Interval Observers for Continuous-Time LPV Systems with L1 /L2 Performance

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Abstract

An approach to interval observer design for Linear Parameter-Varying (LPV) systems is proposed. It is assumed that the vector of scheduling parameters in LPV models is not available for measurement. Two different interval observers are constructed for nonnegative systems and for a generic case. Stability conditions are expressed in terms of matrix inequalities, which can be solved with respect to the observer gains using standard numerical solvers. Applying L_1/L_2 framework the robustness and estimation accuracy with respect to model uncertainty are analyzed. The efficiency of interval estimation for LPV models is demonstrated through numerical experiments for a microfluidic system and an academic example.

I. Introduction

State estimation is an important issue in many engineering fields [1], [2], [3]. Estimated states may be required, for example, for control design or fault detection. This problem has been widely studied in the literature and many solutions already exist for linear systems and a number of nonlinear structures. For the latter situation, the observer design problem is solvable if the system model can be transformed into a canonical form, which may be a hard assumption to satisfy in many applications. To solve the problem, an appealing approach is based on the LPV transformation of the nonlinear system [4], [5], [6], [7]. Note that Takagi-Sugeno decomposition can be another alternative solution to deal with nonlinear systems and to obtain the equivalent representation by a compact set of linear state space models with nonlinear weighting functions satisfying the convex sum property [8], [9]. In the presence of uncertainty (unknown parameters or/and external disturbances) the design of a conventional estimator, converging to the ideal value of the state, cannot be realized. However, an interval estimation may still remain feasible: an observer can be constructed that, using input-output information, evaluates the set of admissible values (interval) for the state at each instant of time. The interval length has to be minimized and it is proportional to the size of the model uncertainty. Despite such a formulation looks like a simplification of the state estimation problem, in fact it is an improvement since the interval mean can be used as the state pointwise estimate, while the interval limits give the admissible deviations from that value (thus, an interval estimator provides a simultaneous accuracy evaluation for bounded uncertainty, which may not have a known statistics).

There are several approaches to design interval/set-membership estimators [10], [11], [12], [13]. This paper continues the trend of interval observer design based on the monotone systems theory [12], [13], [14], [15], [16]. In such a way the main restriction for the interval observer design consists in providing cooperativity of the interval estimation error dynamics by a proper design. Such a complexity has been recently overcame in [17], [15], [18] for LTI systems. In those studies, it has been shown that under some mild conditions, by applying a similarity transformation, a Hurwitz matrix could be transformed to a Hurwitz and Metzler one (cooperative). An interval observer design for the systems with non-constant matrices dependent on measurable input-output signals and time has been presented in [19], where a constant similarity transformation matrix representing a given interval of matrices to an interval of Metzler matrices is used. Thus that method can be applied to design interval observers for LPV systems with a measurable vector of scheduling parameters (as noted in [19]).

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The objective of this paper is to propose interval observers for LPV systems with an unmeasurable vector of scheduling parameters. In some practical applications, one or more candidate scheduling parameters cannot be directly measured, or its (their) measurement(s) is (are) not judged sufficiently reliable. For example, on-board measurement of mass of aircraft is approximate and in some situations its practical use as scheduling parameter can be problematic for LPV modeling [20]. This estimation problem has been addressed in [21], [22], [14], taking also into account a tight bound computation on the size of the estimated interval. The contribution of this paper is considered to be the development of an improved structure of interval observers and stability conditions, in order to compute the bounding solutions as accurate as possible. Optimization of the observer gains is addressed using L_1/L_2 setting [23], [24], [25].

The paper is organized as follows. An introduction to the theory of interval estimation is given in Section 2. Two interval observers, for a nonnegative LPV system and for a generic one, are presented in Section 3. Two examples of numerical experiments are given in Section 4.

II. Preliminaries

Euclidean norm for a vector $x \in \mathbb{R}^n$ will be denoted as $|x|$, and for a measurable and locally essentially bounded input $u : \mathbb{R}_+ \to \mathbb{R}$ ($\mathbb{R}_+ = \{\tau \in \mathbb{R} : \tau \geq 0\}$) the symbol $||u||_{[t_0,t_1]}$ denotes its L_∞ norm:

$$||u||_{[t_0,t_1]} = ess\sup\{|u(t)|, t \in [t_0, t_1]\},$$

if $t_1 = +\infty$ then we will simply write $||u||$. We will denote as $\mathcal{L}_\infty$ the set of all inputs u with the property $||u|| < \infty$. Denote the sequence of integers $1, ..., k$ as $\overline{1,k}$. The symbols I_n, $E_{n\times m}$ and E_p denote the identity matrix with dimension $n \times n$, the matrix with all elements equal 1 with dimensions $n \times m$ and $p \times 1$ respectively. For a matrix $A \in \mathbb{R}^{n\times n}$ the vector of its eigenvalues is denoted as $\lambda(A)$, $||A||_{max} = \max_{i=\overline{1,n},j=\overline{1,n}} |A_{i,j}|$ (the elementwise maximum norm, it is not sub-multiplicative) and $||A||_2 = \sqrt{\max_{i=\overline{1,n}} \lambda_i(A^{\mathrm{T}}A)}$ (the induced L_2 matrix norm), the relation $||A||_{max} \leq ||A||_2 \leq n||A||_{max}$ is satisfied between these norms.

For two vectors $x_1, x_2 \in \mathbb{R}^n$ or matrices $A_1, A_2 \in \mathbb{R}^{n\times n}$, the relations $x_1 \leq x_2$ and $A_1 \leq A_2$ are understood elementwise. The relation $P \prec 0$ ($P \succ 0$) means that the matrix $P \in \mathbb{R}^{n\times n}$ is negative (positive) definite. Given a matrix $A \in \mathbb{R}^{m\times n}$, define $A^+ = \max\{0, A\}$, $A^- = A^+ - A$ (similarly for vectors) and denote the matrix of absolute values of all elements by $|A| = A^+ + A^-$.

Lemma 1. [26] *Let $x \in \mathbb{R}^n$ be a vector variable, $\underline{x} \leq x \leq \overline{x}$ for some $\underline{x}, \overline{x} \in \mathbb{R}^n$.*

(1) *If $A \in \mathbb{R}^{m\times n}$ is a constant matrix, then*

$$A^+\underline{x} - A^-\overline{x} \leq Ax \leq A^+\overline{x} - A^-\underline{x}. \tag{1}$$

(2) *If $A \in \mathbb{R}^{m\times n}$ is a matrix variable and $\underline{A} \leq A \leq \overline{A}$ for some $\underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$, then*

$$\begin{aligned} &\underline{A}^+\underline{x}^+ - \overline{A}^+\underline{x}^- - \underline{A}^-\overline{x}^+ + \overline{A}^-\overline{x}^- \leq Ax \\ &\leq \overline{A}^+\overline{x}^+ - \underline{A}^+\overline{x}^- - \overline{A}^-\underline{x}^+ + \underline{A}^-\underline{x}^-. \end{aligned} \tag{2}$$

Furthermore, if $-\overline{A} = \underline{A} \leq 0 \leq \overline{A}$, then the inequality (2) can be simplified: $-\overline{A}(\overline{x}^+ + \underline{x}^-) \leq Ax \leq \overline{A}(\overline{x}^+ + \underline{x}^-)$.

A matrix $A \in \mathbb{R}^{n\times n}$ is called Hurwitz if all its eigenvalues have negative real parts, it is called Metzler if all its elements outside the main diagonal are nonnegative. Any solution of the linear system

$$\begin{aligned} \dot{x} &= Ax + B\omega(t),\ \omega : \mathbb{R}_+ \to \mathbb{R}^q_+, \\ y &= Cx + D\omega(t), \end{aligned} \tag{3}$$

with $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$ and a Metzler matrix $A \in \mathbb{R}^{n\times n}$, is elementwise nonnegative for all $t \geq 0$ provided that $x(0) \geq 0$ and $B \in \mathbb{R}^{n\times q}_+$ [27], [28]. The output solution $y(t)$ is nonnegative if $C \in \mathbb{R}^{p\times n}_+$ and $D \in \mathbb{R}^{p\times q}_+$. Such dynamical systems are called cooperative (monotone) or nonnegative if only initial conditions in $\mathbb{R}^n_+$ are considered [27], [28].

The L_1 and L_∞ gains for nonnegative systems (3) have been studied in [23], [24], for this kind of systems these gains are interrelated.

Lemma 2. [23], [24] *Let the system (3) be nonnegative (i.e. A is Metzler, $B \geq 0$, $C \geq 0$ and $D \geq 0$), then it is asymptotically stable if and only if there exist $\lambda \in \mathbb{R}_+^n \setminus \{0\}$ and a scalar $\gamma > 0$ such that the following Linear Programming (LP) problem is feasible:*

$$\begin{bmatrix} A^T\lambda + C^T E_p \\ B^T\lambda - \gamma E_q + D^T E_p \end{bmatrix} < 0.$$

Moreover, in this case the L_1 gain of the operator $\omega \to y$ is lower than γ.

The conventional results and definitions on L_2/L_∞ stability for linear systems can be found in [25].

III. Main results

Consider an LPV system:

$$\dot{x} = [A_0 + \Delta A(\rho(t))]x + b(t),\ y = Cx + v(t),\ t \geq 0, \tag{4}$$

where $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^p$ is the output available for measurements, $\rho(t) \in \Pi \subset \mathbb{R}^r$ is the vector of scheduling parameters with a known Π, $\rho \in \mathcal{L}_\infty^r$. The values of the scheduling vector ρ are not available for measurements, and only the set of admissible values Π is known. The matrices $A_0 \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{p\times n}$ are known, the matrix function $\Delta A : \Pi \to \mathbb{R}^{n\times n}$ is piecewise continuous and also known for a given value of ρ. The signals $b : \mathbb{R}_+ \to \mathbb{R}^n$ and $v : \mathbb{R}_+ \to \mathbb{R}^p$ are the external input and measurement noise respectively.

The following assumptions will be used in this work.

Assumption 1. *$x \in \mathcal{L}_\infty^n$; $\underline{b}(t) \leq b(t) \leq \overline{b}(t)$ and $|v(t)| \leq V$ for all $t \geq 0$ and some known $\underline{b}, \overline{b} \in \mathcal{L}_\infty^n$ and $V > 0$.*

Assumption 2. *$\underline{\Delta A} \leq \Delta A(\rho) \leq \overline{\Delta A}$ for all $\rho \in \Pi$ and some known $\underline{\Delta A}, \overline{\Delta A} \in \mathbb{R}^{n\times n}$.*

Assumption 1 means that the state $x(t)$ of the system (4) is bounded, the measurement noise $v(t)$ has an upper bound V and the input $b(t)$ belongs to a known bounded interval $[\underline{b}(t), \overline{b}(t)]$ for all $t \in \mathbb{R}_+$. It is also assumed that the matrix $\Delta A(\rho)$ belongs to the interval $[\underline{\Delta A}, \overline{\Delta A}]$ for all $t \geq 0$, which is easy to compute for a given set Π (in a polytopic case, for example).

A. Problem statement

The objective of this work is to design an interval observer for the system (4).

Note that, in a related work [29], an interval observer for such a class of systems has been proposed in the context of systems stabilization (see also the first work on application of interval observers for stabilization of LPV systems [30], another version is given in [31]), where the stability of the interval observer was ensured by a proper choice of control input dependent on the observer state. In other words, in [29] stability of the interval estimation error dynamics is not established in the observer design step. Therefore, a direct and effective observer design is still awaited. In the present work, only the interval observer is synthesized and some special conditions on the observer gain are imposed in order to guarantee stability in contrast to [29]. Another interval observer for such a LPV system has been formulated in [21], [22], [14], however the conditions of cooperativity of the estimation error dynamics and its stability are restrictive in those works. For the case of measured vector $\rho(t)$ in (4), an interval observer was proposed in [19] using a static transformation of coordinates and the result of Lemma 4 (see below).

Before introduction of interval observer equations note that for a matrix $L \in \mathbb{R}^{n\times p}$ the system (4) can be rewritten as follows:

$$\dot{x} = [A_0 - LC]x + \Delta A(\rho(t))x + L[y - v(t)] + b(t),$$

and according to Lemma 1 and Assumption 2 we have for all $\rho \in \Pi$:

$$\begin{aligned} \underline{\Delta A}^+\underline{x}^+ - \overline{\Delta A}^+\underline{x}^- - \underline{\Delta A}^-\overline{x}^+ + \overline{\Delta A}^-\overline{x}^- &\leq \Delta A(\rho)x \\ \leq \overline{\Delta A}^+\overline{x}^+ - \underline{\Delta A}^+\overline{x}^- - \overline{\Delta A}^-\underline{x}^+ + \underline{\Delta A}^-\underline{x}^- & \end{aligned} \tag{5}$$

provided that $\underline{x} \leq x \leq \overline{x}$ for some $\underline{x}, \overline{x} \in \mathbb{R}^n$.

B. Nonnegative LPV systems

Let us start the analysis with a simplified (but widely met in applications) case of nonnegative system (4).

Assumption 3. *$x(t) \in \mathbb{R}^n_+$ and $\underline{b}(t) \in \mathbb{R}^n_+$ for all $t \geq 0$; $\underline{\Delta A} = 0$.*

Under assumptions 1 and 3 we also have that $b(t), \overline{b}(t) \in \mathbb{R}^n_+$ for all $t \geq 0$. Note that the condition $b(t) \in \mathbb{R}^n_+$ is required for the system (4) with $\Delta A(\rho(t)) \equiv 0$ to be nonnegative [27]. The last condition $\underline{\Delta A} = 0$ simply means that A_0 is the minimal value of $A_0 + \Delta A(\rho)$ for $\rho \in \Pi$ and $\overline{\Delta A} \geq 0$, this condition can be always satisfied under Assumption 2 and a suitable shift of A_0, $\underline{\Delta A}$ and $\overline{\Delta A}$ (for nonnegative systems such a restriction simplifies the notation). Implicitly this assumption may imply that the matrix $A_0 + \Delta A(\rho)$ is Metzler for all $\rho \in \Pi$ (for a nonnegative system the state matrix (or nonlinearity) has to satisfy a monotonicity condition [28]).

Denote by $\underline{x}(t)$ and $\overline{x}(t)$ the lower and upper bound estimates of the state $x(t)$ respectively. Let us introduce two observer gain matrices $\underline{L}, \overline{L} \in \mathbb{R}^{n \times p}$, whose values will be specified later, then an interval observer for the nonnegative system (4) is given by:

$$\begin{aligned} \dot{\underline{x}} &= [A_0 - \underline{L}C]\underline{x} \\ &\quad + \max\{0, \underline{L}y - |\underline{L}|VE_p\} + \underline{b}(t), \\ \dot{\overline{x}} &= [A_0 - \overline{L}C + \overline{\Delta A}]\overline{x} + \overline{L}y + |\overline{L}|VE_p + \overline{b}(t). \end{aligned} \tag{6}$$

Note that it is a *linear* system and a similar structure of interval observer has been proposed in [14], [22], where there is no $\max\{\cdot\}$ function in the first equation, whose introduction improves the estimation accuracy. Indeed, under conditions $\underline{L}C \geq 0$ and $x(t) \geq 0$, the item $\underline{L}[y(t) - v(t)] = \underline{L}Cx(t)$ is nonnegative, but its accessible lower bound $\underline{L}y(t) - |\underline{L}|VE_p$ can take negative values in general, then introduction of the function $\max\{\cdot\}$ improves the tightness of the bound.

The conditions ensuring stability and interval estimation through (6) are stated below.

Theorem 1. *Let assumptions 1–3 be satisfied, the matrices $A_0 - \underline{L}C$, $A_0 - \overline{L}C$ be Metzler and the following constraints be verified for some $\lambda_1, \lambda_2 \in \mathbb{R}^n_+ \setminus \{0\}$:*

$$\begin{gathered} [A_0 - \underline{L}C]^T \lambda_1 + Z^T E_s < 0, \\ [A_0 - \overline{L}C + \overline{\Delta A}]^T \lambda_2 + Z^T E_s < 0, \\ \lambda_i - \gamma E_n < 0,\ i = 1, 2, \\ \underline{L}C \geq 0,\ \overline{L}C \geq 0 \end{gathered}$$

for a scalar $\gamma > 0$ and $Z \in \mathbb{R}^{s \times n}_+$, $0 < s \leq n$. Let $\underline{x}(0) \leq x(0) \leq \overline{x}(0)$, then the solutions of (4), (6) satisfy

$$0 \leq \underline{x}(t) \leq x(t) \leq \overline{x}(t) \quad \forall t \geq 0 \tag{7}$$

and $\underline{x}, \overline{x} \in \mathcal{L}^n_\infty$. In addition, L_1 gain of the operators $\underline{b} \to Z\underline{x}$ and $\overline{b} \to Z\overline{x}$ is less than γ.

Proof. According to the introduced conditions $\underline{b}(t) \in \mathbb{R}^n_+$ and $A_0 - \underline{L}C$ is Metzler, therefore the $\underline{x}$ subsystem in (6) is cooperative (the signal $\max\{0, \underline{L}y - |\underline{L}|VE_p\} + \underline{b}(t)$ is nonnegative for all positive times), then $\underline{x}(t) \in \mathbb{R}^n_+$ for all $t \geq 0$. Consider dynamics of interval estimation errors $\underline{e} = x - \underline{x}$ and $\overline{e} = \overline{x} - x$:

$$\begin{aligned} \dot{\underline{e}} &= [A_0 - \underline{L}C]\underline{e} + \sum_{i=1}^{3} \underline{d}_i(t), \\ \dot{\overline{e}} &= [A_0 - \overline{L}C]\overline{e} + \sum_{i=1}^{3} \overline{d}_i(t), \end{aligned}$$

where

$$\begin{aligned}\underline{d}_1(t) &= \Delta A(\rho(t))x,\\ \underline{d}_2(t) &= \underline{L}[y-v(t)]-\max\{0,\underline{L}y-|\underline{L}|VE_p\},\\ \underline{d}_3(t) &= b(t)-\underline{b}(t);\end{aligned}$$

$$\begin{aligned}\overline{d}_1(t) &= \overline{\Delta A}\overline{x}-\Delta A(\rho(t))x,\\ \overline{d}_2(t) &= \overline{L}v(t)+|\overline{L}|VE_p,\\ \overline{d}_3(t) &= \overline{b}(t)-b(t).\end{aligned}$$

The inputs $\underline{d}_3(t)$, $\overline{d}_2(t)$, $\overline{d}_3(t)$ are nonnegative for all $t\geq 0$ due to Assumptions 1. The fact that $\underline{d}_2(t)\geq 0$ can be proven in two steps. First, if $\underline{L}y(t)\leq|\underline{L}|VE_p$, then $\underline{L}[y(t)-v(t)]=\underline{L}Cx(t)\geq 0$ due to $\underline{L}C\geq 0$ and $x(t)\in\mathbb{R}_+^n$. Second, if in contrast $\underline{L}y(t)>|\underline{L}|VE_p$, then $\underline{d}_2(t)=|\underline{L}|VE_p-\underline{L}v(t)\geq 0$ by Assumption 1. Finally, to show that $\underline{d}_1(t)$ and $\overline{d}_1(t)$ remain nonnegative while the relation (7) is satisfied let us recall that by assumptions 2 and 3

$$0\leq\Delta A(\rho(t))\leq\overline{\Delta A}\quad\forall\rho\in\Pi,$$

then $\Delta A(\rho(t))x\geq 0$ and $[\overline{\Delta A}-\Delta A(\rho(t))]x\geq 0$ for $x\in\mathbb{R}_+^n$. Note that if $0\leq x\leq\overline{x}$, then $\overline{\Delta A}[\overline{x}-x]\geq 0$ that implies

$$\overline{\Delta A}\overline{x}\geq\overline{\Delta A}x\geq\Delta A(\rho(t))x\geq 0$$

and $\underline{d}_1(t)$ and $\overline{d}_1(t)$ are nonnegative whereas the relation (7) is satisfied. However, the relation (7) is valid at time $t=0$ ($\underline{e}(0),\overline{e}(0)\in\mathbb{R}_+$) and using induction arguments it is preserved for all $t\geq 0$ by cooperativity of dynamics of the estimation errors $\underline{e}$, $\overline{e}$ ($A_0-\underline{L}C$ and $A_0-\overline{L}C$ are Metzler matrices and all $\underline{d}_i(t)\geq 0$, $\overline{d}_i(t)\geq 0$ for $i=\overline{1,3}$), therefore (7) is satisfied.

Let us prove boundedness of the variables $\overline{x}(t)$ and $\underline{x}(t)$ for all $t\geq 0$. Note that the $\underline{x}$- and $\overline{x}$- subsystems in (6) are uncoupled and the inputs in these systems $\max\{0,\underline{L}y-|\underline{L}|VE_p\}+\underline{b}(t)$ and $\overline{L}y+|\overline{L}|VE_{pn}+\overline{b}(t)$ respectively are bounded by applying Assumption 1. Therefore, $\underline{x},\overline{x}\in\mathcal{L}_\infty^n$ if the matrices $A_0-\underline{L}C$ and $A_0-\overline{L}C+\overline{\Delta A}$ are Hurwitz. According to Lemma 2, asymptotic stability of (6) is equivalent to feasibility of constraints given in this theorem. In addition, in this case the interval observer has L_1 gain less than γ from the inputs $\underline{b}$, $\overline{b}$ to auxiliary outputs $Z\underline{x}$, $Z\overline{x}$ respectively. □

The matrix Z and the L_1 gain stability conditions are introduced in order to be able to improve/regulate the accuracy of interval estimation for some part of variables (for example, the matrix Z can select all state coordinates excluding the measured variables). Recall that for a nonnegative system (6), existence of an L_1 gain implies existence of an L_∞ gain [23], [24].

Note that the conditions of Theorem 1 imposed on the gains $\underline{L},\overline{L}$ can be formulated in terms of the following linear programming problem for the case $C\geq 0$ (see also [22]): it is required to find $\lambda_1,\lambda_2\in\mathbb{R}^n$ and $w_1,w_2\in\mathbb{R}_+^p$ (λ_i, w_i, $i=1,2$ are the decision variables) such that

$$\begin{gathered}A_0^{\mathrm{T}}\lambda_1-C^{\mathrm{T}}w_1+Z^{\mathrm{T}}E_s<0,\\ [A_0+\overline{\Delta A}]^{\mathrm{T}}\lambda_2-C^{\mathrm{T}}w_2+Z^{\mathrm{T}}E_s<0,\\ \lambda_i-\gamma E_n<0,\ i=1,2,\\ \lambda_i>0,\ w_i\geq 0,\ i=1,2,\end{gathered}$$

then $\underline{L},\overline{L}\in\mathbb{R}_+^{n\times p}$ are solutions of the equations $w_1=\underline{L}^{\mathrm{T}}\lambda_1$, $w_2=\overline{L}^{\mathrm{T}}\lambda_2$. A possible approach is to select $\underline{L}=\lambda_1 w_1^{\mathrm{T}}|\lambda_1|^{-2}$ and $\overline{L}=\lambda_2 w_2^{\mathrm{T}}|\lambda_2|^{-2}$. Next, the inequalities above have to be accompanied with constraints for a diagonal matrix $S\in\mathbb{R}_+^{n\times n}$:

$$\begin{gathered}A_0-\underline{L}C+S\geq 0,\ A_0-\overline{L}C+S\geq 0,\\ \underline{L}=\lambda_1 w_1^{\mathrm{T}}|\lambda_1|^{-2},\ \overline{L}=\lambda_2 w_2^{\mathrm{T}}|\lambda_2|^{-2},\end{gathered}$$

which guarantee the Metzler property of the matrices $A_0-\underline{L}C$, $A_0-\overline{L}C$. The obtained set of inequalities can be rewritten as follows:

$$
\begin{gathered}
A_0^{\mathrm{T}}\lambda_1 - C^{\mathrm{T}}w_1 + Z^{\mathrm{T}}E_s < 0,\\
[A_0 + \overline{\Delta A}]^{\mathrm{T}}\lambda_2 - C^{\mathrm{T}}w_2 + Z^{\mathrm{T}}E_s < 0,\\
|\lambda_i| = \gamma,\ \lambda_i > 0,\ w_i \geq 0,\ i = 1, 2,\\
\underline{L} = \lambda_1 w_1^{\mathrm{T}}\gamma^{-2},\ \overline{L} = \lambda_2 w_2^{\mathrm{T}}\gamma^{-2},\\
A_0 - \underline{L}C + S \geq 0,\ A_0 - \overline{L}C + S \geq 0,\ S \geq 0,
\end{gathered}
$$

which can be resolved with respect to the variables λ_1, λ_2, w_1, w_2 and S using a numerical solver.

Remark 1. This problem solution can also be formulated as a minimization of the parameter γ, in this case the obtained gains $\underline{L}, \overline{L}$ will ensure the best accuracy of interval estimation in L_1 sense. Note that the transfer functions $\underline{b} \to Z\underline{x}$ and $\overline{b} \to Z\overline{x}$ for (6) correspond to the transfer functions $b - \underline{b} \to Z\underline{e}$ and $\overline{b} - b \to Z\overline{e}$ for the estimation errors, thus the gain γ actually determines the interval estimation accuracy for (6). According to the constraints formulated above, the problem of finding optimal gains $\underline{L}, \overline{L}$ is nonlinear. A near optimal solution can be obtained iteratively, i.e. by an appropriate decreasing of the value of γ while the LP problem has a solution $\underline{L}, \overline{L}$.

The result of Theorem 1 can be useful for estimation in a large scale nano- or micro- system, which is described by the Chemical Master equation under the signal or parameter uncertainties [32], [33] (see an example of numerical simulations in Section IV).

C. Transformation of coordinates

The requirement that the matrices $A_0 - \underline{L}C$, $A_0 - \overline{L}C$ have to be Metzler can be relaxed by means of a change of coordinates $z = Tx$ with a nonsingular matrix T such that the matrices $\underline{F} = T(A_0 - \underline{L}C)T^{-1}$, $\overline{F} = T(A_0 - \overline{L}C)T^{-1}$ are Metzler. Let us recall two results dealing with transformation of a linear system to a cooperative form.

Lemma 3. [15] *Given the matrices $A \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{p\times n}$. If there is a matrix $L \in \mathbb{R}^{n\times p}$ such that the matrices $A - LC$ and Y have the same eigenvalues, then there is a matrix $S \in \mathbb{R}^{n\times n}$ such that $Y = S(A - LC)S^{-1}$ provided that the pairs $(A - LC, \chi_1)$ and (Y, χ_2) are observable for some $\chi_1 \in \mathbb{R}^{1\times n}$, $\chi_2 \in \mathbb{R}^{1\times n}$.*

This result was used in [15] to design interval observers for LTI systems with a Metzler matrix Y (the main difficulty is to prove the existence of a *real* matrix S, and to provide a constructive approach of its calculation).

Lemma 4. [19] *Let $D \in \Xi \subset \mathbb{R}^{n\times n}$ be a matrix variable satisfying the interval constraints $\Xi = \{D \in \mathbb{R}^{n\times n} : D_a - \Delta \leq D \leq D_a + \Delta\}$ for some $D_a^T = D_a \in \mathbb{R}^{n\times n}$ and $\Delta \in \mathbb{R}_+^{n\times n}$. If for some constant $\mu \in \mathbb{R}_+$ and a diagonal matrix $\Upsilon \in \mathbb{R}^{n\times n}$ the Metzler matrix $Y = \mu E_{n\times n} - \Upsilon$ has the same eigenvalues as the matrix D_a, then there is an orthogonal matrix $S \in \mathbb{R}^{n\times n}$ such that the matrices $S^T DS$ are Metzler for all $D \in \Xi$ provided that $\mu > n||\Delta||_{max}$.*

This result was used in [19] to design interval observers for linear time-varying systems (for the case of a measured vector of scheduling parameters ρ).

Therefore, the matrix T can be found using the results of Lemma 3 (looking for $\underline{L} = \overline{L} = L$) or Lemma 4. Note that if we would like to preserve non-negativity of the new state vector z, then it is required to find a nonnegative matrix T. In this case for a matrix $L \in \mathbb{R}^{n\times p}$ and $T \geq 0$ the system (4) in the coordinates z can be rewritten as follows:

$$\dot{z} = T[A_0 - LC]T^{-1}z + \Delta F(\rho(t))x + TL[y - v(t)] + \beta(t),$$

where $\beta(t) = Tb(t)$ and $\Delta F(\rho(t)) = T\Delta A(\rho(t))$ with $\underline{\beta}(t) = T\underline{b}(t)$, $\overline{\beta}(t) = T\overline{b}(t)$ by Lemma 1, $0 \leq \Delta F(\rho) \leq \overline{\Delta F} = T\overline{\Delta A}$ for all $\rho \in \Pi$. The interval observer (6) saves its structure in the new coordinates:

$$
\begin{aligned}
\dot{\underline{z}} &= \underline{F}\underline{z} + T\max\{0, \underline{L}y - |\underline{L}|VE_p\} + \underline{\beta}(t),\\
\dot{\overline{z}} &= [\overline{F} + \overline{\Delta F}]\overline{z} + T(\overline{L}y + |\overline{L}|VE_p) + \overline{\beta}(t).
\end{aligned}
$$

The stability conditions and the proof for this interval observer follows Theorem 1. Unfortunately it may be hard to find a nonnegative matrix T [34], in such a case the variable z is not nonnegative and the results presented in the next subsection can be applied.

D. Generic LPV systems

For the case of a non positive LPV system (4), the following interval observer structure is proposed:

$$
\begin{aligned}
\dot{\underline{x}} &= [A_0 - \underline{L}C]\underline{x} + [\underline{\Delta A}^+\underline{x}^+ - \overline{\Delta A}^+\underline{x}^- \\
&\quad - \underline{\Delta A}^-\overline{x}^+ + \overline{\Delta A}^-\overline{x}^-] + \underline{L}y - |\underline{L}|VE_p + \underline{b}(t), \\
\dot{\overline{x}} &= [A_0 - \overline{L}C]\overline{x} + [\overline{\Delta A}^+\overline{x}^+ - \underline{\Delta A}^+\overline{x}^- \\
&\quad - \overline{\Delta A}^-\underline{x}^+ + \underline{\Delta A}^-\underline{x}^-] + \overline{L}y + |\overline{L}|VE_p + \overline{b}(t).
\end{aligned} \tag{8}
$$

Note that due to the presence of $\underline{x}^+, \underline{x}^-, \overline{x}^+$ and $\overline{x}^-$, the interval observer (8) is a globally Lipschitz *nonlinear* system. In addition, in (8) the dynamics of $\underline{x}$ and $\overline{x}$ is coupled. A similar observer, in a less general form has been presented in [21], but the stability conditions developed below differ from [21].

Theorem 2. *Let assumptions 1, 2 be satisfied and the matrices $A_0 - \underline{L}C$, $A_0 - \overline{L}C$ be Metzler. Then the relations (7) are satisfied provided that $\underline{x}(0) \le x(0) \le \overline{x}(0)$. If there exist $P \in \mathbb{R}^{2n\times 2n}$, $P = P^T \succ 0$ and $\gamma > 0$ such that the following Riccati matrix inequality is verified*

$$G^TP + PG + 2\gamma^{-2}P^2 + \gamma^2\eta^2 I_{2n} + Z^TZ \prec 0,$$

where $\eta = 2n||\overline{\Delta A} - \underline{\Delta A}||_{max}$, $Z \in \mathbb{R}^{s\times 2n}$, $0 < s \le 2n$ and

$$G = \begin{bmatrix} A_0 - \underline{L}C + \underline{\Delta A}^+ & -\underline{\Delta A}^- \\ -\overline{\Delta A}^- & A_0 - \overline{L}C + \overline{\Delta A}^+ \end{bmatrix},$$

then $\underline{x}, \overline{x} \in \mathcal{L}_\infty^n$. In addition, the operator $\begin{bmatrix} \underline{b} \\ \overline{b} \end{bmatrix} \to Z \begin{bmatrix} \underline{x} \\ \overline{x} \end{bmatrix}$ in (8) has an L_2 gain less than γ.

Proof. Consider dynamics of the interval estimation errors $\underline{e} = x - \underline{x}$ and $\overline{e} = \overline{x} - x$:

$$
\begin{aligned}
\dot{\underline{e}} &= [A_0 - \underline{L}C]\underline{e} + \sum_{i=1}^{3}\underline{d}_i(t), \\
\dot{\overline{e}} &= [A_0 - \overline{L}C]\overline{e} + \sum_{i=1}^{3}\overline{d}_i(t),
\end{aligned}
$$

where $\underline{d}_3(t)$, $\overline{d}_2(t)$, $\overline{d}_3(t)$ are the same as they have been defined in the proof of Theorem 1 and

$$
\begin{aligned}
\underline{d}_1(t) &= \Delta A(\rho(t))x - \underline{\Delta A}^+\underline{x}^+ \\
&\quad - \overline{\Delta A}^+\underline{x}^- - \underline{\Delta A}^-\overline{x}^+ + \overline{\Delta A}^-\overline{x}^-, \\
\underline{d}_2(t) &= |\underline{L}|VE_p - \underline{L}v(t), \\
\overline{d}_1(t) &= \overline{\Delta A}^+\overline{x}^+ - \underline{\Delta A}^+\overline{x}^- \\
&\quad - \overline{\Delta A}^-\underline{x}^+ + \underline{\Delta A}^-\underline{x}^- - \Delta A(\rho(t))x.
\end{aligned}
$$

The proof that the relation (7) is satisfied for the observer (8) under the introduced restrictions is similar to the proof of Theorem 1: the matrices $A_0 - \underline{L}C$, $A_0 - \overline{L}C$ are Metzler and the inputs $\underline{d}_i(t), \overline{d}_i(t)$ are nonnegative while (7) is satisfied. By construction $\underline{x}(0) \le x(0) \le \overline{x}(0)$, then (7) holds.

Let us show that the variables $\overline{x}(t)$ and $\underline{x}(t)$ stay bounded for all $t \ge 0$ in (4), (8). For this purpose let us rewrite the equations (8) as follows:

$$\begin{aligned}\dot{\underline{x}} &= [A_0 - \underline{L}C + \underline{\Delta A}^+]\underline{x} - \underline{\Delta A}^-\overline{x} + \underline{f}(\underline{x},\overline{x})\\ &\quad +\underline{L}y - |\underline{L}|VE_p + \underline{b}(t),\\ \dot{\overline{x}} &= [A_0 - \overline{L}C + \overline{\Delta A}^+]\overline{x} - \overline{\Delta A}^-\underline{x} + \overline{f}(\underline{x},\overline{x})\\ &\quad +\overline{L}y + |\overline{L}|VE_p + \overline{b}(t),\end{aligned}$$

where

$$\underline{f}(\underline{x},\overline{x}) = \Delta^-\overline{x}^- - \Delta^+\underline{x}^-,\ \overline{f}(\underline{x},\overline{x}) = \Delta^+\overline{x}^- - \Delta^-\underline{x}^-,$$
$$\Delta^+ = \overline{\Delta A}^+ - \underline{\Delta A}^+,\ \Delta^- = \overline{\Delta A}^- - \underline{\Delta A}^-.$$

It is not a cooperative system and the variables $\underline{x},\overline{x}$ are interrelated, but the inputs $\underline{L}y-|\underline{L}|VE_p+\underline{b}(t)$ and $\overline{L}y+|\overline{L}|VE_p+\overline{b}(t)$ in these subsystems are bounded following Assumption 1, and boundedness of $\underline{x},\overline{x}$ is predefined by the linear part and the functions $\underline{f}$, $\overline{f}$. To prove boundedness of solutions of the observer (8), introduce the system

$$\dot{\xi} = G\xi + \phi(\xi) + \delta,$$

where

$$\xi = \begin{bmatrix} \underline{x} \\ \overline{x} \end{bmatrix},\ \phi(\xi) = \begin{bmatrix} \underline{f}(\underline{x},\overline{x}) \\ \overline{f}(\underline{x},\overline{x}) \end{bmatrix},$$

and $\delta \in \mathcal{L}_\infty^{2n}$ is an auxiliary input representing influence of y, v and $\underline{b},\overline{b}$. Since $|\phi(\xi)| \le \eta|\xi|$ the function ϕ is globally Lipschitz. Indeed,

$$\phi(\xi) = \mathcal{A}\begin{bmatrix} \overline{x}^- \\ \underline{x}^- \end{bmatrix},\ \mathcal{A} = \begin{bmatrix} \Delta^- & -\Delta^+ \\ \Delta^+ & -\Delta^- \end{bmatrix},$$

then $|\phi(\xi)| \le ||\mathcal{A}||_2|\xi|$. By definition $||\mathcal{A}||_2 \le 2n||\mathcal{A}||_{max}$ and $||\mathcal{A}||_{max} \le ||\overline{\Delta A} - \underline{\Delta A}||_{max}$. Let us consider a Lyapunov function $V = \xi^{\mathrm{T}}P\xi$, whose time derivative takes the form:

$$\begin{aligned}\dot{V} &= \xi^{\mathrm{T}}[G^{\mathrm{T}}P + PG]\xi + 2\xi^{\mathrm{T}}P[\phi(\xi)+\delta]\\ &\le \xi^{\mathrm{T}}[G^{\mathrm{T}}P + PG]\xi + 2\gamma^{-2}\xi^{\mathrm{T}}P^2\xi\\ &\quad +\gamma^2\phi(\xi)^{\mathrm{T}}\phi(\xi) + \gamma^2\delta^{\mathrm{T}}\delta\\ &\le \xi^{\mathrm{T}}[G^{\mathrm{T}}P + PG + 2\gamma^{-2}P^2 + \gamma^2\eta^2 I_{2n}\\ &\quad +Z^{\mathrm{T}}Z]\xi - \xi^{\mathrm{T}}Z^{\mathrm{T}}Z\xi + \gamma^2\delta^{\mathrm{T}}\delta\\ &\le -\xi^{\mathrm{T}}Q\xi - \xi^{\mathrm{T}}Z^{\mathrm{T}}Z\xi + \gamma^2\delta^{\mathrm{T}}\delta,\end{aligned}$$

where existence of a matrix $Q \in \mathbb{R}^{2n\times 2n}$, $Q = Q^{\mathrm{T}} \succ 0$ follows from Riccati matrix inequality introduced in the theorem's conditions. Then $\overline{x}(t)$ and $\underline{x}(t)$ stay bounded for all $t \ge 0$, and the operator $\delta \to Z\xi$ has L_2 gain less than γ [25]. □

As in subsection III-C for the interval observer (6), a transformation of coordinates T can also be used for (8) in order to relax the requirement of Theorem 2 that the matrices $A_0 - \underline{L}C$, $A_0 - \overline{L}C$ should be Metzler.

The Riccati matrix inequality from Theorem 2 can be reformulated in terms of LMIs with respect to $\underline{L}$, $\overline{L}$ and P. Indeed, $G = D - \Lambda\Upsilon$, where

$$D = \begin{bmatrix} A_0 + \underline{\Delta A}^+ & -\underline{\Delta A}^- \\ -\overline{\Delta A}^- & A_0 + \overline{\Delta A}^+ \end{bmatrix},\ \Lambda = \begin{bmatrix} \underline{L} & 0 \\ 0 & \overline{L} \end{bmatrix},$$
$$\Upsilon = \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix}.$$

Then the Riccati inequality can be rewritten as follows:

$$D^{\mathrm{T}}P+PD-\Upsilon^{\mathrm{T}}W^{\mathrm{T}}-W\Upsilon+2\gamma^{-2}P^{2}+\gamma^{2}\eta^{2}I_{2n}+Z^{\mathrm{T}}Z\prec 0,$$

where $W=P\Lambda$ is a new variable (W also has to be declared block-diagonal as Λ). Using the Schur complement we obtain an equivalent LMI:

$$\left[\begin{array}{cc} 0.5\gamma^{2}I_{2n} & P\\ P & \Upsilon^{\mathrm{T}}W^{\mathrm{T}}+W\Upsilon-D^{\mathrm{T}}P\\ & -PD-\gamma^{2}\eta^{2}I_{2n}-Z^{\mathrm{T}}Z \end{array}\right]\succ 0,$$

which has to be verified with a linear constraint (verification of Metzler property)

$$P\left[\begin{array}{cc} A_{0} & 0\\ 0 & A_{0} \end{array}\right]-W\Upsilon+PS\geq 0$$

for a sufficiently big diagonal matrix $S\in\mathbb{R}_{+}^{2n\times 2n}$ and a diagonal P (an *additional* restriction, the conditions given in the formulation of theorem are less conservative), then $\left[\begin{array}{cc} \underline{L} & 0\\ 0 & \overline{L} \end{array}\right]=P^{-1}W$. These linear inequalities can be solved using LMI toolboxes, as it is done in examples below.

Remark 2. It is important to stress that the above LMIs admit the L_2 optimization problem solution since the parameter γ^2 enters linearly, thus the optimal observer gains $\underline{L}$, $\overline{L}$ can be calculated.

IV. Examples

In this section we consider two numerical examples to show validity of conditions of theorems 1 and 2.

A. An academic LPV system

Consider a nonlinear system:

$$\begin{aligned}\dot{x} &= \left[\begin{array}{ccc} \epsilon\cos t & 1+\epsilon\sin x_{3} & \varepsilon\sin x_{2}\\ \varepsilon\sin x_{3} & -0.5+\epsilon\sin t & 1+\epsilon\cos 2t\\ \varepsilon\sin x_{2} & 0.3+\varepsilon\cos 2t & -1+\epsilon\sin t \end{array}\right]x\\ &+\left[\begin{array}{c} 6\cos x_{1}\\ \sin t+0.1\sin x_{3}\\ -\cos 3t+0.1\sin 2x_{2} \end{array}\right],\ y=x_{1}+v(t),\end{aligned}$$

where $\epsilon=0.01$ and $\varepsilon=0.001$. We assume that $V=0.1$, and for simulation we selected $v(t)=V(\sin 5t+\cos 3t)/2$. For initial conditions $|x_i(0)|\leq 5$ the system has bounded solutions. This system can be presented in the form of (4) for

$$A_{0}=\left[\begin{array}{ccc} 0 & 1 & 0\\ 0 & -0.5 & 1\\ 0 & 0.3 & -1 \end{array}\right],\ \overline{\Delta A}=\left[\begin{array}{ccc} \epsilon & \epsilon & \varepsilon\\ \varepsilon & \epsilon & \epsilon\\ \varepsilon & \varepsilon & \epsilon \end{array}\right]=-\underline{\Delta A},$$

$$\underline{b}(t,y)=\left[\begin{array}{c} 6\underline{f}(y)\\ \sin t-0.1\\ -\cos 3t-0.1 \end{array}\right],\ \overline{b}(t,y)=\left[\begin{array}{c} 6\overline{f}(y)\\ \sin t+0.1\\ -\cos 3t+0.1 \end{array}\right],$$

$$\underline{f}(y)=\begin{cases} \cos y\cos V & \text{if }\cos y\geq 0\\ \cos y & \text{if }\cos y<0 \end{cases}-|\sin y|\sin V,$$

$$\overline{f}(y)=\begin{cases} \cos y & \text{if }\cos y\geq 0\\ \cos y\cos V & \text{if }\cos y<0 \end{cases}+|\sin y|\sin V$$

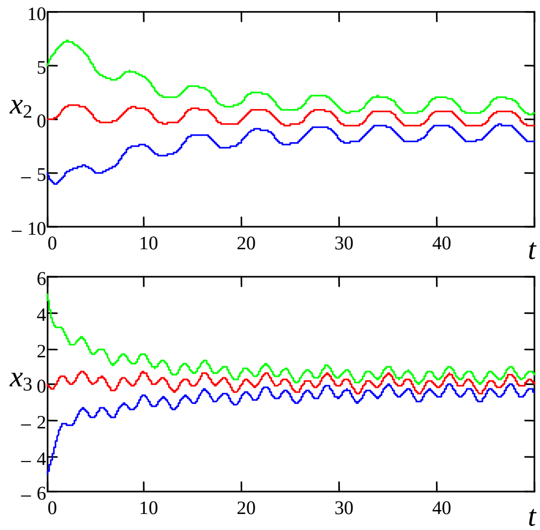

Figure 1. The results of simulations for an LPV system

and a properly selected ρ, clearly assumptions 1 and 2 are satisfied. The optimization of LMIs formulated after Theorem 2 gives the L_2 optimal solution

$$\underline{L} = \begin{bmatrix} 82.923 \\ -3e-4 \\ -4e-4 \end{bmatrix}, \ \overline{L} = \begin{bmatrix} 97.16 \\ -2e-5 \\ -1e-5 \end{bmatrix}$$

for $\gamma = 31.4$ (YALMIP toolbox [35] of MATLAB has been used), where the matrix Z is selecting the variables x_2 and x_3. The results of interval simulations for the variables x_2 and x_3 are given in Fig. 1 (the variable x_1 is omitted since it is available for measurements).

B. Nonnegative droplet-based microfluidic system

Following [32], [33] consider a model of droplet-based microfluidic system derived from the Chemical Master equation:

$$\begin{aligned} \dot{P}_0 &= -\kappa(t)P_0, \ P_0(0) = 1, \\ \dot{P}_i &= \kappa(t)[P_{i-1} - P_i], \ P_i(0) = 0, \ i = \overline{1, N}, \end{aligned}$$

where P_i, $i = \overline{0, N}$ is the probability that a droplet contains i crystals, and $\kappa(t)dt$ is the probability that a critical nucleus will form during an infinitesimal time interval dt. This model evaluates the crystal growth process in time. According to [32] $\kappa(t) = J(S(t))V(t)$, where $S(t)$ is the supersaturation and $V(t)$ is the droplet volume, both of them are assumed to be available from (noisy) measurements, but the function J is not exactly known. Therefore, we will assume that for the function $\kappa(t)$ only a lower $\underline{\kappa}(t)$ and an upper $\overline{\kappa}(t)$ bounds are available. In addition, for simplicity of presentation we assume that the functions $\underline{\kappa}(t)$, $\overline{\kappa}(t)$ are piecewise constant, i.e. there exist intervals $[t_j, t_{j+1})$, $j = 0, K$ such that $\underline{\kappa}(t) = \underline{\kappa}_j$, $\overline{\kappa}(t) = \overline{\kappa}_j$ for all $t \in [t_j, t_{j+1})$, $t_0 = 0$. For this system there is no measurement of the state ($C = 0$).

Thus it is a time-varying autonomous linear system, but since the exact value of $\kappa(t)$ is not known, then the LPV framework has to be used. One of the main difficulties with this system is that the number of subsystems N (the possible of number of crystals in a droplet) can be sufficiently large. And the only way to predict/evaluate a possible state of the crystal growth process in a droplet is based on estimation for Chemical Master equation.

The observer (6) on each interval $[t_j, t_{j+1})$, $j = \overline{0, K}$ can be rewritten as follows:

$$\begin{aligned} \dot{\underline{x}} &= A_0\underline{x}, \\ \dot{\overline{x}} &= [A_0 + \overline{\Delta A}]\overline{x}, \end{aligned}$$

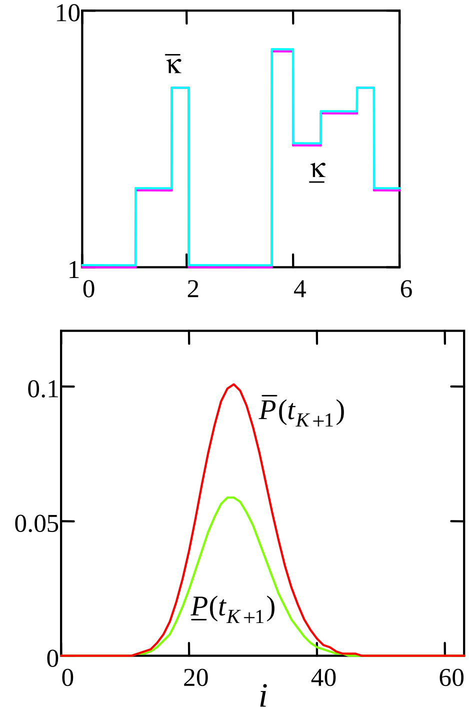

Figure 2. The results of simulations for a nonnegative microfluidic system

where

$$A_0 = \begin{bmatrix} -\overline{\kappa}_j & 0 & \dots & 0 & 0 \\ \underline{\kappa}_j & -\overline{\kappa}_j & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \underline{\kappa}_j & -\overline{\kappa}_j \end{bmatrix},$$

$$A_0 + \overline{\Delta A} = \begin{bmatrix} -\underline{\kappa}_j & 0 & \dots & 0 & 0 \\ \overline{\kappa}_j & -\underline{\kappa}_j & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \overline{\kappa}_j & -\underline{\kappa}_j \end{bmatrix}$$

and all conditions of Theorem 1 are satisfied for $\underline{L} = \overline{L} = 0$ (no measurements). Thus, we can iteratively apply the obtained interval observer (6) on each interval $[t_j, t_{j+1}), j = 0, K$ in order to reconstruct the distribution $P_i(t_{K+1})$, $i = \overline{0, N}$ at the end of the process of crystallization, starting from a fixed initial distribution ($P_0(0) = 1$ and $P_i(0) = 0$ for $i = \overline{1, N}$).

For $\underline{\kappa}(t)$, $\overline{\kappa}(t)$ given in the top of Fig. 2 and $N = 64$ the results of interval estimation of $P_i(t_{K+1})$, $i = \overline{0, N}$ are shown in Fig 2. As we can conclude from these results, even a small difference in $\underline{\kappa}(t)$, $\overline{\kappa}(t)$ may lead on a short time interval ($t_{K+1} = 6$) to a big deviations of $\underline{P}_i(t_{K+1})$ and $\overline{P}_i(t_{K+1})$, $i = \overline{0, N}$.

V. Conclusion

The problem of state estimation for continuous-time LPV systems with unmeasurable vector of scheduling parameters is studied. Two classes of models are considered: general LPV systems and nonnegative ones. The cooperativity and stability of proposed interval observers are expressed in terms of matrix inequalities, which are nonlinear in a common case. However, under some additional mild restrictions these inequalities can be represented as LP or LMIs. The problem of optimal observer gains computation in the L_1/L_2 sense is analyzed. Efficiency of the proposed observers is demonstrated on numerical simulations. Reduction of the conservatism of the proposed LMIs and interval observers is a direction of future investigations.

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