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# Percolation on a non-homogeneous Poisson blob process

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We present the main results of a study for the existence of vacant and occupied unbounded connected components in a non-homogeneous Poisson blob process. The method used in the proofs is a multi-scale percolation comparison.

**Keywords:** Poisson blob model, continuum percolation, phase transition, multi-scale percolation

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## 1 Introduction

One of the most well known examples of phenomena that introduces and motivates the study of continuum percolation is the process of the ground getting wet during a period of rain. At each point hit by a raindrop, one sees a circular wet patch. Right after the rain begins to fall what one sees is a small wet region inside a large dry region. At some instant, so many raindrops have hit the ground that the situation changes from that to a small dry region inside a large wet region. Typically, the parameter in which there is a phase transition behaviour is the density of the raindrops.

Continuum percolation models in which each point of a two-dimensional homogeneous Poisson point process is the centre of a disk of given (or random) radius  $r$ , have been extensively studied. In this note we present phase transition results for a sequence of Poisson point process which defines Poisson Boolean models and whose rates depend on the past. In order to prove our results we rely on a multi-scale percolation structure. General reference for percolation and continuum percolation are the books of Grimmett [2] and Meester and Roy [3]. A nice example of the use of multi-scale percolation technique can be found in Fontes *et al* [1].

## 2 Model and phase transition results

Let  $\beta > 0$  be fixed number. Define  $A_0 = \emptyset$ . Having defined the sets  $A_0, A_1, \dots, A_n$ , define the process  $X_{n+1}$  as the non-homogeneous Poisson point process with intensity function given by:

$$f_{n+1}(x) = \exp(-\beta|B(x, n+1) \setminus \cup_{k=0}^n A_k|) \quad (1)$$

where  $B(a, r)$  is the square of length  $r$  having centre at  $a$  and  $|C|$  is the area (Lebesgue measure) of the set  $C$ .

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Let  $\{x_i^{(n+1)} : i \geq 1\}$  be the set of points from the process  $X_{n+1}$ . Define the set  $A_{n+1} = \cup_{i=1}^{\infty} B(x_i^{(n+1)}, n+1)$  as the random set covered by the boxes from the process  $X_{n+1}$ . Define the total covered set  $A_{\infty} = \cup_{n=0}^{\infty} A_n$ .

The fundamental question in continuum percolation theory is about the existence of unbounded connected components. That is why we ask the following questions about the random set  $A_{\infty}$  and its complement, the set  $A_{\infty}^c$ . Let  $A$  be the component of  $A_{\infty}$  which contains the origin. If the origin is not contained in  $A_{\infty}$ , this is the empty set. Define

$$\theta(\beta) = \mathbb{P}_{\beta}(A \text{ is unbounded}). \quad (2)$$

It is clear that  $\theta(\beta)$  is a decreasing function of  $\beta$ . Hence define the critical parameter  $\beta_c$  as follows:

$$\beta_c = \sup\{\beta > 0 : \theta(\beta) > 0\}. \quad (3)$$

The following result holds

**Theorem 1.**

$$0 < \beta_c < \infty.$$

Similar questions can also be asked the complement set of  $A_{\infty}$ . Define  $C$  as the component of  $(A_{\infty})^c$  which contain the origin. Define the vacant percolation probability as

$$\theta^*(\beta) = \mathbb{P}_{\beta}(C \text{ is unbounded}). \quad (4)$$

In this case, we have that  $\theta^*(\beta)$  is an increasing function of  $\beta$ . Hence define the critical parameter  $\beta_c^*$  as follows:

$$\beta_c^* = \inf\{\beta > 0 : \theta^*(\beta) > 0\}. \quad (5)$$

We also prove the following theorem

**Theorem 2.**

$$0 < \beta_c^* < \infty.$$

It is clear that  $X_1$  is actually an homogeneous Poisson process with intensity  $\exp(-\beta|B(0, 1)|)$ . Thus,  $A_1$  will contain the covered set of a Poisson Boolean model with radius random variable being degenerate at  $1/2$  and intensity  $\exp(-\beta|B(0, 1)|)$ . Thus, if  $\exp(-\beta|B(0, 1)|) > \lambda_c$ , the probability that the origin is contained in an unbounded component of  $A_1$  is positive, where  $\lambda_c$  is the critical intensity of the Poisson Boolean model with radius being degenerate at  $1/2$ . Therefore, we have  $\theta(\beta) > 0$  for this  $\beta$ . Hence, we have that  $\beta_c > 0$ . A similar argument also holds for  $\beta_c^*$  and we can easily show that,  $\beta_c^* > 0$ .

This is an announcement of results from a joint work with P. Ferrari, L. Fontes, S. Popov and A. Sarkar. The proofs rely on a multi-scale comparison argument to prove that the probability of certain events related to the existence of an unbounded connected component is exponentially close to 1 for large values of  $\beta$ .

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