# Quantification in Frame Semantics with Hybrid Logic

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**Abstract.** This paper aims at integrating logical operators into frame-based semantics. Frames are semantic graphs that allow to capture lexical meaning in a fine-grained way but that do not come with a natural way to integrate logical operators such as quantifiers. The approach we propose starts from the observation that modal logic is a powerful tool for describing relational structures, hence frames. We use its hybrid logic extension in order to incorporate quantification and thereby allow for inference and reasoning. We develop a type theoretic compositional semantics using this approach, formulated within Abstract Categorial Grammar.

## 1 Frames and Lexical Semantics

Frames emerged as a representation format of conceptual and lexical knowledge [10,4,15]. They are commonly presented as semantic graphs with labelled nodes and edges, such as the one in Fig. 1, where nodes correspond to entities (individuals, events, ...) and edges correspond to (functional or non-functional) relations between these entities. In Fig. 1 all relations except *part-of* are meant to be functional. Frames can be formalized as extended typed feature structures [18,12,14], but a reformulation in first order logic is also straightforward [12]. This conception of frames is therefore not to be confused with the somewhat simpler FrameNet frames (see [17]).

Recent work has addressed the composition of lexical frames on the sentential level by means of an explicit syntax-semantics interface [12]. However, the integration of logical operators remains a desideratum. While [13] presents an experiment with a seamless intergration of "quantifier frames", [16] suggests to keep frames and logical operators separate. We follow the latter general approach in this paper.

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# 2 Hybrid Logic and Semantic Frames

## 2.1 Hybrid Logic

With the notations of [2], $\mathsf{Rel}$ is a set of relational symbols, $\mathsf{Prop}$ a set of propositional variables, $\mathsf{Nom}$ a set of nominals, and $\mathsf{Svar}$ a set of state variables ($\mathsf{Stat} = \mathsf{Nom} \cup \mathsf{Svar}$). The language of formulas is $\mathsf{Forms} ::= \top \mid p \mid s \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \langle R\rangle\phi \mid \mathbf{\exists}\phi \mid @_s\phi \mid \downarrow x.\phi \mid \exists x.\phi$ where $p \in \mathsf{Prop}$, $s \in \mathsf{Stat}$, $R \in \mathsf{Rel}$ and $\phi, \phi_1, \phi_2 \in \mathsf{Forms}$. A *model* $\mathcal{M}$ is a triple $\langle M, (R^{\mathcal{M}})_{R\in\mathsf{Rel}}, V\rangle$ such that $M$ is a non-empty set, each $R^{\mathcal{M}}$ is a binary relation on $M$, and the valuation $V : \mathsf{Prop} \cup \mathsf{Nom} \longrightarrow \wp(M)$ is such that if $i \in \mathsf{Nom}$ then $V(i)$ is a singleton. An assignment $g$ is a mapping $g : \mathsf{Svar} \longrightarrow M$. For an assignment $g$, $g^x_m$ is an assignment that differs from $g$ at most on $x$ and $g^x_m(x) = m$. For $s \in \mathsf{Stat}$, we also define $[s]^{\mathcal{M},g}$ to be the only $m$ such that $V(s) = \{m\}$ if $s \in \mathsf{Nom}$ and $[s]^{\mathcal{M},g} = g(s)$ if $s \in \mathsf{Svar}$.

Let $\mathcal{M}$ be a model, $w \in M$, and $g$ an assignment for $\mathcal{M}$. The *satisfaction relation* is defined as follows:

| | |
|---|---|
| $\mathcal{M}, g, w \vDash \top$ | |
| $\mathcal{M}, g, w \vDash s$ | iff $w = [s]^{\mathcal{M},g}$ for $s \in \mathsf{Stat}$ |
| $\mathcal{M}, g, w \vDash \neg\phi$ | iff $\mathcal{M}, g, w \nvDash \phi$ |
| $\mathcal{M}, g, w \vDash \phi_1 \wedge \phi_2$ | iff $\mathcal{M}, g, w \vDash \phi_1$ and $\mathcal{M}, g, w \vDash \phi_2$ |
| $\mathcal{M}, g, w \vDash \langle R\rangle\phi$ | iff there is a $w' \in M$ such that $R^{\mathcal{M}}(w, w')$ and $\mathcal{M}, g, w' \vDash \phi$ |
| $\mathcal{M}, g, w \vDash p$ | iff $w \in V(p)$ for $p \in \mathsf{Prop}$ |
| $\mathcal{M}, g, w \vDash @_s\phi$ | iff $\mathcal{M}, g, [s]^{\mathcal{M},g} \vDash \phi$ for $s \in \mathsf{Stat}$ |
| $\mathcal{M}, g, w \vDash \downarrow x.\phi$ | iff $\mathcal{M}, g^x_w, w \vDash \phi$ |
| $\mathcal{M}, g, w \vDash \exists x.\phi$ | iff there is a $w' \in M$ such that $\mathcal{M}, g^x_{w'}, w \vDash \phi$ |
| $\mathcal{M}, g, w \vDash \mathbf{\exists}\phi$ | iff there is a $w' \in M$ such that $\mathcal{M}, g, w' \vDash \phi$ |

We also define $\mathbf{\forall}\phi \equiv \neg\mathbf{\exists}(\neg\phi)$ (i.e., $\mathcal{M}, g, w \vDash \mathbf{\forall}\phi$ iff $\forall w'\ \mathcal{M}, g, w' \vDash \phi$)[3] and $\phi \implies \psi \equiv (\neg\phi) \vee \psi$. A formula $\phi$ is:

- *satisfiable* if there is a model $\mathcal{M}$, and an assignment $g$ on $\mathcal{M}$, and a state $w \in M$ such that $\mathcal{M}, g, w \vDash \phi$
- *globally true* in a model $\mathcal{M}$ under an assignment $g$ if it is satisfiable at all states of the model, i.e., $\mathcal{M}, g, w \vDash \phi$ for all $w \in M$. We write $\mathcal{M}, g \vDash \phi$

## 2.2 Feature Structures

In [12], semantic frames are introduced as *base-labelled feature structure with types and relations*. This definition extends the standard definition of feature structures in two respects: In addition to features, proper relations between nodes can be expressed. Moreover, it is not required that every node is accessible from

[3] According to the satisfaction relation, $\downarrow$ and $\exists$ bind state variables without changing the current evaluation state. [7] shows that they define a distinct hierarchy from the one we get using $\mathbf{\exists}$ (or some other binder $\Sigma$). It also shows that the fragment using operators from the two hierarchies is as expressive as the most expressive fragment.

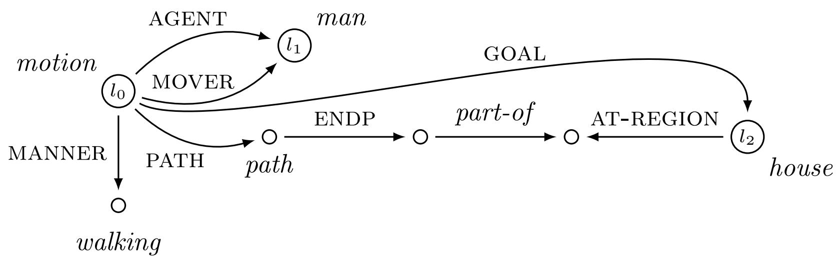



**Fig. 1.** Frame for the meaning of *the man walked to the house* (adapted from [12])

a single root node via a feature path; instead, it is required that every node is accessible from one of the base-labelled nodes. Semantic frames defined in this way can be seen as finite first-order structures which conform to a signature consisting of a set $\mathsf{Label} \cup \mathsf{Type}$ of unary relation symbols and a set $\mathsf{Feat} \cup \mathsf{Rel}$ of binary relation symbols subject to the constraints that the members of Label denote singletons, the members of Feat denote *functional* relations, and that the above accessibility condition holds. In the example frame of Fig. 1, symbols inside nodes ($l_0$, $l_1$, ...) indicate base labels, symbols attached to nodes (*man*, *motion*, ...) belong to Type, members of Feat are marked by small caps (AGENT, ENDP, ...), and *part-of* is the only member of Rel occurring in the frame.

Structures of this kind can easily be turned into Kripke structures by treating the interpretation of the members of $\mathsf{Label} \cup \mathsf{Type}$ by a separate valuation function. Semantic frames, or feature structures, provide thus a natural application domain for modal languages and, in particular, for hybrid extensions because of the need to cope with node labels and feature path re-entrancies [5]. Under the formal set-up of Section 2.1, Type corresponds to Prop, Label corresponds to Nom, and Feat is subsumed under Rel. (The functionality of the members of Feat must be enforced separately.) The semantic frame of Fig. 1 is a model that satisfies the formula (1) at the element named by $l_0$.

(1) $$l_0 \wedge \mathit{motion} \wedge \langle \text{AGENT} \rangle (l_1 \wedge \mathit{man}) \wedge \langle \text{MOVER} \rangle l_1 \wedge \langle \text{GOAL} \rangle (l_2 \wedge \mathit{house}) \wedge \langle \text{MANNER} \rangle \mathit{walking} \wedge (\exists v\ w. \langle \text{PATH} \rangle (\mathit{path} \wedge \langle \text{ENDP} \rangle v) \wedge @_{l_2}(\langle \text{AT-REGION} \rangle w) \wedge @_v(\langle \mathit{part\text{-}of} \rangle w))$$

The logical framework of [12] does not provide means for explicit quantification. As a consequence, the referential entities of the domain of discourse are implicitly treated as definite, which is reflected by the crucial role of nominals in (1).[4] In

[4] Hybrid logic with nominals but without quantification over states also allows [3] to describe semantic dependency graphs. Natural language quantification is encoded using RESTR and BODY relations. However, it is not clear how to compute relations between these representations (e.g., how to check that *John kisses Mary* holds in case *every man kisses Mary* holds).

the following, we will show how this limitation can be overcome by employing hybrid languages.

## 3 Type-Theoretic Semantics with Frames

We now provide the type-theoretic syntax-semantics interface allowing for a compositional building of the meanings. We describe it using the ACG [9] framework. As we are concerned in this article with semantic modeling and quantification rather than with parsing, we use higher-order types for quantified noun-phrases.

The models we are considering are *semantic frames* instead of arbitrary first-order models. So we first present some models in which we consider the sentences (2a–4a). When the model is the frame of Fig. 2(a), we expect (2a) to be true as there is a *kissing* event with AGENT and THEME attributes linking to persons named (represented by the NAME attribute) *John* and *Mary* resp. (3a) is expected to be false, as well as (4a) with the object wide scope reading as there is a person named *Paul* (resp. named *Peter*) who is AGENT of a single *kissing* event whose THEME is a person named *Sue* (resp. *Mary*). On the other hand, the subject wide scope reading of (4a) is expected to be true. (5a) shows how state storing with the ↓ operator correctly interacts with the @ operator used in specifying node sharing (for instance that the state linked with a GOAL relation in the verb semantic recipe has to be specified by the *PP*). This sentence is expected to be true (both readings) in the model given by the frame of Fig. 2(b).

(2) a. *John kisses Mary*

b. $\mathbf{\exists}(\mathit{kissing} \wedge \langle \text{AGENT} \rangle (\mathit{person} \wedge \langle \text{NAME} \rangle \mathit{John}) \wedge$
$\langle \text{THEME} \rangle (\mathit{person} \wedge \langle \text{NAME} \rangle \mathit{Mary}))$

(3) a. *Every man kisses Mary*

b. $\mathbf{\forall}(\downarrow i.\mathit{man} \implies \mathbf{\exists}(\mathit{kissing} \wedge \langle \text{AGENT} \rangle i \wedge$
$\langle \text{THEME} \rangle (\mathit{person} \wedge \langle \text{NAME} \rangle \mathit{Mary})))$

(4) a. *Every man kisses some woman*

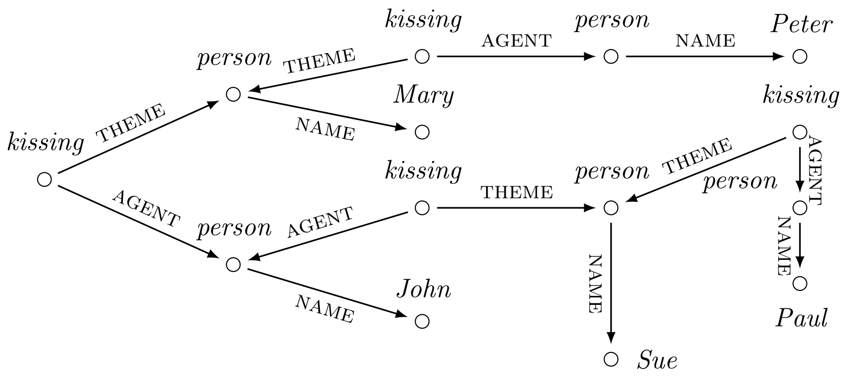
b. $\mathbf{\forall}(\downarrow i.\mathit{man} \implies \mathbf{\exists}(\downarrow i'.\mathit{woman} \wedge \mathbf{\exists}(\mathit{kissing} \wedge \langle \text{AGENT} \rangle i \wedge \langle \text{THEME} \rangle i')))$

c. $\mathbf{\exists}(\downarrow i.\mathit{woman} \wedge \mathbf{\forall}(\downarrow i'.\mathit{man} \implies \mathbf{\exists}(\mathit{kissing} \wedge \langle \text{AGENT} \rangle i' \wedge \langle \text{THEME} \rangle i)))$

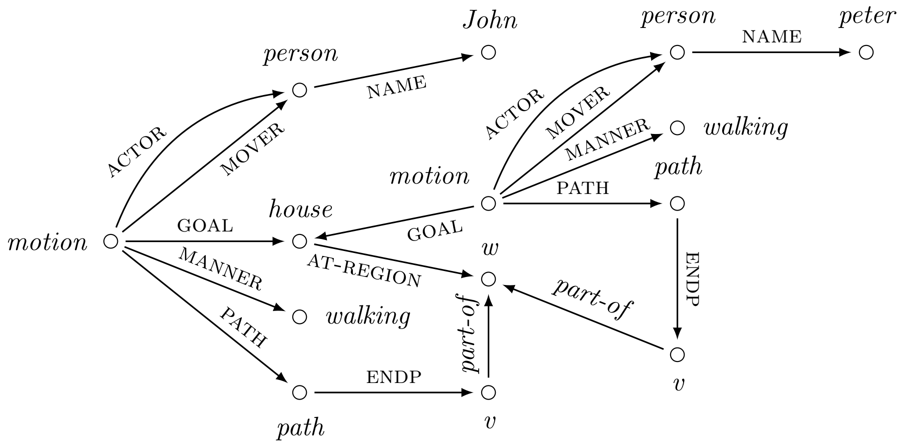
(5) a. *Every man walked to some house*

b. $\mathbf{\forall}(\downarrow i.\mathit{man} \implies (\mathbf{\exists}(\downarrow i'.\mathit{house} \wedge$
$(\exists a\ g.\mathbf{\exists}(\mathit{motion} \wedge \langle \text{AGENT} \rangle a \wedge \langle \text{MOVER} \rangle a \wedge \langle \text{GOAL} \rangle g \wedge$
$\langle \text{PATH} \rangle \mathit{path} \wedge \langle \text{MANNER} \rangle \mathit{walking} \wedge @_a i \wedge$
$(\exists r\ v\ w.\mathit{event} \wedge \langle \text{PATH} \rangle (\mathit{path} \wedge \langle \text{ENDP} \rangle v) \wedge$
$@_r(\langle \text{AT-REGION} \rangle w) \wedge @_v(\langle \mathit{part\text{-}of} \rangle w) \wedge @_r(g \wedge i')))))))$

c. $\mathbf{\exists}(\downarrow i.\mathit{house} \wedge (\mathbf{\forall}(\downarrow i'.\mathit{man} \implies$
$(\exists a\ g.\mathbf{\exists}(\mathit{motion} \wedge \langle \text{AGENT} \rangle a \wedge \langle \text{MOVER} \rangle a \wedge \langle \text{GOAL} \rangle g \wedge$
$\langle \text{PATH} \rangle \mathit{path} \wedge \langle \text{MANNER} \rangle \mathit{walking} \wedge @_a i' \wedge$
$(\exists r\ v\ w.\mathit{event} \wedge \langle \text{PATH} \rangle (\mathit{path} \wedge \langle \text{ENDP} \rangle v) \wedge$
$@_r(\langle \text{AT-REGION} \rangle w) \wedge @_v(\langle \mathit{part\text{-}of} \rangle w) \wedge @_r(g \wedge i)))))))$




(a) Quantification




(b) Quantification and node sharing

**Fig. 2.** Frame samples

As in [12], the syntax-semantics interface we propose builds a *frame description* out of a sentence in natural language. This frame description is a logical formula that is checked against the possible models, and the sentence is true w.r.t. a model $\mathcal{M}$ in case this model satisfies the logical formula. More precisely, given a sentence $s$ and its semantic representation $[\![s]\!]$, we say that $s$ *is true* iff for all assignments $g$, $\mathcal{M}, g \vDash [\![s]\!]$ (i.e., $[\![s]\!]$ is globally true in $\mathcal{M}$ under any assignment).

We use the following syntactic types: *NP*, *S*, *N*, and *PP* and the following syntactic type assignments:

| | | |
|---|---|---|
| John, Mary : $NP$ | kisses : $NP \to NP \to S$ | to, into : $NP \to PP$ |
| man, woman, house : $N$ | every, some : $N \to (NP \to S) \to S$ | walked : $PP \to NP \to S$ |

| | |
|---|---|
| $event, kissing, motion, person, John, Mary, \ldots : t$ | $@ : t \to t \to t$ |
| $\wedge : t \to t \to t$    $\Longrightarrow : t \to t \to t$ | $\mathbf{\exists}, \mathbf{\forall} : t \to t$    $\downarrow, \exists : (t \to t) \to t$ |

**Table 1.** Constant terms of the semantic language

| | | | |
|---|---|---|---|
| $S, NP, N$ | $:= t$ | $PP$ | $:= t \to t$ |
| John | $:= John$ | Mary | $:= Mary$ |
| man | $:= man$ | woman | $:= woman$ |
| house | $:= house$ | | |
| some | $:= \lambda P\ Q.\mathbf{\exists}(\downarrow i.P \wedge (Q\ i))$ | every | $:= \lambda P\ Q.\mathbf{\forall}(\downarrow i.P \Longrightarrow (Q\ i))$ |
| kisses | $:= \lambda o\ s.\mathbf{\exists}(kissing \wedge \langle \text{AGENT} \rangle s \wedge \langle \text{THEME} \rangle o)$ | | |
| walked | $:= \lambda pp\ s.\exists a\ g.\mathbf{\exists}(motion \wedge \langle \text{AGENT} \rangle a \wedge \langle \text{MOVER} \rangle a \wedge \langle \text{GOAL} \rangle g$ $\wedge \langle \text{PATH} \rangle path \wedge \langle \text{MANNER} \rangle walking \wedge @_a s \wedge (pp\ g))$ | | |
| to | $:= \lambda n\ g.\exists r\ v\ w.event \wedge \langle \text{PATH} \rangle (path \wedge \langle \text{ENDP} \rangle v) \wedge$ $@_r \langle \text{AT-REGION} \rangle w \wedge @_v \langle part\text{-}of \rangle w \wedge @_r (g \wedge n)$ | | |
| into | $:= \lambda n\ g.\exists r\ v\ w.event \wedge \langle \text{PATH} \rangle (path \wedge \langle \text{ENDP} \rangle v) \wedge$ $@_r \langle \text{IN-REGION} \rangle w \wedge @_v \langle part\text{-}of \rangle w \wedge @_r (g \wedge n)$ | | |

**Table 2.** Semantic interpretation of constants

Table 1 shows the semantic constants we use, including logical operators and quantifiers. We follow [12] in the semantics and meaning decomposition of locomotion verbs.

Then we can use the semantic interpretation given in Table 2. For sake of conciseness and explanatory purposes, we use a single type $t$ to denote modal formulas. This is not completely satisfactory as we can build terms that are not in Forms. (In principle, any proposition could specify the $@ : t \to t \to t$ operator. But in our lexicon example, we of course restrict the first parameter to state variables.) A more faithful encoding could use the standard parametrization of the propositions with a $s$ type for states, or use a dedicated hybrid type-theory [1]. Then the following equalities hold, where $t_{2b}$ is the term in (2b), $t_{3b}$ is the term in (3b), etc.:

(6) $$[\![\text{kisses Mary John}]\!] = t_{2b}$$

(7) $$[\![(\text{every man})\ (\lambda x.\text{kisses Mary}\ x)]\!] = t_{3b}$$

(8) $$[\![(\text{every man})\ (\lambda x.(\text{some woman})\ (\lambda y.\text{kisses}\ y\ x))]\!] = t_{4b}$$

(9) $$[\![(\text{some woman})\ (\lambda y.(\text{every man})\ (\lambda x.\text{kisses}\ y\ x))]\!] = t_{4c}$$

In (10) and (11), we have an interaction of the storing for quantification and path equalities compositionally deriving from the verb and the preposition. In the verb semantics, the path equalities specify that the MOVER and the AGENT attributes of the event are the same, and that the information provided by the *pp* argument should hold for the GOAL $g$. In its semantics, the preposition contributes on the one hand to the main event (as the *event* proposition is evaluated at the current state) and on the other hand by specifying that the $g$ state (meant to be the target node of the verb that the proposition modifies, here the target of the GOAL attribute) should be identified to the $n$ argument (the noun phrase which is argument of the preposition).

(10) $[\![(\text{every man})\ (\lambda x.(\text{some house})(\lambda y.\text{walked}\ (\text{to}\ y)\ x))]\!] = t_{5b}$

(11) $[\![(\text{some house})(\lambda y.(\text{every man})\ (\lambda x.\text{walked}\ (\text{to}\ y)\ x))]\!] = t_{5c}$

## 4 Conclusion and Perspectives

We used hybrid logic as a means to integrate logical operators with frame semantics. A type theoretic semantics was presented that shows how to compositionally derive different quantifier scope readings. This approach has much in common with [16], which combines data semantics with frame semantics. The exact relation between the two approaches needs to be spelled out in future work. We see applications of our approach of using hybrid logic for frame semantics in the context of various formalisms; we plan in particular to pursue this approach in the framework of Lexicalized Tree Adjoining Grammars (LTAG) [12].

We also think that the compositional account we presented allows us to consider an embedding within a underspecified representation language. The object language (in the sense of [8]) would be the hybrid logic language instead of the usual first-order logic language, following a standard modeling of scope ambiguity in LTAG.

Finally, we plan to investigate the computational properties of the framework we propose with respect to the hybrid inferential systems [6] and the specific properties induced by the frame models we consider, typically the functionality of the attribute relations [19].

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