network: blocking rate minimization, outage probability minimization and load balancing SON, as presented in [4]. We focus on downlink ftp type traffic model in which each user enters the network, downloads a file from its serving cell and then leaves the network.

*a) Load balancing:* The SON adjusts the BS's pilot powers in order to balance the loads between neighboring cells. The corresponding ODE is given by

$$\dot{P}_s = P_s(\rho_1(\mathbf{P}) - \rho_s(\mathbf{P})), \quad \forall s = 1..I \quad (28)$$

where  $\mathbf{P} \in \mathbb{R}^I$  is the vector of BSs' pilot powers, and  $\rho$  - their corresponding loads. This SON converges to a set on which all loads are equal as shown in [4, Theorem 4]. We use an equivalent formulation in order to update the pilots directly in dB:

$$\dot{P}_{\text{dB}_s} = \rho_1(\mathbf{P}) - \rho_s(\mathbf{P}) \quad \forall s = 1 \dots I. \quad (29)$$

where  $P_{\text{dB}_s}$  is the pilot power of BS  $s$  in decibels.

b) *Blocking rate minimization*: This SON adjusts the admission threshold in order to reach a given blocking rate target  $\bar{B} > 0$ . Consider  $x_s \in \mathbb{R}^+$  such that the admission threshold of BS  $s$  is  $\lfloor x_s \rfloor$ , where  $\lfloor \cdot \rfloor$  denotes the floor function. A new user finding the cell with  $n$  users is blocked with probability  $P(n)$ , where  $P(n) \rightarrow 1$  when  $n \rightarrow x_s$  and  $P(n) \rightarrow 0$  when  $n \rightarrow 0$ . The update equation for the blocking rate minimization SON is

$$x_{s,t+1} = [x_{s,t} + \epsilon_t(B_s(x_t) - \bar{B}_s + N_t)]_{[0, x_{\text{max}}]}^+ \quad (30)$$

where  $x_t$  is the vector of the admission thresholds of all the BSs considered at time  $t$ ,  $x_{\text{max}}$  - a sufficiently large value and  $N_t$  - a martingale difference noise introduced by measuring  $B[x_t]$ . The equivalent ODE is

$$\dot{x}_s = B_s(x) - \bar{B}_s. \quad (31)$$

$x_s \rightarrow B_s(x)$  is a decreasing function of  $x_s$  and  $\lim_{x_s \rightarrow \infty} B_s(x) = 0$ . So for any blocking rate target  $0 < \bar{B}_s < 1$ , we have  $B_s(0) \geq \bar{B}_s$  and there exists a finite  $x_0 \in \mathbb{N}$  such that  $\forall x \geq x_0; B_s(x) \leq \bar{B}_s$  and  $\forall x \leq x_0; B(x) \geq \bar{B}_s$ . By projecting the right hand side of (30) on any interval containing  $[0, x_0]$ , we ensure that  $\sup_t \|x_t\| < \infty$ .

Now considering the function  $V(x) = \max(0, |x - x_0| - \delta)$  for  $\delta$  sufficiently small, we can see that  $V(\cdot)$  is a Lyapunov function for (31). Indeed, we have

- $\forall x \in [0, +\infty), V(x) \geq 0$ .
- $H = \{x \in [0, +\infty), V(x) = 0\} \neq \emptyset$  because it contains  $x_0$ .
- $\dot{V}(x) = \frac{\partial V}{\partial x} \dot{x} = \begin{cases} -(B(x) - \bar{B}_s) & \text{if } x < x_0 - \delta \\ B(x) - \bar{B}_s & \text{if } x > x_0 + \delta \\ 0 & \text{if } x \in [x_0 - \delta, x_0 + \delta] \end{cases} \leq 0$ .
- $V(x) \rightarrow +\infty$  when  $x \rightarrow +\infty$ .

This implies that  $H$  is globally asymptotically stable for (31).

c) *Outage Probability Minimization*: The aim of this SON mechanism is to adjust the transmit data power in order to reach a target outage probability. The outage probability considered is expressed as

$$K_s = \frac{1}{|Z_s|} \int_{Z_s} \mathbb{1}_{\{R_s(r) \geq R_{\text{min}}\}}(r) dr \quad (32)$$

where  $Z_s$  is the area covered by BS  $s$ ,  $R_{\text{min}}$  - a minimum data rate and  $R_s(r)$  - the peak data rate obtained at position  $r$  when served by BS  $s$ . The SA algorithm modeling the actual control loop is

$$D_s[k+1] = D_s[k] - \epsilon_k(K_s(\mathbf{D}[k]) - \bar{K} + N_k^s) \quad (33)$$

where  $N_k^s$  is a martingale difference noise and  $D_s$  is the transmit data power of BS  $s$ . The limiting ODE representing the mean behaviour of SA (33) is then

$$\dot{D}_s = -(K_s(D) - \bar{K}). \quad (34)$$

This ODE is stable if there exists an admissible data power  $D_s^*$  such that  $K_s(D_s^*) = \bar{K}$ . Indeed,  $(K_s(\cdot) - \bar{K})^2$  would then be a Lyapunov function for (34) since  $\frac{\partial K_s}{\partial D_s} > 0$ . As a consequence, the SA (33) converges to invariant sets of (34), which means that the mechanism is standalone-stable.

## B. Numerical Results

Consider a hexagonal network with 19 cells with omnidirectional antennas as shown in Figure 2. A wrap-around model is used to minimize truncation effects of the computational domain. It is achieved by surrounding the original network with 6 of its copies while performing the simulation within the original 19 cells.

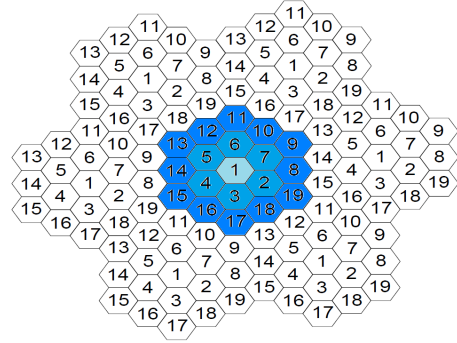


Fig. 2. 19 cells hexagonal network with wrap-around

Table I lists the parameters used in the simulations including the environment, the network, the numerical simulation parameters and the KPIs' targets used by the SON mechanisms. The users arrive in the network according to a Poisson process with a certain arrival rate given in Table I. A hotspot is placed at the center of the network with an additional arrival rate also given in Table I. The hotspot provides initially unbalanced loads in the network, which is of interest for the load balancing SON.

We activate the three SONs in each of the 7 BSs located at the center of the map and observe the stability of the SONs, with and without the coordination mechanism.

We first derive the matrix  $A$  using closed form formulas of the corresponding KPIs and then compute their derivatives using finite differences as in (10). By choosing an adequate step size, this method yields very accurate results. However, closed-form expressions of the KPIs do not always exist, in which case estimations of the KPIs would be used instead, based on measurements from each user that arrive in the network. The stability matrix obtained through linearization already reveals instability since not all of its eigenvalues are negative, and hence the coordination step is inevitable. We then derive the stability matrix  $C$ . Finally coordination is applied using  $\dot{\theta} = CF(\theta)$ .



TABLE I. NETWORK AND TRAFFIC CHARACTERISTICS

Network parameters	
Number of stations	19 (with wrap-around)
Cell layout	hexagonal omni
Intersite distance	500 m
Bandwidth	20MHz
Channel characteristics	
Thermal noise	-174 dBm/Hz
Path loss (d in km)	$128 + 36.4 \log_{10}(d)$ dB
Traffic characteristics	
Arrival rate	40 users/s
Service type	FTP
Average file size	10 Mbits
Hotspot additional arrival rate	2 users/s
Hotspot position	center of BS 1 cell
Hotspot diameter	330 m
Simulation parameters	
Spatial resolution	20 m x 20 m
Time per iteration	6 s
Minimum SINR for coverage	0 dB
Target outage probability	18%
Target blocking rate	2%

We plot the KPIs evolutions for the coordinated (in blue) and non-coordinated (in red) systems (Figures 3 to 5). The coordinated system clearly performs better. The loads and the blocking rates are lower. The outage probabilities in the non-coordinated system diverge. The most loaded BS outage probability is near zero while that of the other BSs is close to one. This is because the decrease in the cell size of the most loaded BS is not accompanied by a decrease of its traffic power. As a result, more interference is produced on its neighbors which have increased their cell size. The objectives related to each SON are satisfied or close to satisfaction. The loads are balanced in about 10 minutes. These results illustrate the usefulness of the distributed SON that benefit from much higher reactivity with respect to a centralized solution.

In Figure 4, we can see that the outage of BS 1 in the coordinated system is low but is off the target set to 18 percent. This is a consequence of the interaction of several SON functions. The operation point defined by the average KPIs can be modified using weights. Each  $F_i$  (corresponding to a distinct SON) is multiplied by a given weight. We now investigate the impact of such weights on the stationary KPIs of the system.

Figures 6 and 7 compare the final values of the KPIs of the coordinated (in blue) and non-coordinated (in red) systems when they reach their permanent state for different weight vectors. For equal weight across all SON functions, we can see in Figure 6 that the self-organized system succeeds more in balancing the loads than reaching the outage target. A closer look at BS1 shows that the power of its traffic channels increases to absorb more traffic while its cell size is reduced leading to a smaller outage probability.

Figure 7 considers the case where more importance is given to the outage, by increasing 20 times the corresponding weight. The outage for the BSs is practically the same as the target,

while the loads are not balanced anymore. We can see that the coordination mechanism reaches a compromise between the different objectives, that can be adjusted by selecting weights to the SON functions to better reflect the network operator policies.

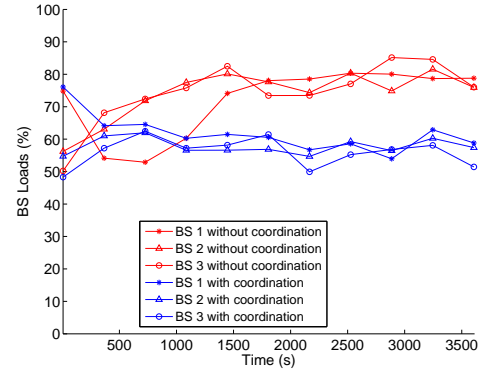


Fig. 3. Impact of Coordination on Loads

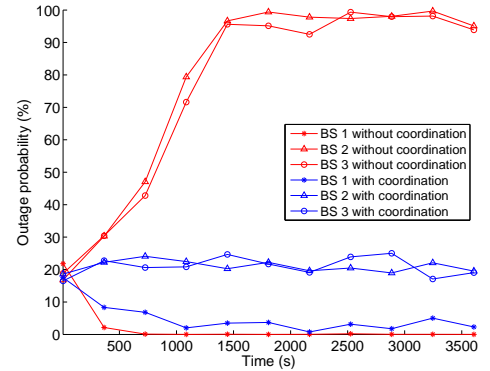


Fig. 4. Impact of Coordination on Coverage Probabilities

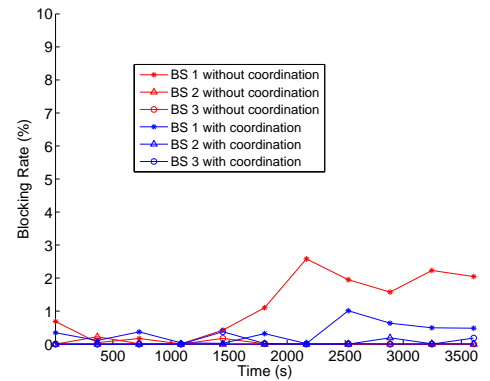


Fig. 5. Impact of Coordination on Blocking Rates

## VII. CONCLUDING REMARKS

In this paper we have studied the problem of coordinating multiple SON entities operating in parallel. Using tools from

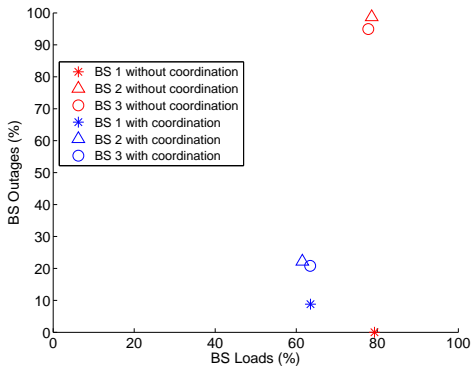


Fig. 6. Stationary KPIs with all SONs equally weighted

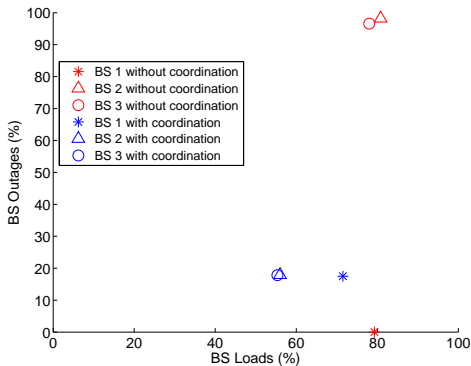


Fig. 7. Stationary KPIs with outage probability prioritized

control theory and Lyapunov stability, we have proposed a coordination mechanism that stabilizes the system. The problem of finding a coordination matrix has been formulated as a convex optimization problem with LMI constraints which ensures that the system of SONs remain distributed. The coordination can be implemented in a distributed fashion and can be scaled with respect to the number of SONs. We have also shown that the coordination solution remains valid in the presence of measurement noise, using stochastic approximation. A practical use case of the coordination method has been presented in a LTE network implementing three distributed SON functionalities deployed in several base stations. It has been shown that the coordination mechanism is necessary to stabilize the network. This use case has also shown that in spite of the linear control assumption, the method remains effective when applied to SON functionalities that are not linear in general.

#### APPENDIX A DEFINITION OF DIAGONAL STRICT CONCAVITY

Diagonal strict concavity is a property introduced in [15] for analyzing equilibrium of  $n$ -person games. Consider  $I$  functions  $\theta \rightarrow g_i(\theta)$  defined on a convex closed bounded set  $S \subset \mathbb{R}^I$ , and  $w_i, i = 1, \dots, I$  some real positive constants. And denote

by

$$JG(\theta) = \begin{bmatrix} w_1 \nabla_1 g_1(\theta) \\ \vdots \\ w_I \nabla_I g_I(\theta) \end{bmatrix}. \quad (35)$$

We say that  $G(\theta) = \sum_{i=1}^I w_i g_i(\theta)$  is diagonally strictly concave for  $\theta \in S$  if for every  $\theta_0, \theta_1 \in S$  we have

$$(\theta_0 - \theta_1)^T JG(\theta_0) + (\theta_0 - \theta_1)^T JG(\theta_1) > 0 \quad (36)$$

#### APPENDIX B MARTINGALES

Martingales are commonly used to characterize noise in stochastic approximation algorithms [19], [22]. We hereby give a succinct definition of martingales and martingale differences along with an insight in why they are useful.

Let  $(\Omega, \mathcal{F}, \mathcal{P})$  denote a probability space, where  $\Omega$  is the sample space,  $\mathcal{F}$  a  $\sigma$ -algebra of subsets of  $\Omega$ , and  $P$  a probability measure on  $(\Omega, \mathcal{F})$ . Let  $\{M_n\}$  be a sequence of real-valued random variables defined on  $(\Omega, \mathcal{F})$ . If  $\mathbb{E}(|M_n|) < \infty$  and

$$\mathbb{E}(M_{n+1} | M_i, i \leq n) = M_n \quad (37)$$

then  $\{M_n\}$  is a martingale sequence. In this case, the sequence  $N_n = M_n - M_{n-1}$  is a martingale difference sequence.

An important result on martingales is the martingale convergence theorem which proves that martingale sequences converge with probability 1. This result is useful to characterize convergence of SA algorithms which model the noise as martingale differences (see [22] and [19]).

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