A Novel 4D Hyperchaotic Attractor with Typical Wings

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Rapid Communication

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The paper presents a new 4D dynamical system exhibiting typical characters. The related 4D hyperchaotic Attractor shows significant wings and scrolls extension. To explore its behavior, the 3D projections of the phase portrait are displayed. The bifurcation diagrams highlight the intricate dynamics of the system for a restricted range of the control parameter.

Key words: 4D system; Phase Portraits; Diagrams of bifurcation; Hyperchaos;

1. Introduction

The paper introduces an intentionally constructed 4D hyperchaotic attractor. At best of our knowledge, its characters and topology are distinct from previous attractor. The related system integrates a modified 2D Lotka-Volterra oscillator [1-2] in its first two equations. Such oscillator mechanism composed the core of some 3D chaotic and 4D hyperchaotic models [3-4-5].

The hyperchaos concept introduced by Rössler [6] allowed the characterization of chaotic behavior when at least two Lyapunov exponents are positive.

In fact, the detection of new 4D hyperchaotic attractors extends our knowledge of system patterns, the outcome of the feedbacks, and the symmetry arrangements.

In the section 2, the new system is presented and the phase portraits displayed. In the section 3, the basic characters of the dynamical behavior are introduced and the bifurcation diagrams computed. Some remarks constitute the conclusion of this short paper.

2. The New Hyperchaotic 4D model

Four nonlinear differential equations govern the new system:

$$\begin{cases} dx/dt = x\,(y - 1) + \alpha\, z \\ dy/dt = \beta\,(x^2 - 1)\, y \\ dz/dt = \varphi\, x + (y - 1)\, z + s\, v \\ dv/dt = \psi\, x\, y \end{cases}$$

where x, y, z, and v the state variables of the model, and α, β, φ, s, and ψ real parameters.

Equations embed four nonlinear terms, three of them are quadratic, i.e. two xy, and yz, and a unique cubic cross-term; i.e. x^2y.

We noticed that the system is not derived – or an expanded version- of previous 4D or 3D chaotic models.

Let the set of parameters C (α, β, φ, s, μ, ψ) = (1, 0.7, -0.1, 1, -0.2), the exploration of the system leads us to determine the equilibria of the system. These points are found by setting:

dx/dt = dy/dt = dz/dt = dv/dt = 0

We obtain the coordinates of three equilibria: the origin S_0 (**0, 0, 0, 0**), S_1 **(1, 0, 1, 1.1),** and S_2 **(-1, 0, -1, - 1.1).**

The eigenvalues $\boldsymbol{\lambda_i}$ and stability features of the related solutions are determined from the characteristic equation $|\boldsymbol{J} - \boldsymbol{\lambda I}| = \mathbf{0}$, where $\boldsymbol{I}$, the unit matrix, and $\boldsymbol{J}$, the Jacobian of the model.

Table 1 reports the stability indexes of the detected equilibria.

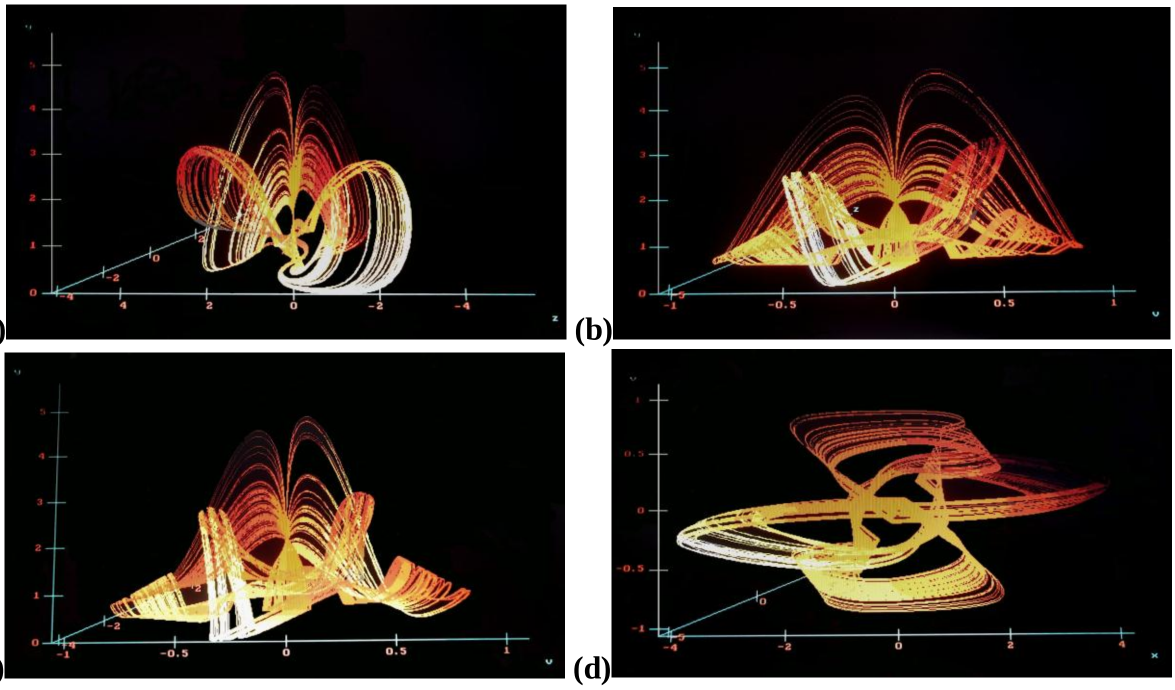
Coordinates of the Equilibium	**The corresponding characteristic equation, $\lvert J - \lambda I\rvert = 0$, and eigenvalues**	**Index (1)**
S_0 (x_0, y_0, z_0, v_0) = (0, 0, 0, 0)	$\lambda\,(0.7 - \lambda)(\lambda^2 + 2\lambda + 1.1) = 0$ $\lambda_1 = 0$ $\lambda_2 = 0.7$ $\lambda_3 = -1 - 0.31\,i$ $\lambda_4 = -1 + 0.31\,i$	**Index-1**
S_1 (x_1, y_1, z_1, v_1) = (1, 0, 1, 1.1)	$\lambda^2\,(\lambda^2 + 2\lambda + 0.9) = 0$ $\lambda_1 = 0$ $\lambda_2 = -1.31$ $\lambda_3 = -0.68$	**Index-0**
S_2 (x_2, y_2, z_2, v_2) = (-1, 0, -1, - 1.1)	$\lambda^2\,(\lambda^2 + 2\lambda + 1.1) = 0$ $\lambda_1 = 0$ $\lambda_2 = -1 - 0.3\,i$ $\lambda_3 = -1 + 0.31\,i$	**Index-0**

(1) Index reports the number of eigenvalues with real parts Re (λ) > 0. From 1 to 4, it indicates the degree of instability. Index-0: null or negative real parts of all eigenvalues of the equilibrium characterize its stability.

Table 1. The Equilibria

With the initial conditions Ic (0.1, 0.1, 0.1, 0.1), a 4D hyperchaotic attractor appears throughout projections into the four 3D phase spaces (fig. 1).

The system exhibit distinguished disposition of several wings and scrolls, and substantiating the novelty of the model.

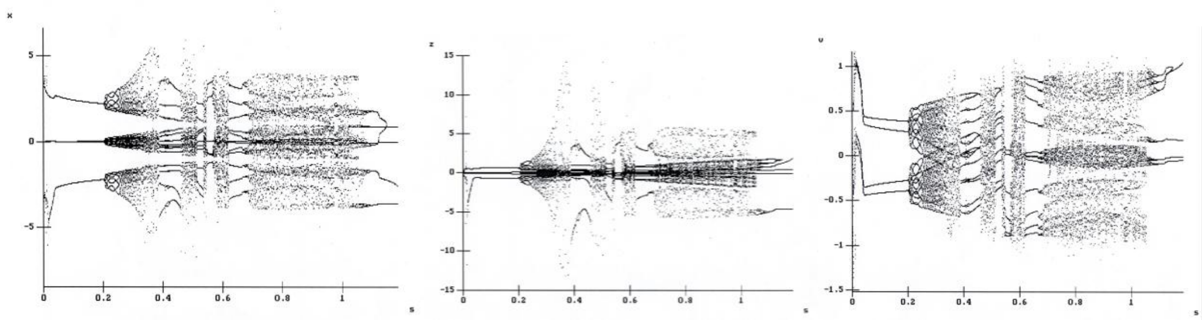


(a) (b) (c) (d)

Fig.1. 4D Attractor for C parameters.
(a) Projection on z-y-x space, **(b)** Projection on v-y-z space, **(c)** Projection on v-y-x space, and **(d)** Projection on x-v-z space

3. Global Behavior of the hyperchaotic System

The computation of the Diagrams of bifurcation could emphasize the range of periodic and non-periodic dynamics. The numerical analysis started with the initial conditions Ic (0.1, 0.1, 0.1, 0.1) showing extremely rich and intricate dynamics (fig. 2). Chaos bubbles and periodic windows indicate the complex envelops of the dynamics.



(a) (b) (c)

Fig.2. Diagrams of bifurcation for Y = 0.5 and the control parameter s ∈]0, 1.2].
(a) Bifurcation diagram of x, **(b)** Bifurcation diagram of z, and **(c)** Bifurcation diagram of v.

4. Concluding Remarks

The introduced 4D system, exhibiting discernable scrolls and wings, displays elegant hyperchaotic attractor with wing symmetries and magnified scrolls.

Exploration of four space dynamics could enhance our understanding of the *science of process* linking ordered and disordered sequences.

The new system could be suitable for digital signal encryption in the communication field when its variants provide a very large set of encryption keys.

Eventually, the system deserves further developments and extended analysis.

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May 26, 2015