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Abstract: This paper presents a decentralized solution to control a leader-follower formation of unicycle wheeled mobile robots allowing collision and obstacle avoidance. It is assumed that only positions and orientations of the robots can be measured and that each robot is influenced by an additive input disturbance. To guarantee the problem solution a supervisory control algorithm is designed and finite-time differentiators are used to estimate the leader velocity. The supervisor orchestrates three control algorithms responsible for the following, rendezvous and collision avoidance manoeuvres. All controls ensure a finite-time regulation for the robots orientation and a practically finite-time fulfilment of the required performance constraints.

1. INTRODUCTION

The control of multi-agent systems has been thoroughly investigated in the last few years (Lewis (2014)). One of the first work in the field can be retrieved in Reynolds (1987) in which a model to simulate consensus is derived starting from the behavior of a herd of animals; a model derived from (Reynolds (1987)) is used in (Vicsek (1995)) to realize a efficient graphic simulation for elements called *boids*. In the literature there are the strategies commonly used to tackle the problem of multi-agent cooperation in robotics: leader-follower techniques where the leader can be physical (Defoort (2008); Gamage (2007); Ji-Wook Kwon (2012); Ghommam (2013)) or virtual (Yongcan (2010); Leonard (2014)), behavior based techniques (Antonelli (2009); Jadbabaie (2003); Sepulchre (2007); Olfati-Saber (2007)) and virtual structure techniques (Mehrjerdi (2011); Rezaee (2014)). In the leader-follower structure, one agent has the role of the leader and all other robots follow it according to a predefined rule. This structure is very sensible to the leader fail of course, in addition the leader has no feedback from the followers. In the virtual structure all the agents have to track the reference of a virtual shape/leader following some geometry constraints, then this strategy has the advantage to be modeled easily but it is not easy to handle it in the case of a need of reconfiguration. In the behavioral structure each agent has the instructions to react to different conditions, consequently a general group behavior is delineated. For the leader-follower approach, usually, the strategies like feedback linearization, backstepping and first order sliding mode control are used to guarantee the convergence to a stable formation, and they all rely on the knowledge of the leader velocity. There exist some works that achieved leader-follower formation without knowing it (Defoort (2008); Ghommam (2013)). The aim of this work is to present an original leader-follower approach for a group of wheeled mobile robots (WMRs), characterized by kinematic non integrable constraints and subject to additive input disturbances. The goal is to move the leader and the followers to a destination point doing that without sharing the leader velocity, and being able to avoid collision between the agents and external obstacles. Despite the classical $l-l$ and $l-\phi$ schemes (Ghommam (2013)), where an angle and one or more distances were given to the agents to achieve the formation and avoid collisions, in the proposed solution just a desired distance to the leader is given to each agent, which means that the leader does not represent the most advanced robot of the formation but more a reference to follow. The goal is reached using the output stabilization and supervisory switching control frameworks: for each agent, except for the leader that is completely autonomous, three controllers are used to regulate two different outputs. The first controller is in charge of achieving the *rendezvous* part, that means to approach the agent to the leader. Once the rendezvous control achieved its task the second one, called the *following* control, assures the follower to maintain the heading and the velocity of the leader. These information, as specified previously, are not available and a finite-time observer is used to get the velocities of the leader. The third controller regulates a second output designed to avoid collisions between agents and obstacles. It is worth to remark that all three controls are robust with respect to additive input disturbances. A supervisor inspired by (Efimov (2006); Guerra (2013)) oversees the switches between three controls. The results are thus presented taking into account the notions of stability for switched systems (Liberzon (2003)) and output-to-state stability (Sontag

(1997)). In this work the communication topology issues are not taken into account, nevertheless the authors tried to achieve the results sharing the less information as possible, i.e. the robots positions and orientations.

2. PROBLEM STATEMENT

Let us consider a group of $N \in \mathbb{R}_+$ unicycle WMRs, in which the input is affected by additive disturbances:

$$\begin{aligned} \dot{x}_i &= \cos(\theta_i)(1 + d_{1,i})v_i \,, \\ \dot{y}_i &= \sin(\theta_i)(1 + d_{1,i})v_i \,, \\ \dot{\theta}_i &= (1 + d_{2,i})\omega_i \,, \end{aligned} \tag{1}$$

where $(x_i, y_i) \in \mathbb{R}^2$ define the Cartesian position of each robot, and $\theta_i \in [0, 2\pi)$ is the orientation of the robots with respect to the world reference frame, v_i and ω_i are the control inputs (the linear velocity and the angular velocity respectively). The additive disturbances on the inputs are unknown, but supposed to be bounded as: $-1 < d_{min} \leq d_{k,i} \leq d_{max}$, $k = 1, 2$, $i = 1, \dots, N$. The aim is to design control laws providing the rendezvous and leader-following (the robots must create a formation around the leader) with collision/obstacle avoidance capability for the specified group of unicycle WMRs. The proposed solution uses a supervisor which articulates the activation of three controls (designed below) depending on the needs. To achieve all the tasks just information about the leader state and other follower positions are used, this forces the use of a derivative estimator to retrieve the information about leader linear and angular velocity. The communication topology considered is static, it means it does not change during the mission. In order to define rendezvous, leader-following and obstacle avoidance goals, two outputs to regulate are defined using the accessible information:

$$z_{1i} = \sqrt{(x_L - x_i)^2 + (y_L - y_i)^2}, \tag{2}$$

$$z_{2i} = \max\left\{0, \frac{1 + \Lambda_i}{1 + d_{ci}} - 1\right\}, \ \Lambda_i > 0, \tag{3}$$

where z_{1i} being the distance from the leader (i.e. (x_L, y_L) is the leader Cartesian position), and z_{2i} is an output function of the distance $d_{ci} = \sqrt{(x_c - x_i)^2 + (y_c - y_i)^2}$ from a point with the coordinates (x_c, y_c) (defined in Section 3.4, and dependent on other robot positions). The first output z_{1i} is used to manage the switch between the rendezvous and following controllers, while the second output z_{2i} is the one designed to tackle the collision/obstacle avoidance part and design the dedicated controller. To proceed, several assumptions must be introduced. Firstly the maximum leader velocity must be smaller than the maximum followers velocities, i.e. $\omega_{L,max} \leq \omega_{i,max}$ and $v_{L,max} \leq v_{i,max}$, where the suffix *max* defines the maximum velocity. Then each follower enters the *following* mode when it reaches a distance δ_i from the leader, which can be different for different robots and bounded: $\delta_{min} < \delta_i < \delta_{max}$, where δ_{min} is tied to the collision avoidance minimum distance and δ_{max} is proportional to the number N of robots. There is a safe distance around each robot λ_i, which ensures absence of collisions. We will also assume that the linear velocity of the leader v_L is nonnegative (i.e. it is moving forward).

2.1 Theoretical Problem Formulation

The problem can be generalized as follows. Consider $N \in \mathbb{R}_+$ dynamical systems

$$\dot{q}_i = f(q_i, u_i, d_i), \ z_{1i} = h_1(q_i), \ z_{2i} = h_{2i}(q_i), \tag{4}$$

where $q_i \in \mathbb{R}^n$ is the state, $q = [q_1^T, \dots, q_N^T]^T$, $u_i \in \mathbb{R}^m$ is the control input and $d_i \in \mathbb{R}^m$ is a disturbance, with $d_i \in \Omega = \{d_i \in \mathcal{L}_m^\infty : ||d_i|| \leq D\}$ for some $D \in \mathbb{R}_+$ ($\mathcal{L}_m^\infty$ denotes the set of essentially bounded functions $d_i : \mathbb{R}_+ \to \mathbb{R}^m$). We want to regulate the outputs $z_{1i} \in \mathbb{R}^{p_1}$ and $z_{2i} \in \mathbb{R}^{p_2}$ assuming that the functions f, h_1 and h_{2i} are continuous and locally Lipschitz. It is needed to design the controls $u_i : \mathbb{R}^n \to \mathbb{R}^m$ guaranteeing that both outputs z_{1i} and z_{2i} will be kept under certain thresholds: i.e. for all $1 \leq i \leq N$ and all initial conditions $q_{i0} \in \mathbb{R}^n$, $q_0 = [q_{10}^T, \dots, q_{N0}^T]^T$, all $d_i \in \Omega$ and $t \geq t_0 \geq 0$:

$$|z_{1i}(t, q_0, d_i)| \leq \sigma_{1i}(\max(\Delta_i, |h_1(q_{i0})|)), \tag{5}$$

$$|z_{2i}(t, q_0, d_i)| \leq \sigma_{2i}(\max(\Upsilon_i, |h_{2i}(q_0)|)), \tag{6}$$

where the values of Δ_i and Υ_i are given (they are related with δ_i and λ_i), whereas σ_{ji}, $j = 1, 2$, are functions from the class $\mathcal{K}$ (continuous strictly increasing functions, $\sigma(0) = 0$). The first output, (5), must be smaller than $\sigma_{1i}(\Delta_i)$, in the case $|h_1(q_{0i})| > \Delta_i$ the trajectory should converge to a subset where $|h_1(q_i)| \leq \sigma_{1i}(\Delta_i)$. In the same way (6) must be smaller than $\sigma_{2i}(\Upsilon_i)$. In the case $|h_{2i}(q_0)| > \Upsilon_i$ the trajectory should converge to a subset where $|h_{2i}(q)| \leq \sigma_{2i}(\Upsilon_i)$. For the designed outputs, the restriction (5) implies that all robots should find their positions sufficiently close to the leader (on the distance $\sigma_{1i}(\Delta_i)$), and a safe distance should be preserved between the robots and obstacles ($\sigma_{2i}(\Upsilon_i)$).

3. THE SUPERVISORY CONTROL

To proceed the following sets have to be defined:

$$X_{\delta_i} = \{q_i \in \mathbb{R}^n : |h_1(q_i)| \leq \delta_i\} \,,$$
$$X_{\Delta_i} = \{q_i \in \mathbb{R}^n : |h_1(q_i)| \leq \Delta_i\}$$

$$X_{\lambda_i} = \left\{j \in \{1, \dots, N\} \setminus \{i\} : \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \leq \lambda_i\right\} \cup \left\{j \in \{1, \dots, N_o\} : \sqrt{(x_{jo} - x_i)^2 + (y_{jo} - y_i)^2} \leq \lambda_i\right\}$$

$$X_{\Lambda i} = \left\{j \in \{1, \dots, N\} \setminus \{i\} : \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \leq \Lambda_i\right\} \cup \left\{j \in \{1, \dots, N_o\} : \sqrt{(x_{jo} - x_i)^2 + (y_{jo} - y_i)^2} \leq \Lambda_i\right\}$$

where N_o is the number of (static) obstacles with the coordinates (x_{jo}, y_{jo}) (the number N_o could be considered finite without any loose of generality), $\{\lambda_i, \Lambda_i, \delta_i, \Delta_i\} \in \mathbb{R}_+^4$ are given values of parameter ($\lambda_i < \Lambda_i$ and $\delta_i < \Delta_i$), whose meaning will be explained below. Once defined those sets we can describe the switching sequence: the controller U_{1i} (following) is activated when $q_i \notin X_{\lambda_i}$ and the distance from the leader is less than the threshold δ_i and it is kept active while the output $h_{1i}(q_i)$ remains less than Δ_i (this is a safety measure to avoid continuous switching between U_{1i} and U_{2i} controllers); the controller U_{2i} (rendezvousing) is active when $q_i \notin X_{\lambda_i}$ and the output $h_{1i}(q_i)$ is greater than δ_i. The third controller U_{3i} (collision/obstacle avoiding) becomes active as soon as $q_i \in X_{\lambda_i}$ and it is kept active until $q_i \notin X_{\Lambda_i}$; also in this case a hysteresis is added to avoid continuous switching, or chattering, between the controllers (Liberzon (2003)). Therefore, the supervisory control law u_i for all $i = 1, \dots N$ can be summarized as follows

$$u_i(t) = U_{p_i(t)i}(q(t)),\ p_i : \mathbb{R}_+ \to \{1, 2, 3\} \tag{7}$$

with the initial conditions

$$t_0 = 0,\ p_i(t_0) = \begin{cases} 1 & \text{if } q_i(t_0) \in X_{\delta_i} \text{and } q(t_0) \notin X_{\lambda_i}, \\ 2 & \text{if } q_i(t_0) \notin X_{\delta_i} \text{and } q(t_0) \notin X_{\lambda_i}, \\ 3 & \text{if } q_i(t_0) \in X_{\lambda_i}, \end{cases}$$

and $p_i(t) = p_i(t_j)$ for $t \in [t_j\ t_{j+1})$, where

$$p_i(t_{j+1}) = \begin{cases} 1 & \text{if } q_i(t_{j+1}) \in X_{\delta_i} \text{and } q(t_{j+1}) \notin X_{\lambda_i} \\ 2 & \text{if } q_i(t_{j+1}) \notin X_{\delta_i} \text{and } q(t_{j+1}) \notin X_{\lambda_i} \\ 3 & \text{if } q_i(t_{j+1}) \in X_{\lambda_i} \end{cases}, \tag{8}$$

with t_j is the generic switching instant defined as follows:

$$t_{j+1} = \arg\ \inf_{t \geq t_j} \begin{cases} q(t) \in X_{\lambda_i} & \text{if } p_i(t_j) \in \{1, 2\}, \\ q(t) \notin X_{\Lambda_i} \text{ and } q_i(t) \in X_{\delta_i} & \text{if } p_i(t_j) \in \{2, 3\}, \\ q(t) \notin X_{\Lambda_i} \text{ and } q_i(t) \notin X_{\delta_i} & \text{if } p_i(t_j) \in \{3\}, \\ q(t) \notin X_{\Lambda_i} \text{ and } q_i(t) \notin X_{\Delta_i} & \text{if } p_i(t_j) \in \{1\}. \end{cases}$$

Now let us define the estimators and all U_{ki}, $k = 1, 2, 3$.

3.1 The Homogeneous Estimator

As specified in Section 2 each follower can access just the position and the orientation of the leader robot; to gather information about the leader velocities v_L and ω_L, linear and angular, an observer is necessary. We will assume that the leader has the following dynamics:

$$\begin{aligned} \dot{x}_L &= \cos(\theta_L) v_L\,, \\ \dot{y}_L &= \sin(\theta_L) v_L\,, \\ \dot{\theta}_L &= \omega_L\,, \end{aligned}$$

where the terms v_L and ω_L may contain perturbations with respect to some reference controls, but for the cooperation objective we need to estimate not the reference controls applied to the leader, but its real inputs v_L and ω_L. If $\dot{x}_L$, $\dot{y}_L$ and $\dot{\theta}_L$ would be available for all followers, then

$$v_L = \sqrt{\dot{x}_L^2 + \dot{y}_L^2},\ \omega_L = \dot{\theta}_L.$$

Thus since x_L, y_L and θ_L are only available, then the estimates of the derivatives of these variables have to be calculated. To estimate the derivatives the following homogeneous finite-time differentiator (Perruquetti (2008)) has been adopted:

$$\begin{aligned} \dot{\xi}_1 &= -\alpha |e|^{0.75} \text{sign}(e) + \xi_2, \\ \dot{\xi}_2 &= -\beta |e|^{0.5} \text{sign}(e),\ \hat{f} = \xi_2, \\ e &= \xi_1 - f, \end{aligned}$$

where ξ_1, ξ_2 are the states of the differentiator, f is the measured signal to be differentiated (i.e x_L, y_L or θ_L), $\hat{\dot{f}} = \xi_2$ is the estimate of derivative we are looking for (i.e. $\hat{\dot{x}}_L$, $\hat{\dot{y}}_L$ and $\hat{\dot{\theta}}_L$). The use of this kind of observer helps also to filter the disturbances and to have a better estimation of both velocities. It has been proven in Perruquetti (2008) that $\max\{|\eta_v|, |\eta_\omega|\} \leq \bar{\eta}$ where $\eta_v = v_L - \hat{v}_L$, $\eta_\omega = \omega_L - \hat{\omega}_L$ are the estimation errors and

$$\hat{v}_L = \sqrt{\hat{\dot{x}}_L^2 + \hat{\dot{y}}_L^2},\ \hat{\omega}_L = \hat{\dot{\theta}}_L.$$

Therefore, in all calculations below the estimates $\hat{v}_L$, $\hat{\omega}_L$ can be used assuming presence of bounded errors η_v and η_ω.

3.2 Following

The *Following* control $U_{1i}=$ (v_i, ω_i), which should be activated when the follower reaches the circle of radius δ_i around the leader, it forces the orientation of the ith robot to track the leader's one. Defining a deviation angle $\epsilon_f = \theta_L - \theta_i$, the dynamics of this error can be easily derived from the WMR model (1) where θ_L is the leader heading:

$$\dot{\epsilon}_f = \dot{\theta}_L - \dot{\theta}_i = \hat{\omega}_L + \eta_\omega - \omega_i(1 + d_{2i}). \tag{9}$$

Select a Lyapunov function $V_f = \frac{1}{2}\epsilon_f^2$ with $\dot{V}_f = \epsilon_f[\hat{\omega}_L + \eta_\omega - \omega_i(1 + d_{2i})]$, then the following control can be proposed:

$$\omega_i = \frac{\hat{\omega}_L + [K_f|\epsilon_f| + \rho_f + \rho_0]\text{sign}(\epsilon_f)}{1 - d_{min}}, \tag{10}$$

where K_f and ρ_f are the design parameters, $\rho_0 = |\hat{\omega}_L| \frac{d_{max} - d_{min}}{1 - d_{min}} + \bar{\eta}$. An upper estimate for ϵ_f can be evaluated:

$$|\epsilon_f(t)| \leq \begin{cases} \left(|\epsilon_0| + \dfrac{\rho_f}{K_f}\right) e^{K_f(t_0 - t)} - \dfrac{\rho_f}{K_f} & \text{if } t < t_0 + \bar{T}^f_{\epsilon_0}, \\ 0 & \text{if } t \geq t_0 + \bar{T}^f_{\epsilon_0}, \end{cases} \tag{11}$$

where t_0 is the instant in which the control is switched on and $\epsilon_0 = \epsilon(t_0)$ is the value of the angle error at t_0 with $\epsilon_0 \in [-\pi, \pi]$, $\bar{T}^f_{\epsilon_0} = -K_f^{-1} \ln \frac{\rho_f}{K_f|\epsilon_0| + \rho_f}$. Thus the *following* control stabilizes the orientation of the robot in a finite time, and this time has the upper bound $\bar{T}_f = \sup_{\epsilon_0 \in [-\pi, \pi]} \bar{T}^f_{\epsilon_0} = K_f^{-1} \ln \left[1 + \frac{K_f \pi}{\rho_f}\right]$. The velocity part of the *following* control v_i cannot be a simple estimation of the leader velocity $\hat{v}_L$ because of the disturbances d_{1i} acting on the follower. To explain the idea of the control used in this case, consider the Lyapunov function $W_f = \frac{1}{2} z_{1i}^2$ with $\dot{W}_f = -z_{1i}(1 + d_{1i}) v_i \cos(\alpha_i - \theta_i) + z_{1i}(\hat{v}_L + \eta_v) \cos(\alpha_i - \theta_L)$, where $\alpha_i = atan\left(\frac{y_L - y_i}{x_L - x_i}\right)$ is the angle between the leader and the follower robots. For the sake of simplicity hereafter, denote $C_\alpha = \cos(\alpha_i - \theta_L)$ then the chosen linear velocity control can be proposed:

$$v_i = \begin{cases} \zeta(z_{1i}) \text{sign}\left(C_\alpha\right) \dfrac{\bar{\eta} + d_{max} \hat{v}_L}{1 - d_{min}} + \hat{v}_L & \text{if } \epsilon_f = 0, \\ 0 & \text{otherwise}, \end{cases}$$

where

$$\zeta(z_1) = \begin{cases} 0 & z_1 < \delta \\ 1 & z_1 > \tilde{\delta} \\ \dfrac{z_1 - \delta}{\tilde{\delta} - \delta} & \delta < z_1 < \tilde{\delta} \end{cases}$$

with $\delta < \tilde{\delta}$ are the design parameters. Substitution of this control gives:

$$\dot{W}_f < 0 \quad \forall z_{1i} > \tilde{\delta}$$

if $\epsilon_f = 0$. While the error ϵ_f goes to zero in the finite time $\bar{T}_f$, then the distance z_{1i} may increase its value by $v_{L,max} \bar{T}_f$. Therefore, the control assures the follower to stay in the zone $z_{1i}(t) \leq \Delta = \delta_i + v_{L,max} \bar{T}_f$ (the control is activated at the instant when $z_{1i} \le \delta_i$). The *following* control can be summarized as follows:

$$U_{1i} = \begin{cases} v_i = \begin{cases} \zeta(z_{1i})\text{sign}\left(C_\alpha\right) \dfrac{\bar{\eta} + d_{max}\hat{v}_L}{1 - d_{min}} + \hat{v}_L & \text{if } \epsilon_f = 0, \\ 0 & \text{otherwise,} \end{cases} \\ \omega_i = \dfrac{\hat{\omega}_L + [K_f|\epsilon_f| + \rho_f + \rho_0]\text{sign}(\epsilon_f)}{1 - d_{min}}. \end{cases} \tag{12}$$

We have proven the following result.

Lemma 1. The controller (12) for the system (1) provides an uniform finite-time stabilization for the variable $\epsilon_f = \theta_L - \theta_i$ (with an upper estimate (10)) and practical output stability for the output z_{1i}.

3.3 Rendezvous

The *Rendezvous* control, $U_{2i} = (v_i, \omega_i)$, assures the robot to approach the leader. Define a desired orientation angle as $\epsilon_{rdv} = \theta_i - \alpha_i$, where $\alpha_i = atan\left(\frac{y_L - y_i}{x_L - x_i}\right)$. Consider a Lyapunov function $W_{rdv} = \frac{1}{2}z_{1i}^2$, whose derivative admits the estimate:

$$\dot{W}_{rdv} = z_{1i}\{C_\alpha(\hat{v}_L + \eta_v) - \cos(\epsilon_{rdv})v_i(1 + d_{1i})\}.$$

To preserve the semi-definitiveness of the function $\dot{W}_{rdv}$ the proposed control v_i has the form:

$$v_i = \begin{cases} \dfrac{\cos(\epsilon_{rdv})[C_\alpha(\hat{v}_L + \eta_v) + \rho_{rdv}]}{1 - d_{min}} & \text{if } |\epsilon_{rdv}| \le \kappa\dfrac{\pi}{2}, \\ 0 & \text{otherwise,} \end{cases} \tag{13}$$

where $0 < \kappa < 1$, $\rho_{rdv} > \frac{\hat{v}_L + \bar{\eta}}{\cos^2(\kappa\frac{\pi}{2})} + \rho_1$ and $\rho_1 > 0$ are design parameters.Then we define the Lyapunov function $V_{rdv} = \frac{1}{2}\epsilon_{rdv}^2$ and evaluate its derivative:

$$\dot{V}_{rdv} = \epsilon_{rdv}\left\{\omega_i - \frac{(x_L - x_i)(\dot{y}_L - \dot{y}_i) - (y_L - y_i)(\dot{x}_L - \dot{x}_i)}{z_{1i}^2}\right\},$$

which brings to the following expression for ω_i:

$$\omega_i = -\frac{S_\alpha\hat{v}_L + |S_\alpha|\bar{\eta} + \sin(\epsilon_{rdv})[1 + d_{max}]v_i}{z_{1i}} - \rho_{rdv}\text{sign}(\epsilon_{rdv}) - \left(K_{rdv} + \frac{[d_{max} - d_{min}]|v_i|}{z_{1i}}\right)\epsilon_{rdv}, \tag{14}$$

where $K_{rdv} > 0$ and $\rho_{rdv} > 0$ are design parameters. Applying this control, the Lyapunov function derivative $\dot{V}_{rdv}$ can be rewritten as follows:

$$\dot{V}_{rdv} \le -2K_{rdv}V_{rdv} - \rho_{rdv}\sqrt{2V_{rdv}}.$$

Therefore, the proposed control stabilizes the variable ϵ_{rdv} heading the robot toward the leader in a finite time, and as for the previous controller the time of orientation can be evaluated referring to the ϵ_{rdv} variable dynamics. The estimation for ϵ_{rdv} is

$$|\epsilon_{rdv}(t)| \le \begin{cases} \left(|\epsilon_0| + \dfrac{\rho_{rdv}}{K_{rdv}}\right)e^{K_{rdv}(t_0 - t)} - \dfrac{\rho_{rdv}}{K_{rdv}} & \text{if } t < t_0 + \bar{T}_{\epsilon_0}^{rdv}, \\ 0 & \text{if } t \ge t_0 + \bar{T}_{\epsilon_0}^{rdv}, \end{cases} \tag{15}$$

where t_0 is the instant in which the control is switched on and $\epsilon_0 = \epsilon(t_0)$ is the value of the angle error at $t_0$with $\epsilon_0 \in [-\pi, \pi]$, $\bar{T}_{\epsilon_0}^{rdv} = -K_{rdv}^{-1}\ln\frac{\rho_{rdv}}{K_{rdv}|\epsilon_0| + \rho_{rdv}}$. Thus the the upper bound of the orientation time is $\bar{T}_{rdv} = \sup_{\epsilon_0 \in [-\pi,\pi]}\bar{T}_{\epsilon_0}^{rdv} = K_{rdv}^{-1}\ln\left[1 + \frac{K_{rdv}\pi}{\rho_{rdv}}\right]$. Then for any $0 < \kappa < 1$ and any initial orientation ϵ_0, the time of reaching the zone where $|\epsilon_{rdv}| \le \kappa\frac{\pi}{2}$ is less than $\bar{T}_{rdv}$. The controller can be resumed as:

$$U_{2i} = \begin{cases} v_i = \begin{cases} \dfrac{\cos(\epsilon_{rdv})[C_\alpha(\hat{v}_L + \eta_v) + \rho_{rdv}]}{1 - d_{min}} & \text{if } |\epsilon_{rdv}| \le \kappa\dfrac{\pi}{2}, \\ 0 & \text{otherwise,} \end{cases} \\ \omega_i = -\dfrac{S_\alpha\hat{v}_L + |S_\alpha|\bar{\eta} + \sin(\epsilon_{rdv})[1 + d_{max}]v_i}{z_{1i}} \\ -\rho_{rdv}\text{sign}(\epsilon_{rdv}) - \left(K_{rdv} + \dfrac{[d_{max} - d_{min}]|v_i|}{z_{1i}}\right)\epsilon_{rdv}, \end{cases} \tag{16}$$

From the inequality $\dot{W}_{rdv} \le -\rho_1\sqrt{2W_{rdv}}$, for the case $|\epsilon_{rdv}| \le \kappa\frac{\pi}{2}$, it follows that for $t \ge t_0 + \bar{T}_{rdv}$ the distance z_{1i} is uniformly decreasing to zero in a finite time. Then the distance z_{1i} may increase on the value $v_{L,max}\bar{T}_{rdv}$ during the orientation phase, and after $t_0 \le t_1 \le t_0 + \bar{T}_{rdv}$ when $|\epsilon_{rdv}(t_1)| \le \kappa\frac{\pi}{2}$ for the first time ($z_{1i}(t_1) \le z_{1i}(t_0) + v_{L,max}\bar{T}_{rdv}$), then:

$$z_{1i}(t) \le \begin{cases} z_{1i}(t_0) + v_{L,max}\bar{T}_{rdv} & \text{if } t_0 \le t \le t_1, \\ z_{1i}(t_1) - \rho_1(t - t_1) & \text{if } t_1 \le t \le t_1 + T_1, \\ 0 & \text{if } t \ge t_1 + T_1, \end{cases} \tag{17}$$

$$T_1 = \frac{z_{1i}(t_1)}{\rho_1} \le \frac{z_{1i}(t_0) + v_{L,max}\bar{T}_{rdv}}{\rho_1}.$$

In this case, to achieve the task the robot has to reach a distance δ_i from the leader. The necessary time to travel till this distance is $\bar{t}_{rdv} = \frac{z_{1i}(t_1) - \delta_i}{\rho_1}$. Considering the worst case scenario the time in which the control (16) will achieve his task would be $T_{rdv} = \bar{t}_{rdv} + \bar{T}_{rdv}$. Thus the following claim has been proven.

Lemma 2. The control (16) provides for the system (1): 1. Uniform finite-time stability with respect to the variable ϵ_{rdv} (see (15)); 2. Uniform boundedness and finite-time convergence with respect to the variable z_{1i} (see (17)); 3. $\exists T_{rdv} \in \mathbb{R}_+$ such that $z_{1i}(T_{rdv}) \le \delta_i$.

3.4 Collision/Obstacle Avoidance

The *Collision/Obstacle Avoidance* control becomes active when either the leader or other robots of the group (or an external obstacle, or all of them) enter the safety zone around a robot, which is specified by the circle of radius λ_i. This control is kept active until all the robots exit a bigger circle of radius Λ_i; the annulus delimited the two radii can be considered as a hysteresis to avoid Zeno/chattering phenomena. To achieve the task, an effective strategy has been designed. Firstly, each robot who finds itself in a collision avoidance condition, evaluates a point (x_c, y_c) as follows:

$$x_c = \frac{1}{M}\sum_{j=1}^{M}x_j,\ y_c = \frac{1}{M}\sum_{j=1}^{M}y_j,$$

where $(x_c, y_c) \in X_{\lambda_i}$ is the medium point among all robots/obstacles participating in the collision avoidance maneuver, with X_{λ_i}defined as in Section 3, $0 < M \le N + N_o$ is the number of robots and obstacles. The point (x_c, y_c) represents the point from which the robot has to go away to exit the collision avoidance conditions. In order to maximize the distance d_{ci} from the point (x_c, y_c) for all participating robots, the following Lyapunov function is introduced $W_{ca}(x) = z_2 = \max\left\{0, \frac{1+\Lambda_i}{1+d_{ci}} - 1\right\}$. Let $\bar{\gamma}_i = atan\left(\frac{y_c - y_i}{x_c - x_i}\right)$ be the angle between the robot and the point (x_c, y_c). The derivative $\dot{W}_{ca} = 0$ if $d_{ci} > \Lambda_i$ (the avoiding is performed), and for $d_{ci} \le \Lambda_i$ it has the form:

$$\dot{W}_{ca} = \frac{v_i \cos(\theta_i - \bar{\gamma}_i)\{1 + d_{1i}\} - \frac{1}{M}\sum_{j=1}^{M} v_j \cos(\theta_j - \bar{\gamma}_i)\{1 + d_{1j}\}}{(\Lambda + 1)^{-1}(1 + d_{ci})^2}. \quad (18)$$

Let us introduce the desired orientation that the robot has to reach to go away from the point (x_c, y_c). It is given by the angle $\gamma_i = \theta_i - (\bar{\gamma}_i + \pi)$, where π is the natural choice to get away from that point. The proposed controller has the form:

$$U_{3i} = \begin{cases} v_i = \begin{cases} v_{max} & \text{if } |\gamma_i| \le k\pi, \\ 0 & \text{otherwise}, \end{cases} \\ \omega_i = \dfrac{-[\rho_{ca} + \rho_2]\text{sign}(\gamma_i)}{1 - d_{min}} \end{cases}, \quad (19)$$

where $\rho_{ca} \ge \frac{v_{max}(1+d_{max})}{d_{ci}}$ and $\rho_2 > 0$. Substituting the control (19) in the derivative of the Lyapunov function $V_{ca} = \frac{1}{2}\gamma_i^2$ we obtain:

$$\dot{V}_{ca} \le -\rho_2\sqrt{2V_{ca}},$$

which gives us a finite time convergence on the variable $\gamma_i(t)$. This time can be evaluated from the estimation of $\gamma_i(t)$:

$$|\gamma_i(t)| \le |\gamma_0| - \rho_2(t - t_0), \quad (20)$$

where $\gamma_0 \in [-\pi, \pi]$ is the initial value of γ_i at the instant t_0 when the collision avoidance control has been switched on. Thus the time, when the condition $|\gamma_i| \le k\pi$ can be verified, is $t_{ca}^{\gamma_0} = \frac{\max\{0, |\gamma_0| - k\pi\}}{\rho_2}$, and for the worst case scenario $t_{ca} = \sup_{\gamma_0 \in [-\pi,\pi]} t_{ca}^{\gamma_0} = \frac{(1-k)\pi}{\rho_2}$. Following this result and (19), the value of λ_i has to satisfy $\lambda_i > t_{ca}v_{max}$ (the maximal movement velocity for the point (x_c, y_c) is v_{max}). Denote $C_{k\pi} = \cos(k\pi)$, then (18) with the control (19) satisfies the estimate:

$$\dot{W}_{ca} \le v_{max}\frac{\Lambda + 1}{(1 + d_{ci})^2}[-\frac{M-1}{M}C_{k\pi}\{1 - d_{min}\} + \frac{M-2}{M}\{1 + d_{max}\}],$$

where the upper bound $M - 2$ on the number of terms in the sum appears since one term leaves for $j = i$ and at least one (in the worst case) has a negative value of $\cos(\theta_j - \bar{\gamma}_i)$. If we can assure that the quantity in the square brackets is negative, then we can assure decreasing W_{ca}. It can be shown that there exist sufficiently small values d_{min}, d_{max} and k close to 1 such that this term is negative (it is easy to see that it is true for $d_{min} = d_{max} = 0$ and $C_{k\pi} > \frac{M-2}{M-1}$, next it will be true by continuity for sufficiently small values of d_{min}, d_{max} and some k). To conclude, z_{2i} may increase during the orientation phase t_{ca} (due to constraint $\lambda_i > t_{ca}v_{max}$ a collision is not possible), but next is decreasing to zero, thus this distance is bounded and there is a finite time $T_{ca} > 0$ such that $z_{2i}(t)$ becomes sufficiently small for $t \ge t_0 + T_{ca}$ and $q(t_0 + T_{ca}) \notin X_{\Lambda_i}$, thus the collision avoiding is finished.

Lemma 3. The system (1) with the control (19) admits the properties: 1. Uniform finite-time stability with respect to the variable γ_i (see the estimate (20)); 2. $\exists d_{min}, d_{max}$ and k such that the variable z_{2i} is bounded and there exists $T_{ca} > 0$ such that $q(t_0 + T_{ca}) \notin X_{\Lambda_i}$.

Remark 4. A static obstacle is considered as a robot, which has zero linear and angular velocities.

3.5 Supervisory control

Summarizing the results obtained so far and using the results of Efimov (2006); Guerra (2013), the following statement can be obtained.

Conjecture 5. The system (4) with the supervisor (8) and controls (7) is forward complete and for all $q_0 \in \mathbb{R}^n$, $d \in \Omega$

$$|z_{1i}(t, q_{0i}, d_i)| \le \max(\Delta_i, |h_1(q_{0i})|),$$
$$|z_{2i}(t, q_{0i}, d_i)| \le \max(\Upsilon_i, |h_{2i}(q_{0i})|)$$

for $t \ge 0$, $\Upsilon_i = \frac{1+\Lambda_i}{1+\lambda_i - t_{ca}v_{max}} - 1$.

4. SIMULATIONS

In the simulations the number of WMRs is $N = 4$, with sampling time $t_s = 0.01$ $[sec]$; the maximum velocity for the leader is set to $v_{L,max} = 0.5$ while the maximum velocity for the followers is $v_{i,max} = 2$. The disturbances have form $d_i = \chi\sin(t) + 0.1 * rand$ where $rand$ is a pseudo-random values drawn from the standard uniform distribution on the open interval $(0, 1)$ with $i \in \{1, 2\}$ and $|\chi| \le 0.5$. The *following* controller has $\delta_i \in \{x \in \mathbb{R} : 0.7 < x < 0.9 + 0.1N\}$, $K_f = 5$ and $\rho_f = 0.01$; for the *rendezvous* control the values are: $\rho_1 = 2$, $\rho_{rdv} = 0.1$. For the *obstacle avoidance* $\rho_2 = 0.1$. Fig. 1 and Fig. 2 represent how the agents behave when the presented strategy is implemented. The leader follows a predefined path, while the followers are placed randomly with random orientation at $t = 0$. Indeed, each agent activates the *following* controller and the formation movement is accomplished. The distance of each robot from the leader is shown in Fig. 2 and the straight horizontal lines of the same colors represent the corresponding values of Δ_i while the black line represents the limit distance beyond which the collision/obstacle avoidance is activated. When the collision avoidance control is not active, the followers reach the *following* mode and remain in it if no external perturbation are applied (as an obstacle could be). If necessary, at the end of the *collision avoidance* maneuver, they switch back to the *rendezvous* control to reach again the minimum distance, which is necessary to switch back in the *following* mode. Thoroughly analyzing Fig. 2 though, it can be noticed that the activation of the collision avoidance controller not always forces the WMR to switch back to the *rendez-vouz* one once the maneuver is accomplished switching back directly to the *following* one.

5. CONCLUSION

A switching-based solution has been presented to the leader-follower formation problem for a group of WMR in the presence of additive input disturbances with obstacle/collision avoidance. A supervisor, able to regulate two different outputs, orchestrates three different controls to regroup the robots (*rendezvous* controller), make them follow the leader (*following* controller) and avoiding collisions/obstacles when necessary during the motion. It is worth to remark that no assumption has been made about *a priori* knowledge of the positions of obstacles or leader velocities. It has been formally shown that each

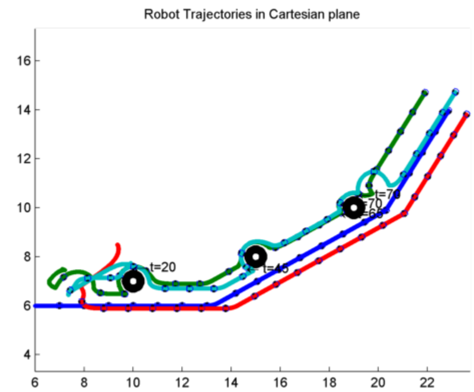

Fig. 1. The path followed in an environment with obstacles, where the blue one is the leader WMR

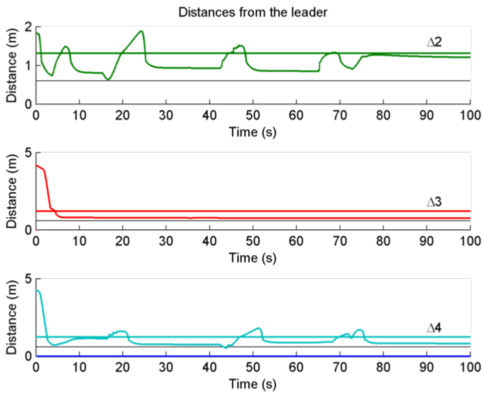

Fig. 2. Distances of each agent from the leader and relative Δ_i values

control robustly achieves the task it is designed for and, in addition, the robots orientations are provided in a finite-time. Simulations are performed for a group of 4 WMRs to prove the effectiveness of the strategy.Future research will consider the case in which the leader takes into account information from the followers to avoid the increasing of the distances during the *rendez-vous* and the *collision avoidance* maneuvers.

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