

Finding a Strong Stable Set or a Meyniel Obstruction in any Graph

Kathie Cameron, Jack Edmonds

► **To cite this version:**

Kathie Cameron, Jack Edmonds. Finding a Strong Stable Set or a Meyniel Obstruction in any Graph. Stefan Felsner. 2005 European Conference on Combinatorics, Graph Theory and Applications (EuroComb '05), 2005, Berlin, Germany. Discrete Mathematics and Theoretical Computer Science, DMTCS Proceedings vol. AE, European Conference on Combinatorics, Graph Theory and Applications (EuroComb '05), pp.203-206, 2005, DMTCS Proceedings. <hal-01184368>

HAL Id: hal-01184368

<https://hal.inria.fr/hal-01184368>

Submitted on 14 Aug 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Finding a Strong Stable Set or a Meyniel Obstruction in any Graph

Kathie Cameron¹ and Jack Edmonds²

¹ *Math Department, Wilfrid University, Waterloo, Ontario, Canada N2L 3C5, kcameron@wlu.ca*

² *jackedmonds@rogers.com*

A strong stable set in a graph G is a stable set that contains a vertex of every maximal clique of G . A Meyniel obstruction is an odd circuit with at least five vertices and at most one chord. Given a graph G and a vertex v of G , we give a polytime algorithm to find either a strong stable set containing v or a Meyniel obstruction in G . This can then be used to find in any graph, a clique and colouring of the same size or a Meyniel obstruction.

Keywords: stable set, independent set, graph colouring, Meyniel graph, perfect graph

A *Meyniel graph* is a graph which does not contain an odd circuit with at least five vertices and at most one chord. Such a circuit is called a *Meyniel obstruction*. Meyniel [6] proved that Meyniel graphs are perfect. Meyniel's theorem can be stated in the following way.

Theorem 1 (Meyniel's Theorem) *For any graph G , either G contains a Meyniel obstruction, or G contains a clique and colouring of the same size, or both.*

We give a polytime algorithm to find, in any graph G , some instance of what Meyniel's Theorem says exists.

Burlet and Fonlupt [1] and Roussel and Rusu [7] gave polytime algorithms for recognizing whether or not a graph is a Meyniel graph. In the case that the graph is Meyniel, they do not find a clique and colouring of the same size. Our algorithm is incomparable with Meyniel graph recognition. It may give a clique and colouring the same size in a non-Meyniel graph without recognizing that the graph is non-Meyniel.

Algorithms for finding a minimum colouring of a Meyniel graph have been given by Hoàng [4], Hertz [3], Roussel and Rusu [8], and Lévêque and Maffray [5]. Any polytime algorithm for finding a minimum colouring in a perfect graph, in particular a Meyniel graph, can be used to find in polytime a clique in the graph which is the same size as the colouring [2, 4]. However, none of these algorithms provide a way to find in any graph an instance of what Meyniel's Theorem asserts to exist. All of them, as well as ours, can be used to find a clique and colouring the same size in any graph which does not have a Meyniel obstruction. However our algorithm can also be described as finding a Meyniel obstruction in any graph which does not have a clique and colouring the same size.

A *stable set* in a graph G is a set of vertices, no two of which are joined by an edge of G . A *strong stable set* is a stable set that contains a vertex of every maximal clique. (By maximal, we mean maximal

with respect to inclusion, not largest.) It is easy to see that a polytime algorithm for finding a strong stable set in a graph can be applied repeatedly to find a colouring of a graph, and it is also then easy to construct a clique of the same size as the colouring.

Theorem 2 (Hoàng [4]) *For any graph G and vertex w of G , either G contains a strong stable set containing w , or G contains a Meyniel obstruction, or both.*

We give a polytime algorithm to find an instance of what Theorem 2 says exists. We now describe the ideas we use for developing this algorithm. As usual, we use P_4 to denote the path with four vertices.

A *nice set* S is a maximal stable set linearly ordered so there is no induced P_4 between any vertex u of S and the pseudonode obtained by identifying all vertices of S which precede u .

Theorem 3 *Every nice set is a strong stable set.*

Note that the definition of nice set is an NP description, but the definition of strong stable set is not.

Algorithm.

Input: Graph G and vertex w of G .

Output: Nice stable set of G containing w or a Meyniel obstruction.

Let $w = u_1$.

Suppose u_1, u_2, \dots, u_k have been chosen. If every vertex of $V(G) - \{u_1, u_2, \dots, u_k\}$ is adjacent to one of u_1, u_2, \dots, u_k , then the chosen vertices form a nice set. Otherwise, choose u_{k+1} to be a vertex of $V(G) - \{u_1, u_2, \dots, u_k\}$ not adjacent to any of u_1, u_2, \dots, u_k and such that it has the largest number of common neighbours with the pseudonode $v(u_1, u_2, \dots, u_k)$ obtained by identifying u_1, u_2, \dots, u_k . If there is a P_4 from $v(u_1, u_2, \dots, u_k)$ to u_{k+1} , then G contains a Meyniel obstruction. To find this circuit, we use a “pseudonode expansion algorithm”, which we cannot describe here. The simple lemmas below help us to find the circuit.

A chord of a circuit C is called *short* if it joins two vertices at distance 2 in C (i.e., if it creates a triangle with C). Two short chords of C are *overlapping* if one is ac and the other is bd , where a, b, c, d are consecutive vertices on C .

Lemma 1 *In an odd circuit of size at least 7 with two chords, either there is an odd circuit of size at least 5 with at most one chord, or the two chords are overlapping short chords..*

Lemma 2 *In an odd circuit of size at least 5 with all chords hitting the same vertex h and at least one of these possible chords missing, there is an odd circuit of size at least 5 with at most one chord, and the chord is short and hits h .*

References

- [1] M. Burlet and J. Fonlupt, Polynomial algorithm to recognize a Meyniel graph, *Topics on Perfect Graphs*, 225-252, North-Holland Math. Stud., 88, North-Holland, Amsterdam, 1984.
- [2] M. Grötschel, L. Lovász and A. Schrijver, Polynomial algorithms for perfect graphs, *Topics on Perfect Graphs*, 325-356, North-Holland Math. Stud., 88, North-Holland, Amsterdam, 1984.

- [3] A. Hertz, A fast algorithm for coloring Meyniel graphs. *J. Combin. Theory Ser. B* **50** (1990), 231-240.
- [4] C. T. Hoàng, On a conjecture of Meyniel, *J. Combin. Theory Ser. B* **42** (1987), 302-312.
- [5] B. Lévêque and F. Maffray, Coloring Meyniel graphs in linear time, preprint, 2004.
- [6] H. Meyniel, The graphs whose odd cycles have at least two chords, *Topics on Perfect Graphs*, 115-119, North-Holland Math. Stud., 88, North-Holland, Amsterdam, 1984.
- [7] R. Roussel and I. Rusu, Holes and dominoes in Meyniel graphs, *Int. J. Found. Computer. Sci.* **10** (1999), 127-146.
- [8] F. Roussel and I. Rusu, An $O(n^2)$ algorithm to color Meyniel graphs, *Discrete Math.* **235** (2001), 107-123.

