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# Open-source numerical simulation tool for two-dimensional neural fields involving finite axonal transmission speed

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## Motivation

This work aims to provide an open source and cross-platform simulation tool that integrates numerically integral-differential equations of the type

$$\left(\eta\frac{\partial^2}{\partial t^2}+\gamma\frac{\partial}{\partial t}+1\right)V(x,t)=I(x,t)+\int_\Omega d^2y\,K(\|x-y\|)\,S\left[V\left(y,t-\frac{\|x-y\|}{c}\right)\right] \tag{1}$$

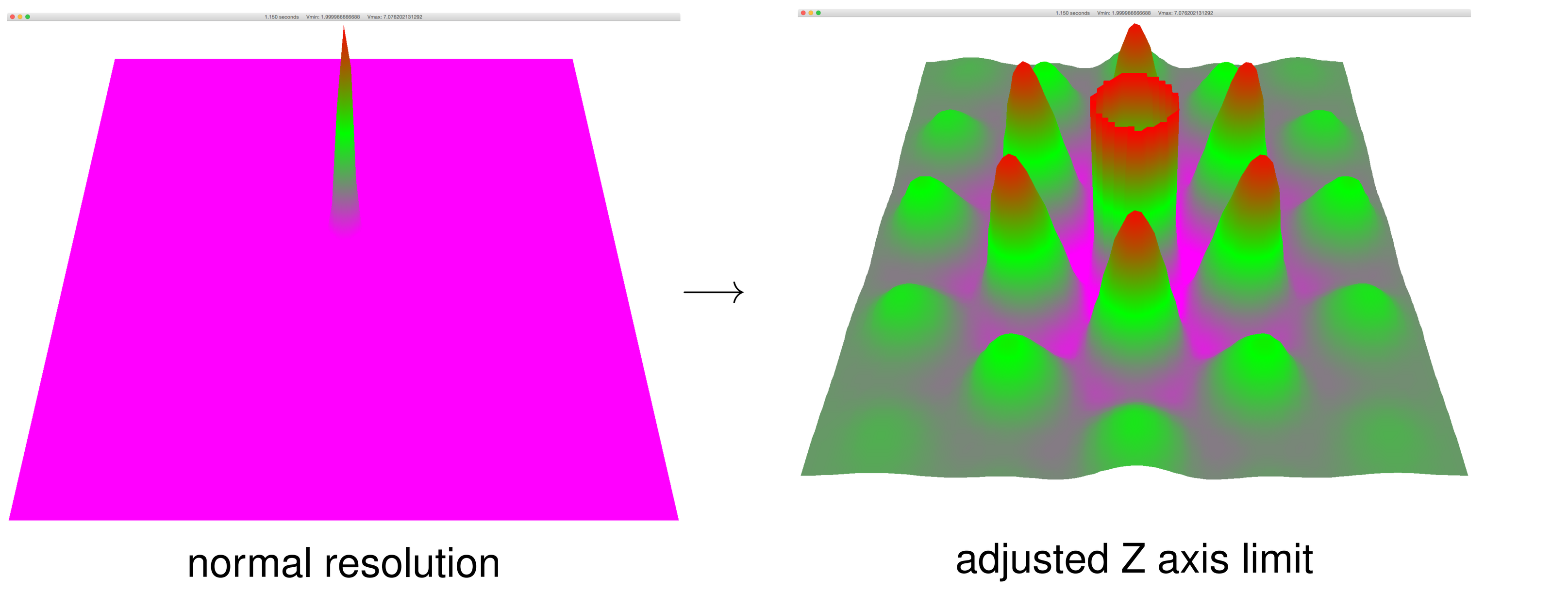
with mean neuron potential $V$ in a two-dimensional quadratic spatial domain $\Omega$ with periodic boundary conditions. The term $I$ denotes the external stimulus, $K$ is the synaptic connectivity kernel and $S$ is the firing rate. Finite axonal transmission speed $c$ induces space-dependent delays.

To this end, we present the *Neural Field Simulator* [1].

## Features

- ***Great usability***
  - Parametrization can be as simple or complex as field model
  - Visualization is easily modified by a keypress
- ***Complete control over Eq. (1) variables***
  - Free choice of values provided by text-based Python interface
- ***Spatio-temporal kernel***
  - Integral renders into a spatial integral and an integral over delays [2]
- ***Optimal acceleration***
  1. Fast Fourier transform in space
  2. Self-writing code based on interface selections
  3. Utilization of graphics processing unit for hardware acceleration
  4. Reduced rate of GPU uploads optimised for visual perception
- ***Output in rich detail***
  - 2D matrices output in 3D whereby $[x,y]\mapsto[z]$

  This allows features normally hidden in neural fields to be magnified and examined




  - Matrices can be moved, rotated, zoomed and colors and axis limits are easily changed
  - Movies and images of simulations can be saved

## Breather

A breather (Fig. 1) is simulated with a spatial grid of $n$=512 squared elements and parameters $dt$=0.001, $\gamma$=1, $\eta$=0, length $l$=30, $V(t$=0$)$=0, $V_{noise}(t>0)=\frac{e^{(a^2+b^2)}\sqrt{\partial t}}{320\pi}$ where $a$ and $b$ are a meshgrid of $[-l/2,\cdots,l/2]$, $I=\frac{20e^{-x^2/32}}{32\pi}$, $K=\frac{-e^{-x/3}}{4.5\pi}$, $S=\frac{1}{1+e^{-10000(V-0.005)}}$ and $c$=500.

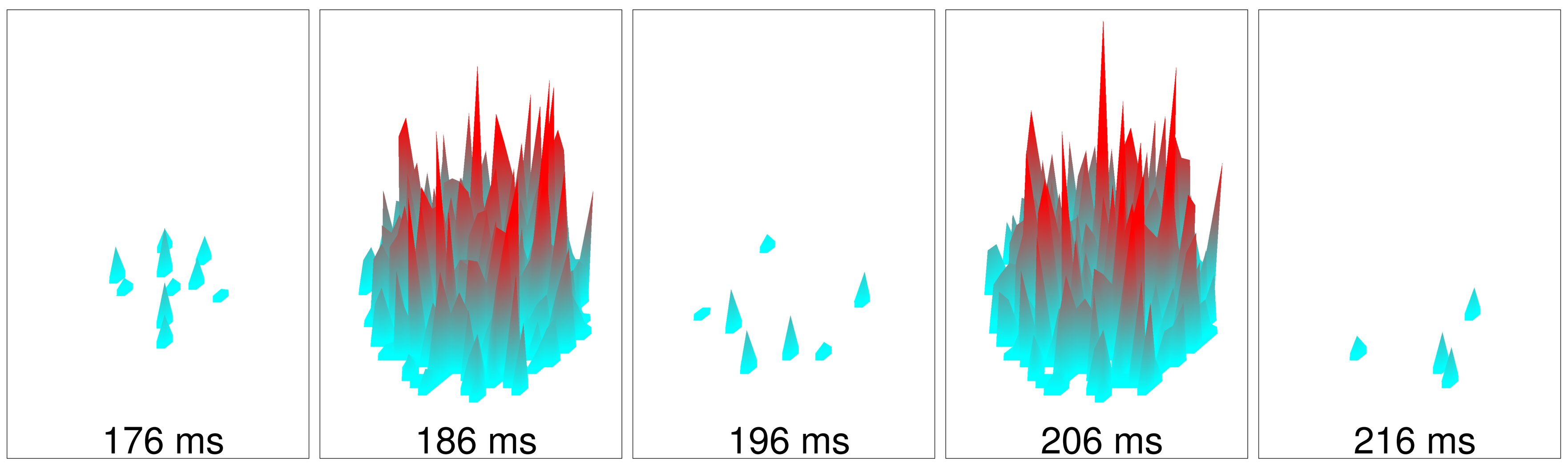



Figure 1: A breather at 10 millisecond intervals with manually set minimum Z axis (=0.0048).

## Turing Pattern

Turing patterns emerge from noisy field voltage (Fig. 2) with terms $dt$=0.01, $\gamma$=1, $\eta$=0, $l$=90, $n$=512, $V(t=0)=5.4+\frac{e^{-x^2/0.02}}{\sqrt{0.02\pi}}$, $I$=0, $K=K_{-}-\sin(\vec{v})/200$ with $K_{-}=\sin(\vec{v})/150$ and $\vec{v}=[-9\pi,\ldots,0]$, $S=\frac{2}{1+e^{-1.24(V-3)}}$ and $c\geq l\sqrt{2}\Delta t$ is infinite (=6364).

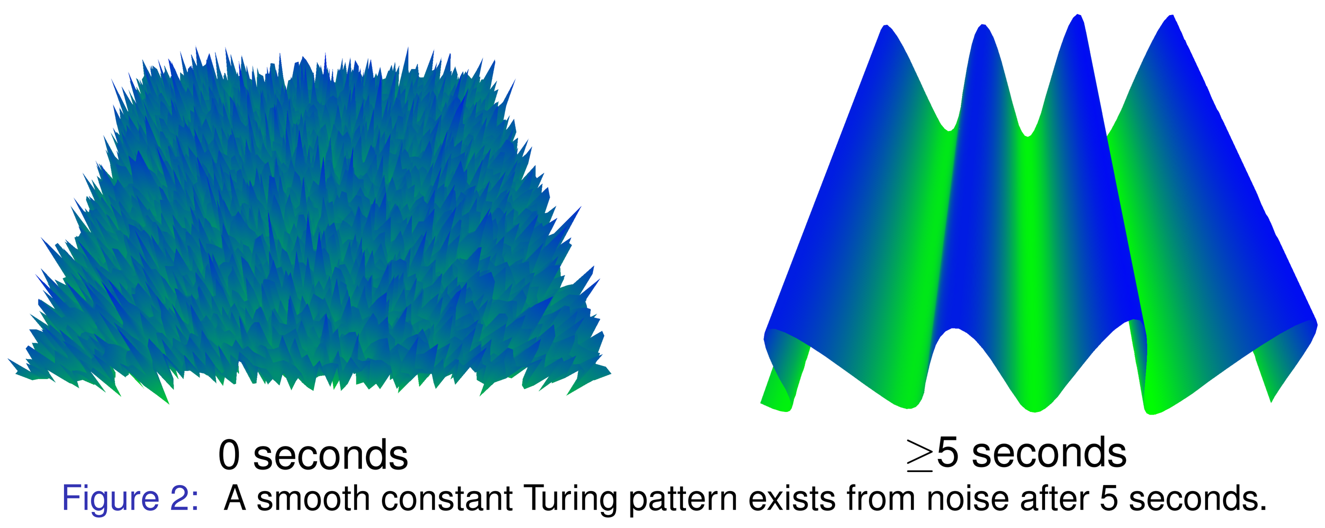



Figure 2: A smooth constant Turing pattern exists from noise after 5 seconds.

## Alternating Roll

Reference [1] presents an alternating roll solution (Fig. 3) with descriptions of the following values: $dt$=0.02, $\gamma$=2, $\eta$=1, $l$=40.3805226, $n$=512, $k_c$=1.0891958379832 $\omega_c$=3.4003003526352 $V(t=0)=3+0.4\sin(k_c a)$, $U_{excite}=0.4\omega_c\cos(k_c b)$ $I$=2.5, $K=\frac{121e^{-x}-235.2e^{-1.4x}}{2\pi}$, $S=\frac{1}{1+e^{-2.856(V-3)}}$ and finite $c$=6.

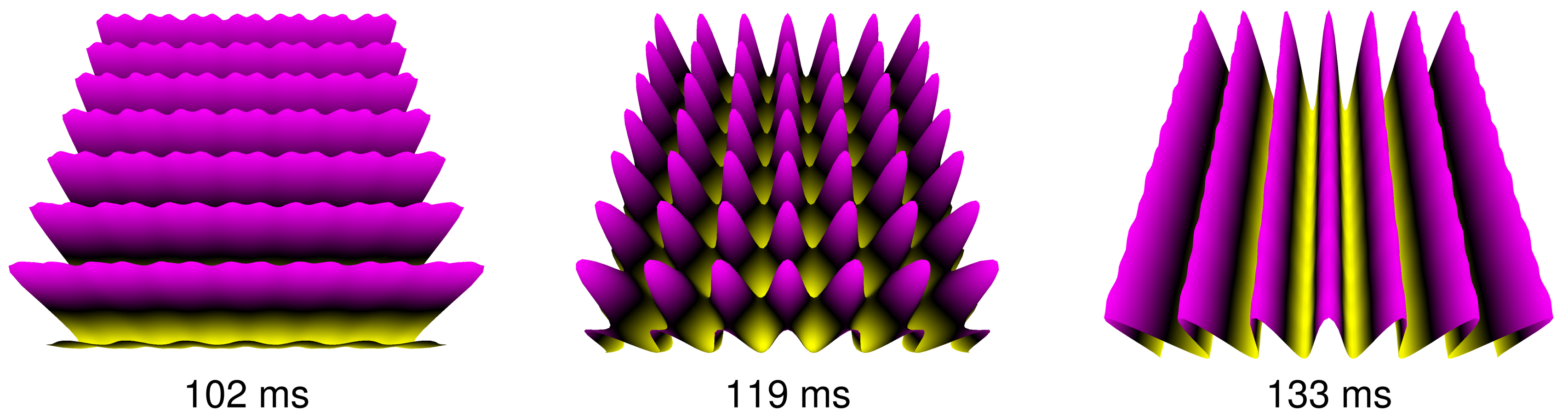



Figure 3: A stable alternating roll continuously transforms between horizontal and vertical line patterns every ≈31 milliseconds.

## Acknowledgments

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## References

[1] http://nfsimulator.gforge.inria.fr
[2] A. Hutt and N. Rougier. Activity spread and breathers induced by finite transmission speeds in two-dimensional neural fields, *Physical Review E* 82 (5):055701 (2010).
[3] K.R. Green and A. Hutt. Dynamic square patterns in a two-dimensional neural field with finite transmission speed. *Society for Industrial and Applied Mathematics* (2015), Submitted for publication.

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