Sound pulse broadening in stressed granular media

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The pulse broadening and decay of coherent sound waves propagating in disordered granular media are investigated. We find that the pulse width of these compressional waves is broadened when the disorder is increased by mixing the beads made of different materials. To identify the responsible mechanism for the pulse broadening, we also perform the acoustic attenuation measurement by spectral analysis and the numerical simulation of pulsed sound wave propagation along one-dimensional disordered elastic chains. The qualitative agreement between experiment and simulation reveals a dominant mechanism by scattering attenuation at the high-frequency range, which is consistent with theoretical models of sound wave scattering in strongly random media via a correlation length.

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I. INTRODUCTION

The structure of jammed granular media is characterized by the inhomogeneous contact networks at the mesoscopic scale [1]. Elastic waves in such media propagate in different ways according to the ratio of the wavelength $\lambda$ to the grain size $d$. At low $\lambda/d$, the propagation of waves is incoherent and strongly scattered, while at high $\lambda/d$, the wave propagation is coherent and ballistic [2]. The scattered waves are the fingerprint of a specific configuration of the contact force network, which can be used to measure the interfacial dissipation on the grain scale and detect the tiny rearrangement of the contact network [3]. On the other hand, the velocity measurement of coherent waves allows one to characterize the elastic properties of granular packings on the macroscopic scale and infer the coordination number within the framework of effective medium theories [4–7]. This latter plays a central role in the granular mechanics [8–11].

Pulse wave propagation provides a convenient way for material characterization and signal transmission, thanks to the possible temporal separation of signals from parasites. In disordered granular packings, both experiments and numerical simulations have shown that coherent wave pulses, compressional or shear, decay in amplitude and broaden in width as they propagate, thus reducing significantly the resolution capacity [2,12–16]. In general, two mechanisms may be responsible for the pulse broadening, i.e., attenuation and dispersion of velocity, which can be analyzed in the frequency domain via the dispersion relationship $k(\omega)$ between the wave number $k$ and the angular frequency $\omega$. Indeed, from $k(\omega) = k'(\omega) + ik''(\omega)$, the phase velocity $V(\omega) = \omega/k'$ and the attenuation $\alpha(\omega) = k''$ can be deduced, respectively. Somfai et al. [14] showed that in (one-dimensional) ordered granular media, the dispersion effect due to discrete lattice governs the shape of the coherent wave front, and leads to a pulse broadening which scales with the source-detector distance $L \propto L^{1/3}$. By introducing the polydispersity in the one-dimensional (1D) chain, their numerical simulations revealed that disorder increases the pulse width. It was also shown in amorphous media that disorder could induce a dispersion of the phase velocity particularly at the high-frequency range with $\lambda \sim d$ [17]. However, at such frequency range, the coherent waves often exhibit strong attenuation dominated by strong scattering thus being responsible for the pulse broadening [18,19]. In granular media, despite some recent numerical studies [14–16], few experimental data are actually available to highlight the interplay between pulse broadening and attenuation.

In this work, we study the propagation of compressional wave pulses through disordered granular packings under stress. A particular attention is focused on the evolution of the pulse width of the coherent sound wave as a function of propagation distance. Various granular media are investigated, including the mixture of glass and (poly)methyl-methacrylate (PMMA) beads with enhanced disorder. To understand the origin of the pulse broadening, we also perform the attenuation measurement by a spectral analysis and numerical simulations of the pulse propagation along disordered 1D elastic chains. The existing models of elastic wave scattering in random media are finally discussed to highlight the attenuation mechanism in granular media.

II. EXPERIMENTS

A. Granular samples

Acoustic measurements are performed using a pulse transmission through granular media under stress [2]. The beads are poured by rain deposition into a rigid cell of inner diameter 30 mm. A plane-wave source (longitudinal) transducer of diameter 30 mm is placed on the top of the cell and the wave transmission is detected by a similar transducer on the bottom; both the source and the detector are in direct contact with the beads (Fig. 1). Before the ultrasonic measurements, two
cycles of loading and unloading are performed to improve the reproducibility of measurements.

Various granular samples are investigated in this study. Table I summarizes the main characteristics like constitutive material (glass, PMMA, or steel), mean size of beads, polydispersity, and sphericity. The beads with a mean diameter larger than 1 mm have a good spherical shape and a low polydispersity, while the smaller ones present sometimes a nonspherical shape (e.g., partial coalescence of two beads) and higher polydispersity. The use of polymeric beads, e.g., PMMA, allows us to evaluate the effect of dissipation mechanisms on the coherent wave front. Indeed, by measuring the absorption time (not shown here) from the time-resolved intensity profile of scattered waves [20,21], we infer a wave dissipation four times larger in PMMA beads than in glass beads at 300 kHz due to the viscoelastic loss.

### B. Impulse response and wave velocity measurement

To obtain the impulse response of the coherent compressional wave in granular media, we may deconvolve the signals transmitted through the media by the signal sent by the source transducer. Figure 2 shows an example of the measured signal and deconvolved one (first part). The source signal [inset of Fig. 2(b)] is obtained by putting the source against the detector via a coupling film. The deconvolution calculation, based on FFT, is performed on the frequency range [1–900 kHz]. As in [14], we characterize the impulse response of the coherent signal by the arrival times associated to three particular points: the peak $A_1$ ($t_1$), the first arrival at 10% of the peak ($t_0$), and the first zero crossing ($t_2$). Complementary measurements of pulse transmission were also performed in a reference medium, i.e., water filling up the above cell (waterproof) in which the shape of received signal does not evolve significantly as the distance source-receiver increases. This observation indicates that the source transducer does generate a quasiplane wave and the effect of edge waves are negligible in our experiments [22].

In pulse transmission experiments, the wave velocity may be determined via the time-of-flight method, making measurements at two different distances. If only one-distance measurement is conducted for the velocity calculation, the different time $t_0$, $t_1$, or $t_2$ leads obviously to different values (Fig. 3). Notice however that the velocity determined with $t_1$ is almost independent of the source-receiver distance, contrary to the other times used. As shown in Fig. 3, this velocity determined under low stress may exhibit a slight increase with the distance source receiver. Such discrepancy could be improved by an alternative method based on the Hilbert transform using a reference signal, for example, the source signal [23]. Nevertheless, the difference between the velocity measured via $t_1$ and that determined by the alternative method is no more than 5%.

### TABLE I. Description of the beads used for granular packings.

<table>
<thead>
<tr>
<th>Material</th>
<th>Glass</th>
<th>Steel</th>
<th>PMMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size $d$ (mm)</td>
<td>5 4 3 2 1.5</td>
<td>0.715 0.35 0.225 0.7 0.615</td>
<td></td>
</tr>
<tr>
<td>Tolerance (mm)</td>
<td>±0.01 ±0.125 ±0.085 ±0.05 ±0.025 ±0.1 ±0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sphericity</td>
<td>good quite good quite good poor poor poor quite good</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The wave velocity measurement has been extensively used to study the nonlinear elasticity of stressed granular media; see, e.g., [4,6]. Figure 4(a) shows that as the confining pressure is decreased, there is an increase of the time of flight, a decay of the pulse amplitude, and a broadening of the pulse. However, by rescaling the time $t/t_1$ and the amplitude $A/A_1$ axes, we find that almost all responses collapse [Fig. 4(b)], indicating that the coherent wave fronts are mainly governed by the time scale $t/t_1$ or $V_p t$. To evaluate the accuracy of this collapse, we compare the normalized width of the signal, $W = (t_1 - t_0)/t_1$ at high and low pressures (inset of Fig. 4). The normalized width at low pressure is always higher than that obtained at high pressure. This difference may arise from the heterogeneity of the contact force networks which is more important at lower confining pressure [24]. For glass bead packings, the collapse of the rescaled responses appears more effective with beads of large size, less polydisperse, and more spherical whose packing structures are expected to be less disordered. These results would suggest a possible dependence of the normalized width on the amount of disorder.

C. Evolution of sound pulse width

Figure 5(a) shows a pulse broadening and decay with the distance of propagation through granular media. The width of the signal, $t_1 - t_0$, scales with the ratio $L/d$ as a power law with exponent of about 1/2 or higher for beads of large size [Fig. 5(b)]. However, if the pulse width is normalized by the propagation time $t_1$, it decreases with the propagation distance $L/d$, according to

$$ W \simeq C_W (L/d)^{-0.5}, $$

where $C_W$ is a prefactor. Moreover, Fig. 5(c) shows that the data of these normalized widths are much less disperse compared to those in Fig. 5(b), ranged in a grey band likely independent of the material property of beads and its mean size $d$. The data located in the upper part of this band are obtained with less spherical small beads. The results obtained from PMMA beads are very similar to those from glass beads, showing that the viscoelastic dissipation does not significantly affect measurements of the normalized pulse width.

To verify whether the normalized width depends on the amount of disorder mentioned above, we investigate the evolution of the prefactor $C_W$ in granular packings with a mixture of glass and PMMA beads. A mixture of the beads of different nature is expected to increase the heterogeneity of stiffness and mass in the granular packings and accordingly the normalized width $W$. For a given volume fraction of glass beads $\phi_{\text{glass}} = V_{\text{glass}}/(V_{\text{glass}} + V_{\text{pmma}})$, we determined the prefactor $C_W$ by fitting the normalized widths measured for three source-receiver distances. Figure 6 shows the evolution of prefactor $C_W$ as a function of the glass beads volume fraction $\phi_{\text{glass}}$. As expected, $C_W$ versus $\phi_{\text{glass}}$ presents a maximum. Instead, the elastic longitudinal modulus $C_{11}$ given by $C_{11} = \rho_m V_p^2$ with $\rho_m$ the packing density of the granular mixture increases monotonically with the volume fraction of glass beads.

D. Attenuation measurement by FFT

To identify the origin of the pulse broadening, we investigate the wave attenuation and velocity dispersion in frequency
FIG. 5. (Color online) (a) Pulse broadening and decay through a glass bead packing of \( d = 0.715 \) mm under stress \( P = 995 \) kPa. The first period (plain line) of coherent wave front is used for the FFT analysis. (b) Width of the pulse measured as a function of the ratio \( L/d \) for different granular media under \( P = 995 \) kPa. (c) Normalized pulse width \( W \) plotted as a function of ratio \( L/d \). Numerical results in 2D disordered granular packings [14] and 1D ordered chain simulation (dashed red line) are added here for comparison. The normalized width scales with \( L \) as \( W \sim C_L(L/d)^{-0.5} \).

FIG. 6. (Color online) Pulse broadening due to disorder: the best fit values of prefactor \( C_W \) and the elastic longitudinal modulus \( C_{11} = \rho m V_p^2 \) obtained as a function of glass beads volume fraction in a glass-PMMA beads mixture \( d = 0.615 \) mm, \( P = 995 \) kPa.

III. SIMULATION AND MODELING

Despite the lack of theoretical models accurately describing the sound propagation and scattering in granular media, we seek to explain our main experimental observations by the simplified numerical simulations and the general models of elastic wave scattering in random media.

A. Pulse broadening versus dispersion relationship

We here consider two distinct mechanisms leading to the pulse broadening, i.e., wave attenuation \( \alpha(\omega) \equiv k''(\omega) \) and velocity dispersion \( V(\omega) \equiv \omega/k'(\omega) \) where \( k(\omega) = k'(\omega) + ik''(\omega) \) is the complex wave number. To this end, we investigate the propagation of a plane wave \( a(x,t) \) along \( x \), related to its inverse Fourier transform \( \tilde{a}(\omega) \) by

\[
a(x,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-i\omega t} \tilde{A}(\omega) e^{ikx} d\omega
\]

with \( \tilde{a} = A(\omega)e^{ikx} \). For a Dirac-like pulse propagation, one has \( A(\omega) = 1 \). Two properties of the Fourier transform are recalled which are useful for the following discussions. (i) Translation: if \( g(t) = f(t-t_0) \), then \( \tilde{g}(\omega) = e^{i\omega t_0} \tilde{f}(\omega) \). (ii) Scaling: if \( g(t) = f(at) \), then \( \tilde{g}(\omega) = |1/a| \tilde{f}(\omega/a) \).

We first study the effect of the velocity dispersion \( V(\omega) = \omega/k'(\omega) \) on the pulse broadening in a 1D mass-spring chain composed of identical spheres of radius \( R \). There is no attenu-
ei will be given by
\[ V = \omega L \]
where the phase velocity \( V \) detected at different distances \( L \) long enough, the wave number can be approximated by
\[ k = \frac{\omega}{V} \]
with \( \omega \) and \( V \) is given by
\[ k \sim \frac{\omega}{V} = \frac{\omega^2}{2\pi c^2} \]
as asymptotics system((SOUND PULSE BROADENING IN STRESSED GRANULAR MEDIA PHYSICAL REVIEW E 91, 022205 (2015)))

Second, we examine a pulse propagation in a 1D random medium where the phase velocity \( V = \omega/k' \) is constant, dispersionless. However, there is an attenuation given by
\[ \alpha = k'(\omega) = \left(\frac{\sigma K L_n}{\pi}\right)^{1/3} k^{1/3} \]
with \( n \) an integer; the underlying physics (and the parameters in the bracket) of such a system is detailed in Sec. III C below. The Fourier transform of the temporal signal is then
\[ \tilde{a}(\omega) = e^{i k' x} e^{-k' x} = e^{i \alpha_1 t} e^{-i (\alpha_1 n)^{3/2}} \]
with \( t_1 = \frac{\omega}{V} \) and \( \alpha_1 = \left(\frac{\omega^2}{2\pi c^2}\right)^{1/3} \).
By considering the properties of the Fourier transform, the temporal signal is \( a(x,t) = \omega s[\omega(t - t_1)] \) where the function \( s \) is the inverse Fourier transform of \( e^{-i \omega t} \). With the rescaled time axis \((t_1 - t)/t_1\), the normalized width of the signal is then \( W \sim (\alpha_1 t_1)^{-1} = (\sigma K L_n)^{3/2}/(\pi c^{3/2}) \). In 1D randomly layered media, one has \( n = 2 \) (see details below in Sec. III C) leading to \( \alpha \sim \omega^2 \) and \( W \) at \((x = L)\)
\[ W \propto L^{-1/2}. \]

**B. Simulations in 1D chains of elastic lattices**

The use of normalized variables allows us to compare our experimental results to numerical simulations. As shown in Fig. 5(c), the numerical results obtained in disordered 2D granular media [14] compare quite well with our experimental results. Moreover, the velocity dispersion in 1D ordered chains leads to a power law scaling of \( W \) on distance as \((L/d)^{-2/3}\) [Eq. (3)] as found in [14]. However, the normalized widths obtained in disordered packing scales with distance as \((L/d)^{-1/2}\) [similar to Eq. (4)] and have also values larger than those deduced from ordered packings.

In order to evaluate the effect of scattering attenuation on the pulse broadening, we perform the simulation in a 1D disordered mass-spring chain in the presence of the correlation length similar to random media by Fouque et al. [19]. More specifically, we consider a 1D chain of mass spring consisted of subchains of length \( L_1 \) and stiffness \( D_1 \) which is uniform inside each subchain [Fig. 8(a)]. Assuming an exponential distribution of the subchain length \( L_1 \),
\[ p(L_1) = \frac{(L_1/L_0)^{2} e^{-L_1/L_0}}{L_0}, \]
we then obtain a mean length of the subchains equal to \( L_0 \). If the inverse of stiffness is given by \( D^{-1} = \bar{D}^{-1} + (1 + \nu_D) \) with \( \bar{D}^{-1} \) the mean value and \( \nu_D \) the fluctuation uniformly distributed in the range \([-\nu_{max}, +\nu_{max}]\), we may readily verify \((\nu_D) = 0\) and the variance \((\nu_D^2) = (\nu_{max})^2/3.\)

To determine the correlation length associated with such a disordered chain, we compute the correlation function of the stiffness fluctuation \((\nu_D(x)\nu_D(x + \Delta x)) via ensemble average between two points separated by a distance \( \Delta x \). Figure 8(c) shows the normalized correlation function \((\nu_D^2) \) versus \( \Delta x \), calculated for the different \( L_0 \). If we defined the correlation length \( L_c \) corresponding to a decrease of the normalized correlation to about 0.4 \((\approx e^{-1})\), we deduce a correlation length \( L_c \approx L_0 \) from Fig. 8(c).

Simulations of the pulse propagation are performing by solving the eigenproblem of the linear spring system at given initial conditions. The equation of motion of the \( N \) beads is given by [Fig. 8(b)]
\[ m_i \ddot{u}_i = D_{i+1}(u_{i+1} - u_i) + D_i(u_{i-1} - u_i). \]
This equation can be expressed in a matrix form \( m_i \ddot{u} = D \cdot u \) where \( u \) is the displacement vector of each bead (only the translational motion is considered here), \( m \) is the constant mass of beads, and \( D \) is a \( N \times N \) matrix (for simplicity \( m \) and \( D \) are here set to 1). Eigenfrequencies \( \omega_n \) of this linear
system are the square roots of the eigenvalues of the matrix $D/m$. The oscillations of beads are given by the superposition of the eigenmodes: $u(t) = \sum_n A_n u_n \cos(\omega_n t)$ where $u_n$ are the eigenvectors of the matrix $D/m$ and the amplitudes $A_n$ are determined by the initial conditions. To prevent the wave reflections at the edges, the source and the detector are placed in the middle of the chain. Assuming that at $t = 0$ the particle displacement located at the source is equal to 1, the propagation of the pulse for various source-detector distances $L$ is then computed with the displacement $u(t)$, averaged over 100,000 configurations.

Figure 9(a) shows the evolution of the coherent wave front where the shape of the wave front evolves during the propagation and tends to a Gaussian-like pulse at long distance. The normalized width of the pulse is plotted in Fig. 9(b) as a function of the source-detector distance for the case $\sigma_D = 0.29$. The variance $\sigma_D$ slightly increases the normalized width and the variation of the mean or correlation length further enhances the effect on $W$. Figure 9(c) depicts the pulsed wave attenuation observed in frequency domain. It scales as $\omega^2$ and $\propto (L/d)^{-2/3}$ for various correlation lengths.

C. Models of elastic wave scattering and dissipation

Finally, we compare our experimental and numerical results with the theoretical models of elastic wave propagation in random or granular media. In these scattering media, the models seek to relate the attenuation of coherent waves to the statistical properties of random media such as the elastic modulus $K$ and the density $\rho$ [18,19,26], for instance introducing a characteristic length $L_c$ of the fluctuation $\nu_K = \Delta K^{-1}/K^{-1}$ (see below).

Consider 1D random layers with the similar statistical properties as those investigated above in 1D disordered chains and free of dispersion and dissipation. Assuming $L_c \ll \lambda \ll L$ and a high level of disorder corresponding to a variance $\langle \nu_K^2 \rangle = \sigma_K^2 \sim 1$, Fouque et al. [19] show that the wave form of a coherent pulse tends to a Gaussian signal at long distance of propagation [Fig. 9(a)]; it propagates at an effective velocity...
tends to L > \lambda_c\) satisfying the variance \(\sigma^2\) given distance \(L/d\) for disordered chains, regarding the normalized width \(\gamma \Gamma / \hat{\rho}\). (7)

\[
\tilde{V} = \sqrt{\frac{\sigma^2 L_c}{\lambda_c}};
\]

\[
a(L,t) = \frac{1}{\sqrt{2\pi w_L^2}} e^{-\left(\frac{-L^2}{2w_L^2}\right)^2}.
\]

where \(w_L = (\gamma L/2\tilde{V}^2)^{1/2}\) governs the pulse width and \(\gamma = \int_0^\infty \langle v_k(0)v_k(x)\rangle dx = \sigma^2_k L_c\) is determined by the product of the variance \(\sigma^2_k\) and the correlation length \(L_c\). Equation (7) implies that the normalized width \(W \sim w_L/(L/\tilde{V})\) scales with propagation distance \(L\) as \((\sigma^2_k L_c/L)^{1/2}\), and that the attenuation of the pulse is given by \(\alpha = \gamma (\sigma^2_k)^2\).

Figure 10 depicts numerical results from simulations in disordered chains, regarding the normalized width \(W\) for various source-detector distances \(L/d\) as a function of a combined parameter \((\sigma^2_k L_c/L)^{1/2}\). More specifically, at a given distance \(L\), we determine \(W\) by varying the variance \(\sigma^2_k\) or/and the correlation length \(L_c\). The stack data show that for high disorder \(W\) scales as \((\sigma^2_k L_c/L)^{1/2}\) as predicted above, while for the weak level of disorder, i.e., \(\sigma^2_k L_c / L < \lambda_c / 2\), \(W\) tends to the constant values as those determined in ordered 1D elastic chains (represented by the horizontal solid lines). In terms of the dispersion relationship for the high level of disorder, the attenuation is given by \(\alpha = \sigma^2_k L_c k^2/4 \sim \omega^2\), consistent with the results shown in Fig. 9(c).

In 3D random media without dispersion and dissipation [18,26], the attenuation may be expressed in the limit case \(k^2 \gamma L \ll 1\), by

\[
\alpha \sim \sigma^2_k L_c^{-1} k^n \sim \omega^n\) (8)

where \(n = 2\) corresponds to the large scale fluctuation \((L_c \gg \lambda)\) with an anisotropic scattering, identical to the above result obtained in 1D random media [see also Fig. 9(c)], and \(n = 4\) corresponds to the small scale fluctuation \((L_c \ll \lambda)\) with an isotropic Rayleigh-like scattering [26].

Regarding the experimental data of attenuation shown in Fig. 7 and in [27], these two mechanisms of scattering invoked here corresponding to \(\alpha \sim \omega^2\) and \(\alpha \sim \omega^4\), respectively, may explain qualitatively the measurements at the high-frequency range (note that the former appears more significant in the present work), due to the heterogeneous and anisotropic structure of the contact force networks. However, at the low-frequency range, we experimentally observe a different behavior of attenuation \(\alpha \sim \omega^4\). As mentioned before, the viscoelastic dissipation related to the particle material seems to be negligible here. Nevertheless, the wave dissipation around the bead contacts via thermoelastic relaxation might be a mechanism responsible for such kind of attenuation, as proposed by Wang and Santamarina [28], when the characteristic time is comparable with \(1/\omega\).

**IV. CONCLUSION**

We have studied the pulse broadening of coherent sound waves propagating in granular media. The evolution of the pulse width of the coherent compressional wave is analyzed in terms of the dispersion relationship, including wave attenuation and velocity dispersion. The pulse broadening measured in various disordered bead packings reveals both a different scaling law on distance of propagation \(L\) and larger values, compared to analytical and numerical results only invoking the dispersion effect as that found in ordered elastic lattices. The numerical simulations carried out in 1D disordered elastic systems composed of subchains show that the pulse broadening may be caused by scattering attenuation. More specifically, the normalized pulse width \(W\) by the wave propagation time scales with distance as \(\propto L^{-1/2}\) instead \(\propto L^{-2/3}\) due to the dispersion, and significantly depends on the product of the inverse stiffness variance \(\sigma^2_k\) and the correlation length \(L_c\). Our experimental and numerical results are consistent with theoretical models of elastic wave scattering in 1D and 3D random continuous media, suggesting a dominant attenuation mechanism \(\alpha \sim \omega^2\) possibly due to an anisotropilcike scattering. Further works are still necessary to highlight the underlying physics of attenuation at the low-frequency range.

For future studies, the effects of disorder via mass or spring fluctuations found by numerical simulation in 1D chains need to be investigated in 2D or 3D disordered elastic networks. In these cases, the additional topological disorder appears and the absence of a reference medium prevents actually the analytical model of sound propagation [5]. Numerical simulation may help us to better understand the elastic wave propagating in granular media.

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