Energy Efficient Power Control Game Applied to Cognitive Radio Networks

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***Abstract*—Motivated by the facts that mobile terminals have a limited battery life, this paper presents a hierarchical game to model distributed joint power and channel allocation for multi-carrier energy efficient cognitive radio systems. A thorough analysis of the existence, uniqueness and characterization of the Stackelberg equilibrium is conducted. We proved that, at the Stackelberg equilibrium, each of the two users transmits on only one carrier depending on the fading channel gains. This results contrast with capacity-based approaches in which a certain number of carriers is exploited depending on the channel gains. Interestingly, we show that, for the vast majority of cases, introducing a certain degree of hierarchy in a multi-carrier system induces a natural coordination pattern where users have incentive to choose their transmitting carriers in such a way that they always transmit on orthogonal channels. Analytical results are provided for assessing and improving the performances in terms of energy efficiency between the non-cooperative game with synchronous decision makers and the proposed Stackelberg game.**

***Index Terms*—Cognitive Radio Networks; Multi-carrier system; Energy Efficiency; Spectrum Coordination; Game Theory; Nash Equilibrium; Stackelberg Equilibrium.**

I. Introduction

Introducing cognitive radio networks (CRNs) along with heterogeneous networks (HetNets) poses a number of challenges in wireless networking and communications. While developing all the components of these networks is important, a major challenge is figuring out how they fit together and work together under realistic conditions. In fact, many licensed and unlicensed frequency bands support heterogeneous wireless networks running different physical and link layer protocols. These networks coordinate the spectrum, either in a centralized manner or in an arbitrary manner. However, the desired autonomous feature of future wireless systems makes the use of a central authority for spectrum management less appealing. Furthermore, an arbitrary manner results in poor performance for some networks and sub-optimal performance in aggregate. To overcome these operational challenges among others that the introduction of future generations of LTE imposes on the spectrum management, we introduce a degree of hierarchy between the primary user (PU) and the secondary user (SU) in the CRN multi-carrier system and show how this can *naturally* lead to a spectrum coordination pattern where users have incentive to transmit on different carriers.

Recent trends in mobile client access recognize energy efficiency as an additional constraint for realizing efficient and sustainable computing [1]. It is a challenge to support these two notions of efficiency while addressing a wide variety of delay and throughput objectives. In a prior work [2], we studied a hierarchical game theoretic model for two-user–two-carrier energy efficient wireless systems. The power control problem is modeled as a Stackelberg game where transmitters choose their control policy selfishly in order to maximize their individual energy efficiency. It was shown that, for the vast majority of cases, users choose their transmitting carriers in such a way that if the leader transmits on a given carrier, the follower have incentive to choose the other carrier. One major motivation of this paper is to extend the original problem in [2] to some general models that can be widely used in practice assuming an arbitrary number of carriers. The multi-dimensional nature of such a problem and the non-quasi-concavity of the energy efficiency function (which we will see later in the paper) make the extension to an arbitrary number of carriers problem much more challenging than the two-carrier case. We provide a thorough comparison between our work and the one addressed in [3] in terms of strategies at equilibrium and energy efficiency. We further provide *tight* bounds on the probability of no coordination.

Note that the Stackelberg formulation arises naturally in many context of practical interest. For example, the hierarchy is inherent to CRNs where the user with the higher priority (*i.e.*, the leader or PU) transmits first, then the user with the lower priority (*i.e.*, the follower or SU) transmits after sensing the spectral environment [4]. It is also natural if the users access to the medium sequentially in an asynchronous manner [5]. Note that there have been many works on Stackelberg games, even in the context of cognitive radio [6], but they do not consider energy-efficiency for the individual utility as defined in [7]. They rather consider transmission rate-type utilities (see *e.g.,* [8]–[10]).

II. System Model

We consider a decentralized multiple access channel with an arbitrary number of carriers $K \geq 2$ composed of a PU (or leader – indexed by 1), having the priority to access the medium, and a SU (or follower – indexed by 2) that accesses the medium after observing the action of the PU. For any user $n \in \{1, 2\}$, the received signal-to-noise plus interference ratio

(SINR) is expressed as

$$\gamma_n^k = \frac{g_n^k p_n^k}{\sigma^2 + \sum_{\substack{m=1 \\ m \neq n}}^{2} g_m^k p_m^k} := p_n^k \widehat{h}_n^k \quad (1)$$

We will call $\widehat{h}_n^k$ the *effective channel gain*, defined as the ratio between the SINR and the transmission power for the users over the k^{th} carrier for $k = \{1, \ldots, K\}$. g_n^k and p_n^k are resp. the fading channel gain and the power control of user n transmitting on carrier k, whereas σ^2 stands for the variance of the Gaussian noise. We statistically model the channel gains g_n^k to be independent identically distributed (i.i.d.) over the Rayleigh fading coefficients. It follows from the above SINR expression that the strategy chosen by a user affects the performance of other users in the network through multiple-access interference.

III. Network energy-efficiency analysis

The system model adopted throughout the paper is based on the seminal paper [7] that defines the energy efficiency framework. The energy efficiency can be concisely captured by an increasing, continuous and S-shaped "efficiency" function $f(\cdot)$ which measures the packet success rate. The following utility function allows one to measure the corresponding tradeoff between the transmission benefit (total throughput over the K carriers) and cost (total power over the K carriers):

$$u_n(\mathbf{p_1}, \mathbf{p_2}) = \frac{R_n \cdot \sum_{k=1}^{K} f(\gamma_n^k)}{\sum_{k=1}^{K} p_n^k} \quad (2)$$

where R_n is the transmission rate of user n and $\mathbf{p_n}$ is the power allocation vector of user n over all carriers, *i.e.,* $\mathbf{p_n} = (p_n^1, \ldots, p_n^K)$. The utility function u_n, that has bits per joule as units, perfectly captures the tradeoff between throughput and battery life and is particularly suitable for applications where energy efficiency is crucial.

IV. The game theoretic formulation

A. The Nash game problem

An important solution concept of the game under consideration is the Nash equilibrium (NE), which is a fundamental concept in non-cooperative strategic games. It is a vector of strategies (referred to hereafter and interchangeably as actions) $\mathbf{p}^{NE} = \{\mathbf{p_1}^{NE}, \mathbf{p_2}^{NE}\}$, one for each player, such that no player has incentive to unilaterally change his strategy. If there exists an $\epsilon > 0$ such that[1] $(1+\epsilon) u_n(\mathbf{p_n}^{\epsilon NE}, \mathbf{p}_{-n}^{\epsilon NE}) \geq u_n(\mathbf{p_n}, \mathbf{p_{-n}}^{\epsilon NE})$ for every action $\mathbf{p_n} \neq \mathbf{p_n}^{\epsilon NE}$, we say that the vector $\mathbf{p}^{\epsilon NE} = \{\mathbf{p_1}^{\epsilon NE}, \mathbf{p_2}^{\epsilon NE}\}$ is an ϵ-Nash equilibrium.

[1]The $-n$ subscript on vector $\mathbf{p}$ stands for "except user n".

B. The hierarchical game formulation

Hierarchical models in wireless networks are motivated by the idea that the utility of the leader obtained at the Stackelberg equilibrium can often be improved over his utility obtained at the Nash equilibrium when the two users play simultaneously. It has been proved in [11], that when only one carrier is available for the players this result is valid for both the leader and the follower. The goal is then to find a Stackelberg equilibrium in this two-step game. In this work, we consider a Stackelberg game framework in which the leader decides first his power allocation vector $\mathbf{p_1}$, and based on this value, the follower adapts his power allocation vector $\mathbf{p_2}$.

Definition 1. *(***Stackelberg equilibrium***):* A vector of actions $\widetilde{\mathbf{p}} = (\widetilde{\mathbf{p}}_\mathbf{1}, \widetilde{\mathbf{p}}_\mathbf{2}) = (\widetilde{p}_1^1, \ldots, \widetilde{p}_1^K, \widetilde{p}_2^1, \ldots, \widetilde{p}_2^K)$ is called Stackelberg equilibrium (SE) if and only if:

$$\widetilde{\mathbf{p}}_\mathbf{1} = \arg\max_{\mathbf{p_1}} u_1(\mathbf{p_1}, \overline{p}_2(\mathbf{p_1})),$$

where for all $\mathbf{p_1}$, we have

$$\overline{p}_2(\mathbf{p_1}) = \arg\max_{\mathbf{p_2}} u_2(\mathbf{p_1}, \mathbf{p_2}),$$

and $\widetilde{\mathbf{p}}_\mathbf{2} = \overline{p}_2(\widetilde{\mathbf{p}}_\mathbf{1})$.

A Stackelberg equilibrium can then be determined using a bi-level approach, where, given the action of the leader, we compute the best-response function of the follower (the function $\overline{p}_2(\cdot)$) and find the actions of the followers which maximize their utilities. We characterize this best-response function by using a result from [3].

V. Characterization of the Stackelberg equilibrium

A. Review of the follower's power allocation vector

We first determine the best-response function of the follower depending on the action of the leader. This result comes directly from Proposition 1 of [3]. For making this paper sufficiently self-contained, we review here the latter proposition.

Proposition 1. *Given the power allocation vector* $\mathbf{p_1}$ *of the leader, the best-response of the follower is given by*

$$\overline{p}_2^k(\mathbf{p_1}) = \begin{cases} \dfrac{\gamma^*(\sigma^2 + g_1^k p_1^k)}{g_f^k}, & \textit{for} \quad k = L_2(\mathbf{p_1}) \\ 0, & \textit{for all} \quad k \neq L_2(\mathbf{p_1}) \end{cases} \quad (3)$$

with $L_2(\mathbf{p_1}) = \arg\max_k \widehat{h}_2^k(p_1^k)$ *and* γ^* *is the unique (positive) solution of the first order equation*

$$x\, f'(x) = f(x) \quad (4)$$

Equation (4) has a unique solution if the efficiency function $f(\cdot)$ is sigmoidal [12]. The last proposition says that the best-response of the follower is to use only one carrier, the one such that the effective channel gain is the best.

B. Characterization of the leader's power allocation vector

Let us first present a result that will allow us to characterize the equilibrium strategies.

Proposition 2. *Denote by B_1 and S_1 two carriers for the leader for which g_1^k is the highest and the second highest respectively, while by B_2 and S_2 the ones with two highest g_2^k (that is, for the follower). If the Stackelberg game has an equilibrium, then it has an equilibrium where the leader transmits over one of the carriers B_1, S_1, while the follower transmits over one of the carriers B_2, S_2.*

Due to the lack of space, proofs and complementary analysis can be found in the technical report [13].

Given this result, we may only concentrate on strategies where each of the players uses one of his two best carriers. The proposition below gives the algorithm to compute the equilibrium power allocations for both players. Before the proposition we introduce additional notation:

$$\hat{\gamma} = \frac{g_2^{B_2} - g_2^{S_2}}{g_2^{S_2}}.$$

Proposition 3. *If $B_1 \neq B_2$ then equilibrium power allocation of each of the players is*

$$\overline{p}_n^k = \begin{cases} \frac{\gamma^* \sigma^2}{g_n^k} & \textit{when } k = B_n \\ 0 & \textit{otherwise} \end{cases}$$

If $B_1 = B_2$ then the equilibrium power allocations of the players are computed in three steps:

1) *If $\hat{\gamma} \leq \gamma^*$ then equilibrium power allocation of the leader is*

$$\overline{p}_1^k = \begin{cases} \frac{\gamma^* \sigma^2}{g_1^k} & \textit{when } k = B_1 \\ 0 & \textit{otherwise} \end{cases}$$

and that of the follower is

$$\overline{p}_2^k = \begin{cases} \frac{\gamma^* \sigma^2}{g_2^k} & \textit{when } k = S_2 \\ 0 & \textit{otherwise} \end{cases}$$

If this is not satisfied steps 2 and 3 have to be made.

2) *Find all the solutions $x \leq \frac{\hat{\gamma}}{1+\gamma^*(1+\hat{\gamma})}$ to the equation*

$$(x - x^2\gamma^*) f'(x) = f(x) \tag{5}$$

If there are solutions different than $x = 0$, choose the one for which $\frac{f(x)(1-x\gamma^*)}{x}$ is the highest. Let β^* be this solution.

3) *Compare four values[2]:*

$$V_{B_1} = \frac{f(\beta^*)(1-\gamma^*\beta^*) g_1^{B_1} R_1}{\beta^* \sigma^2 (1+\gamma^*)}, \quad W_{B_1} = \frac{f(\hat{\gamma}) g_1^{B_1} R_1}{\hat{\gamma}\sigma^2},$$

$$U_{S_1} = \frac{f(\gamma^*) g_1^{S_1} R_1}{\gamma^* \sigma^2}, \quad V_{B_1}^0 = f'(0) \frac{g_1^{B_1} R_1}{\sigma^2 (1+\gamma^*)}$$

[2]Of course V can only be computed if β^* exists.

If V_{B_1} is the greatest, then equilibrium power allocations of the leader and the follower are

$$\overline{p}_1^k = \begin{cases} \frac{\beta^*(1+\gamma^*)\sigma^2}{g_1^k(1-\gamma^*\beta^*)} & \textit{when } k = B_1 \\ 0 & \textit{otherwise} \end{cases}$$

and

$$\overline{p}_2^k = \begin{cases} \frac{\gamma^*(1+\beta^*)\sigma^2}{g_2^k(1-\gamma^*\beta^*)} & \textit{when } k = B_2 \\ 0 & \textit{otherwise} \end{cases}$$

Next, if W_{B_1} is the greatest, then equilibrium power allocations of the leader and the follower are

$$\overline{p}_1^k = \begin{cases} \frac{\hat{\gamma}\sigma^2}{g_1^k} & \textit{when } k = B_1 \\ 0 & \textit{otherwise} \end{cases}$$

and

$$\overline{p}_2^k = \begin{cases} \frac{\gamma^*\sigma^2}{g_2^k} & \textit{when } k = S_2 \\ 0 & \textit{otherwise} \end{cases}$$

If U_{S_1} is the greatest, then equilibrium power allocation of the leader is

$$\overline{p}_1^k = \begin{cases} \frac{\gamma^*\sigma^2}{g_1^k} & \textit{when } k = S_1 \\ 0 & \textit{otherwise} \end{cases}$$

and that of the follower is

$$\overline{p}_2^k = \begin{cases} \frac{\gamma^*\sigma^2}{g_2^k} & \textit{when } k = B_2 \\ 0 & \textit{otherwise} \end{cases}$$

Finally, if $V_{B_1}^0$ is (the only) greatest, then the game has no equilibrium.

Remark 1. Note that the equilibria computed above are unique as long as channel gains for different carriers are different and as long as $V_{B_1} \neq W_{B_1} \neq U_{S_1}$. Also the response of the follower at equilibrium is unique as long as channel gains for different carriers are different and W_{B_1} is not the greatest value in step 3) of the algorithm described by the theorem[3]. The matter of uniqueness of the follower's response is obviously very important, as in case there are multiple best responses to an equilibrium strategy of the leader, the follower has no incentive to follow his equilibrium policy. In our case the equilibrium strategy can be imposed to the follower when he has multiple best responses to the leader's policy by using a simple trick: whenever W_{B_1} appears to be the greatest in step 3), the leader has to use power infinitesimally smaller than that prescribed by his equilibrium policy. This gives him a minimally smaller utility, but at the same time makes the best response of the follower unique.

The next proposition characterizes the degenerate case when there is no equilibrium in the Stackelberg game.

[3]Note that, in case there are multiple equilibria, because $V_{B_1} = U_{S_1} > W_{B_1}$, the response of the follower to both equilibrium strategies of the leader is unique.

Proposition 4. *Whenever $B_1 = B_2$, $\hat{\gamma} > \gamma^*$ and*

$$f'(0) > \max\left\{ \frac{f(\hat{\gamma})(1+\gamma^*)}{\hat{\gamma}}, \frac{f(\gamma^*)(1+\gamma^*)}{\gamma^*}\frac{g_1^{S_1}}{g_1^{B_1}}, \frac{f(\beta^*)(1-\gamma^*\beta^*)}{\beta^*} \right\}, \tag{6}$$

the Stackelberg game has no equilibrium, but for any $\epsilon > 0$ there are ϵ-equilibria of the form

$$\overline{p}_1^k(\epsilon) = \begin{cases} \alpha(\epsilon) & \text{when } k = B_1 \\ 0 & \text{otherwise} \end{cases}$$

for the leader and

$$\overline{p}_2^k(\epsilon) = \begin{cases} \frac{\gamma^*(\sigma^2 + g_1^k\alpha(\epsilon))}{g_2^k} & \text{when } k = B_2 \\ 0 & \text{otherwise} \end{cases}$$

for the follower, where $\alpha(\epsilon)$ is an arbitrarily small value, guaranteeing that the utility of the leader is within ϵ form V_0.

This is important to notice that the case considered in Proposition 4 is indeed possible for some sigmoidal function f.

On the other hand, any of the two following assumptions:
(A1) $f'(0^+) = 0$,
(A2) $f'(0^+) > 0$ and $\frac{f'(0^+)}{f'(0^+)} > 2\gamma^*$,
implies that (6) is never satisfied, and so the game under consideration always has an equilibrium. In particular, for the most standard form of f,

$$f(x) = (1 - e^{-x})^M, \qquad M > 1$$

not only there always exists an equilibrium in the Stackelberg model (because f satisfies (A1)), but also the procedure in Proposition 3 slightly simplifies, as:

1) Equation (4) can be written as $Mx = e^x - 1$,
2) Equation (5) can be written as $M(x - x^2\gamma^*) = e^x - 1$, moreover it has exactly one positive solution.

VI. Performance Evaluation of the Stackelberg Approach

This section is dedicated to present some important properties of the Stackelberg equilibrium we derived in the previous section.

A. Spectrum coordination

In this section, we shall look for values of channel gains of users that may lead the leader and the follower to use the same carrier.

Proposition 5. *The set of $g_n^1, \ldots, g_n^K$, $n = 1, 2$ for which there is no coordination between the users (and they both use the same carrier) is a proper subset of the set G_0 of g_n^ks satisfying:*

$$B_1 = B_2 \text{ and } g_n^{B_n} \geq (1 + \gamma^*)g_n^{S_n};\ \text{for } n = 1, 2. \tag{7}$$

Note that G_0 is exactly the set of g_1^ks for which there is no coordination in the simultaneous-move game considered in [3]. Thus, introducing hierarchy in the game induces more spectrum coordination than there was in the simultaneous-move scenario.

In the next proposition, we will show that the probability of no coordination between the players is always small and decreases fast as the number of carriers grows.

Proposition 6. *The probability that there is no coordination between the players is bounded above by*

$$(1+\gamma^*)\mathcal{B}(1+\gamma^*, K)\left[\frac{K-1}{K} + (1+\gamma^*)\mathcal{B}(1+\gamma^*, K)\right] \sim O(K^{-(1+\gamma^*)}) \tag{8}$$

where $\mathcal{B}$ denotes the Beta function, which is the exact probability of no coordination in the simultaneous-move version of the model.

B. Payoffs comparison

The leader is not worse off on introducing hierarchy (which is always the case in Stackelberg games if both the leader and the follower use their equilibrium policies), but the follower loses on it in some cases. The proposition below gives more insights on what the latter depends on.

Proposition 7. *For any sigmoidal function f the following three situations are possible:*

1) *$B_1 \neq B_2$. Then, for both the leader and the follower, the payoff in the simultaneous-move game is the same as in the Stackelberg game.*
2) *Both players use the same carrier $B_1 = B_2$ in equilibria (or ϵ-equilibria) of simultaneous-move and Stackelberg games. Then, the payoff of the follower in the Stackelberg game is always bigger than that in the simultaneous-move game.*
3) *$B_1 = B_2$ and both players change the carriers they use in equilibrium: the leader from S_1 in the simultaneous-move game to B_1 in the Stackelberg game, the follower from B_2 in the simultaneous-move game to S_2 in the Stackelberg game. Then, the payoff of the follower in the Stackelberg game is smaller than that in the simultaneous-move game.*

C. Comparison between leading and following

It is known, from [11], that if there is only one carrier available for the players, it is always better to be the follower than to be the leader. The situation changes when the number of carriers increases.

Proposition 8. *Suppose that the Stackelberg game has exact equilibria both when player 1 is the leader and when he is the follower. Then the utility at Stackelberg equilibrium of player 1 if he is the leader is not less than his utility if he is the follower iff one of the following conditions is satisfied:*

1) *$B_1 \neq B_2$.*
2) *$B_1 = B_2$ and $\min\{\frac{g_1^{B_1}}{g_1^{S_1}}, \frac{g_2^{B_2}}{g_2^{S_2}}\} \leq 1 + \gamma^*$*

3) $B_1 = B_2$ *and for* $i = 1, 2,\ j \neq i$,

$$f\left(\frac{\frac{g_i^{B_i}}{g_i^{S_i}} - 1}{\frac{g_i^{B_i}}{g_i^{S_i}} - 1}\right) \geq \max\left\{\frac{f(\gamma^*)g_j^{S_j}}{\gamma^* g_j^{B_j}}, \frac{f(\beta^*)(1-\gamma^*\beta^*)}{\beta^*(1+\gamma^*)}\right\}$$

4) $B_1 = B_2$,

$$\frac{f(\beta^*)(1-\gamma^*\beta^*)}{\beta^*(1+\gamma^*)} \geq \max\left\{\frac{f(\gamma^*)g_1^{S_1}}{\gamma^* g_1^{B_1}}, \frac{f\left(\frac{g_2^{B_2}}{g_2^{S_2}} - 1\right)}{\frac{g_2^{B_2}}{g_2^{S_2}} - 1}\right\}$$

and

$$\frac{f\left(\frac{g_1^{B_1}}{g_1^{S_1}} - 1\right)}{\frac{g_1^{B_1}}{g_1^{S_1}} - 1} \geq \max\left\{\frac{f(\gamma^*)g_2^{S_2}}{\gamma^* g_2^{B_2}}, \frac{f(\beta^*)(1-\gamma^*\beta^*)}{\beta^*(1+\gamma^*)}\right\}$$

5) $B_1 = B_2$,

$$\frac{f(\gamma^*)g_1^{S_1}}{\gamma^* g_1^{B_1}} \geq \max\left\{\frac{f\left(\frac{g_2^{B_2}}{g_2^{S_2}} - 1\right)}{\frac{g_2^{B_2}}{g_2^{S_2}} - 1}, \frac{f(\beta^*)(1-\gamma^*\beta^*)}{\beta^*(1+\gamma^*)}\right\}$$

$$\frac{f(\beta^*)(1-\gamma^*\beta^*)}{\beta^*(1+\gamma^*)} \geq \max\left\{\frac{f(\gamma^*)g_2^{S_2}}{\gamma^* g_2^{B_2}}, \frac{f\left(\frac{g_1^{B_1}}{g_1^{S_1}} - 1\right)}{\frac{g_1^{B_1}}{g_1^{S_1}} - 1}\right\}$$

and

$$\frac{g_1^{B_1}}{g_1^{S_1}} \leq \frac{1+\beta^*}{1-\gamma^*\beta^*}$$

6) $B_1 = B_2$,

$$\frac{f\left(\frac{g_2^{B_2}}{g_2^{S_2}} - 1\right)}{\frac{g_2^{B_2}}{g_2^{S_2}} - 1} \geq \max\left\{\frac{f(\gamma^*)g_1^{S_1}}{\gamma^* g_1^{B_1}}, \frac{f(\gamma^*)(1-\gamma^*\beta^*)}{\gamma^*(1+\gamma^*)}\right\}$$

and

$$\frac{f(\beta^*)(1-\gamma^*\beta^*)}{\beta^*(1+\gamma^*)} \geq \max\left\{\frac{f(\gamma^*)g_2^{S_2}}{\gamma^* g_2^{B_2}}, \frac{f\left(\frac{g_1^{B_1}}{g_1^{S_1}} - 1\right)}{\frac{g_1^{B_1}}{g_1^{S_1}} - 1}\right\}$$

Although the formulation of the proposition is rather complicated, its general meaning is simple. It states that if best carriers of the players are different or at least one of the players has two good carriers (that is – the second best carrier is not much worse then the best one), then it is profitable to be the leader. If each of the players has only one good carrier and the same for both, their situation reduces to that when only one carrier is available, and so every user can obtain better energy efficient utility by decreasing its priority from leading to following. It is worth noting though that if g_n^k are i.i.d. Rayleigh variables, one of the two first cases of Proposition 8 will occur with probability significantly bigger than $1-(1+\gamma^*)\mathcal{B}(1+\gamma^*, K)\left[\frac{K-1}{K} + (1+\gamma^*)\mathcal{B}(1+\gamma^*, K)\right]$, and so it will be very close to 1 even for small values of K.

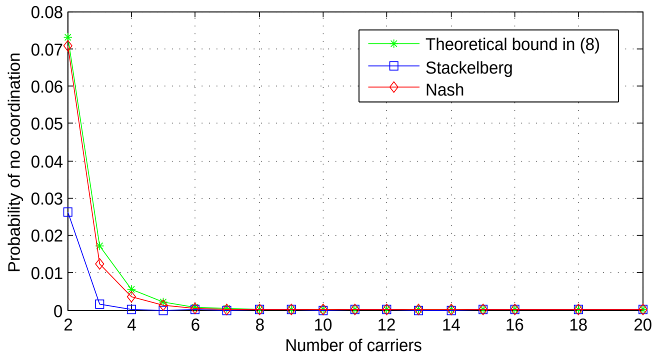

Fig. 1. The probability of no coordination between the players as a function of the the number of carriers.

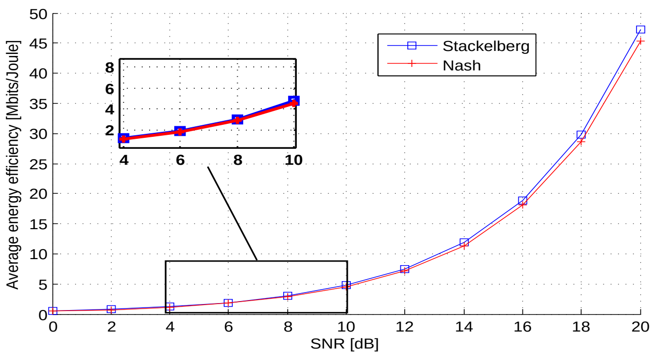

Fig. 2. Average energy efficiency for $K = 20$ carriers.

VII. Simulations

We consider the well-known energy efficiency function in power allocation games, $f(x) = (1-e^{-x})^M$, where $M = 100$ is the block length in bits. Note that we have simulated 10000 scenario to remove the random effects from Rayleigh fading. We have considered that $SNR = \frac{1}{\sigma^2}$ and that the rate of all the users is $R = 1$ Mbps.

In Figure 1, we plot the probability of no coordination for the Nash and the Stackelberg models. As expected, we remark that the probability of no coordination between the users decreases as the number of carriers grows, which is somehow intuitive. In order to assess the accuracy of the theoretical bound derived in Eq. (8), we also plot the simulated probability of no correlation. Fig. 1 shows that the simulated curve of the probability of no coordination at the Nash equilibrium and the theoretical curve in Eq. (8) match pretty well. Now, when we look at the Stackelberg equilibrium in Fig. 1, it is clearly illustrated that the probability of no coordination at the Stackelberg equilibrium is upper-bounded by the theoretical one in Eq. (8), which confirms the accuracy of the results.

On the other hand, such an allocation is in stark contrast with that of capacity-maximizing schemes. Indeed, it is com-

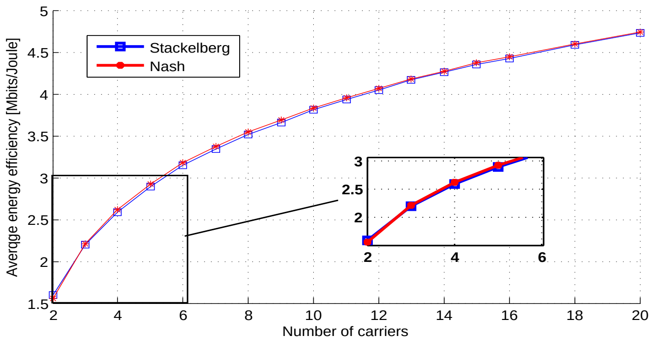

Fig. 3. Average energy efficiency at SNR = 10 dB.

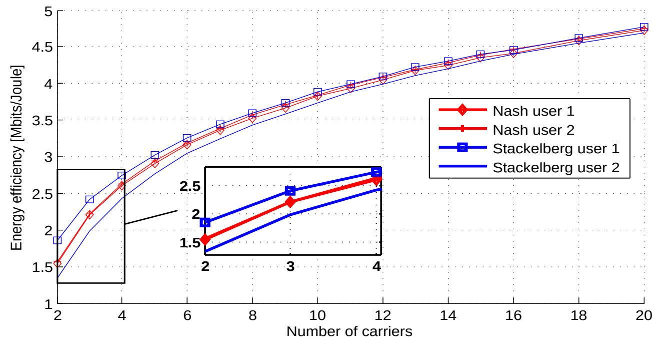

Fig. 4. Per-user energy efficiency at SNR = 10 dB.

monly known that maximizing the users' sum capacity leads to a water-filling power allocation [14], while for multi-cell interfering links, the capacity-optimal power allocation has been shown to be binary [15]. We emphasize that, in the latter both cases, a certain number of carriers are exploited depending on the channel gains, whereas, when we come up to maximize energy efficiency, each user uses only one carrier depending on his fading channel gain.

Figure 2 depicts the average energy efficiency as a function of the SNR in dB for $K = 20$ carriers. We see that the Stackelberg model performs almost the same than the Nash model. This suggests that both models are not very sensitive with respect to the SNR.

Fig. 3 depicts the average energy efficiency at the equilibrium for increasing number of carriers K. Again, we remark that the Stackelberg model performs almost the same than the Nash model.

We then resort to plot the energy efficiency per user in Fig. 4. Interestingly, we see that, at the Stackelberg equilibrium, the energy efficiency of the follower in the Stackelberg game is smaller than that in the Nash game. This suggests that, for the vast majority of cases, Situation 3) in Prop. 7 is more likely to occur for a low number of carriers K. As K increases, Situation 1) in Prop. 7 is more likely to occur yielding the same energy efficiency for both the leader and the follower in the Stackelberg game as in the Nash game. This is justified by the fact that with probability $1/K$, resp. $(K-1)/K$, users have the same, resp. different, best channels. It is then easy to see that, for low K, users are more likely to have the same best channels and interference is an issue in this case yielding to Situation 3) in Prop. 7, whereas, for sufficiently large K, users are more likely to have different best channels yielding to Situation 1) in Prop. 7. Moreover, Fig. 4 also shows that ***leading*** is much better than ***following*** which corresponds to what Prop. 8 points out. Notice that this contrasts with the result in [11] which studies single carrier hierarchical games.

VIII. Conclusion

We have proposed a hierarchical game to model distributed joint power and channel allocation for multi-carrier energy efficient cognitive radio systems. We have formulated the problems of resource allocation and energy efficiency as a two-stage Stackelberg game. We have established the existence of the Stackelberg equilibrium and give its formal expression. As opposed to [11] (which studies single carrier hierarchical games), we have shown that, when we come up to study multi-carrier hierarchical games, the degree of freedom increases and ***leading*** becomes better than ***following***.

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